Many-body scar state intrinsic to periodically driven system

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The violation of the Floquet version of the eigenstate thermalization hypothesis is rigorously discussed with realistic Hamiltonians. Our model is based on the PXP-type interactions without disorder. We rigorously prove the existence of many-body scar states in the Floquet eigenstates that appear only at specific periods of driving by showing the explicit expressions of the wave functions. This is equivalently the first rigorous proof of the breakdown of the Floquet eigenstate thermalization hypothesis. Through the exact expression of the scar states, the underlying physical mechanism is clarified. Using the underlying mechanism, various driven Hamiltonians with Floquet-scar states can be systematically engineered. We then discuss Floquet-scar states that can be checked through time evolution of observables in a chain of Rydberg atoms.

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I. INTRODUCTION

In the past few decades, significant progress has been made in the in-depth understanding of the thermalization phenomenon in isolated systems [1–8]. The eigenstate thermalization hypothesis (ETH) is one of the most important keywords in this subject for static systems, because if it holds, the thermodynamic property in the isolated systems is consistently explained [9–12]. The ETH states that any single eigenstate is *thermalized* in the sense that an expectation value of any local observable is equal to the value calculated by the canonical ensemble with the corresponding temperature. When a periodic driving is applied to the system, the system generally heats up. In this case, the standard ETH is replaced by another hypothesis known as the Floquet ETH, which states that any single Floquet eigenstate is thermalized with an infinite temperature [13–17]. Both the ETH and the Floquet ETH have been intensively studied, and the affirmative results of many numerical simulations corroborate their validity, as long as the system is nonintegrable [14,17–21]. However, exceptions exist for both the ETH and the Floquet ETH. The most common example is the many-body localization, which is a phenomenon driven by disorder [5,22,23]. In the system exhibiting such phenomena, all eigenstates inside the target energy shell are nonthermal states, which are protected by an extensive number of emergent local integrals of motion.

Recently, a new type of violation of the ETH has been found in the PXP model in the framework of the *many-body scar state* [24–31]. The PXP model is an effective model derived from the transverse Ising model that describes the experimental setup of a chain of Rydberg atoms [32]. The many-body scar states have been numerically proposed as nonthermal eigenstates which consistently explain the long-time oscillations observed in experiments [27,30,32]. In addition, Lin and Motrunich found the explicit expressions for several nonthermal eigenstates using the matrix product form [33]. The exact results done in their work and related works [30,34–38] ensure the existence of many-body scars in the thermodynamic limit and clarify the underlying physics on the wave functions. Crucial characteristics of the scar states include that (i) those are not induced by local integral of motion and (ii) the number of scar states is at most polynomial order with respect to the system size, and those are embedded in the overwhelming number of chaotic eigenstates. The first property is in stark contrast to the conventional theories on the breakdown of the ETH such as the many-body localized systems. The second property indicates that the scar states are exceptional eigenstates.

In this paper, we discuss the existence of similar *Floquet many-body scar states* in driven Hamiltonian systems. Here the Floquet many-body scar states imply nonthermal eigenstates of the Floquet operator. The Floquet eigenstates possess all information on the long-time behavior in the dynamics, and hence it is generically very hard to make a precise statement about them, especially in the thermodynamic limit. A numerical approach cannot give accurate results due to the limitation of the system size, and a simple analysis such as the Floquet-Magnus expansion is not available in the thermodynamic limit because it diverges [16,17]. Hence we definitely need rigorous analysis of the Floquet eigenstates, similar to the rigorous

work by Lin and Motrunich for understanding the scar states in the static Hamiltonian. We consider experimentally realistic systems with arbitrary size where the parameters can be tuned. We then analytically look for the Floquet version of many-body scar states which satisfy the two key properties seen in the static Hamiltonian's case, i.e., the properties (i) and (ii) above [39]. With this motivation, we provide a complete proof of the breakdown of the Floquet ETH by showing the existence of such nontrivial Floquet many-body scars. Our model uses the PXP-type interactions, where any local conserved quantities are absent [24,25]. We exactly derive the explicit expressions of the many-body Floquet scar eigenstates for specific driving periods. Numerical calculation shows that the other states satisfy the Floquet ETH. Through the derivation of the exact Floquet scar states, underlying mechanisms to have the Floquet scar states are clarified. In addition, we engineer various Hamiltonians with Floquet scar states. Since our model is based on the PXP-type of interactions [32,40–42], simple cases of our Floquet scar states should be experimentally feasible with a chain of Rydberg atoms.

II. FLOQUET-INTRINSIC MANY-BODY SCAR STATE

Let $H(t)$ be a time-dependent many-body Hamiltonian, which is periodic in time with the period T . The Floquet operator for a single period is given by

$$F = \mathcal{T} e^{-i \int_0^T dt H(t)}, \quad (1)$$

where $\mathcal{T}$ is a time-ordering operator. We set $\hbar$ to be unity. For simplicity, we consider the following time dependence in the Hamiltonian:

$$H(t) = \begin{cases} H_1 & 0 \leq t < T/2 \\ H_2 & T/2 \leq t < T \end{cases}. \quad (2)$$

Hereinafter, we assume that H_1 and H_2 do *not* commute with each other and that both of the Hamiltonians are nonintegrable. The Floquet operator is now simply written as $F = e^{-iH_2 T/2} e^{-iH_1 T/2}$. According to the Floquet ETH, the Floquet Hamiltonian H_F defined from the relation $e^{-iH_F T} = F$ is generally a random Hamiltonian, whose eigenstates are the states with an infinite temperature. Although explicitly describing the Floquet Hamiltonian is difficult, the Floquet Hamiltonian is generally thought to be far from an integrable Hamiltonian.

To make our objective more explicit, we classify the possible Floquet scar states into two cases. The first is a trivial case where we have the simultaneous eigenstates of H_1 and H_2 ; such states automatically become the eigenstates of the Floquet operator and can be demonstrated, e.g., with a frustration-free Hamiltonian. The second is a more nontrivial case, which is investigated in this study. In this class, the Floquet scar states are the eigenstates of F , but not of H_j ($j = 1, 2$). To distinguish this class of scars from the first class, we term the second one a *Floquet-intrinsic scar state*. We below focus on the latter scar state.

III. MODEL AND NUMERICAL DEMONSTRATION

We construct a model to investigate the Floquet-intrinsic scar state; we use a time-dependent version of the PXP model in this study. We set the following combination of

Hamiltonians:

$$\begin{aligned} H_1 &= \sum_{j=2}^{N/2} P_{j-1} X_j P_{j+1} + V_1, \\ H_2 &= \sum_{j=N/2+1}^{N-1} P_{j-1} X_j P_{j+1} + V_2, \end{aligned} \quad (3)$$

where N represents the size of the system, which is an even number. The operators X_j , Y_j , and Z_j are the x , y , and z components of the Pauli operators, respectively, at the site j . Let $|\uparrow\rangle_j$ and $|\downarrow\rangle_j$ be the eigenstates of Z_j with the eigenvalues $+1$ and -1 , respectively. The operator P_j is a projection operator onto a down spin state at the site j , i.e., $P_j := (1 - Z_j)/2$. We consider a constrained Hilbert space without any adjacent up states, e.g., $|\uparrow\rangle_j |\uparrow\rangle_{j+1}$. The Hamiltonians H_1 and H_2 represent the PXP model acting on the left and the right halves of the system, respectively. The terms V_1 and V_2 determine the boundary conditions. We use the open boundary conditions setting $V_1 = X_1 P_2$ and $V_2 = P_{N-1} X_N$.

We have two remarks on the present setup. First, we note that uncommutability between the two Hamiltonians arises only from the edge of each Hamiltonian. In this sense, this driving may be a small perturbation from the static PXP model. However, even this type of small perturbation causes a significant effect on the long-time scale physics leading to the Floquet ETH. We demonstrate this *generic aspect* in the Supplemental Materials using the simple system [45]. Second, the PXP model is originally derived as an effective model from the transverse Ising model that describes a chain of Rydberg atoms [32]. The PXP interaction is switched on and off by turning the laser on and off [32,42,46].

As an indicator of a nonthermal (or thermal) state, we use the entanglement entropy $S_{N/2} := -\text{Tr}_1 \rho_1 \log \rho_1$, where ρ_1 is the reduced density matrix obtained by taking the partial trace with respect to the sites $j \in [N/2 + 1, N]$. Thermal states have a large amount of entanglement entropy which increases in proportion to the system size N [6,47–50]. If the entanglement entropy is small and independent of N , the state is an nonthermal state.

Now let us discuss the entanglement entropy as a function of the period T . We numerically calculate the entanglement entropies for all Floquet eigenstates. We use the system sizes $N = 16$ and 20 . Figure 1 shows the T -dependence of the minimum value of the entanglement entropies among all the eigenstates; interestingly, a resonance-like phenomenon is observed. The inset shows a magnified plot around $T \sim 44.4$, where vanishing entanglement entropy is observed, indicating a scar state. An important question is whether this is the state intrinsic to the Floquet operator or a simultaneous eigenstate of the static Hamiltonian. To address this question, we consider the entanglement entropies for all Floquet eigenstates at a fixed period 44.4 and compare them with the entanglement entropies for the eigenstates of $H_1 + H_2$. If the Floquet scars are the simultaneous eigenstates for H_j ($j = 1, 2$), one will see the coincidence of entanglement entropies between them. In Fig. 2 we present them as a function of the expectation value of $H_1 + H_2$ for each eigenvalue. In the figure the orange dots and the blue points represent the results for Floquet eigenstates and eigenstates of $H_1 + H_2$, respectively.

FIG. 1. T -dependence of the *minimum entanglement entropies* of the Floquet eigenstates for $N = 16$. For each period T , we calculate the entanglement entropy $S_{N/2}$ for all the Floquet eigenstates which are computed through the exact diagonalization method. We plot the minimum value among them. For $T = 8.88m$ with $m = 1, 2, \dots$, we find zero entanglement.

Almost all Floquet eigenstates are thermal, as indicated by large values of entanglement entropies. The saturated values are close to the theoretical value estimated from the reduced density matrix with an infinite temperature, i.e., $S_{N/2} \sim 4.4$ [51]. Remarkably, exceptions for four states are seen, which have zero entanglement entropies (two states are degenerated at energy = 0, and only three points are visible in the figure).

FIG. 2. Entanglement entropies of the Floquet eigenstates at $T = 10\sqrt{2}\pi$ with $N = 20$ (orange dots). As a reference, we also show the entanglement entropies for the eigenstates of the static PXP Hamiltonian $H_1 + H_2$ (blue dots). The x and y axes are the energy expectation values of the PXP Hamiltonian and the entanglement entropies, respectively, where $|n\rangle$ is either a Floquet eigenstate of F or an eigenstate of $H_1 + H_2$. We observe the zero entanglement entropy for the four Floquet eigenstates at $x = \pm\sqrt{2}$ and 0 (there are two points at $x = 0$). The entanglement entropies of the other Floquet eigenstates are almost saturated, $S_{N/2} \simeq 4.4$ [51], indicating the Floquet ETH. For the energy eigenstates of the static PXP model, we see many low-entangled states, which are the many-body scar states found in Ref. [24].

By contrast, all eigenstates of $H_1 + H_2$ have finite entanglement entropies. Hence, the Floquet scars observed here are not the simultaneous eigenstates of the static Hamiltonian but the Floquet-intrinsic scar state.

IV. EXACT DESCRIPTION OF THE FLOQUET-INTRINSIC SCAR STATE

We make the above numerical indication *rigorous* in the following by providing an explicit description of the many-body scar state. We eventually show the following form $|\text{FS}_{\alpha,\beta}\rangle$ for the specific periods $T = t_m := 2\sqrt{2}\pi m$ with a positive integer m :

$$|\text{FS}_{\alpha,\beta}\rangle = |\Phi_{1,\alpha}\rangle \otimes |\downarrow\rangle_{N/2} |\downarrow\rangle_{N/2+1} \otimes |\Phi_{2,\beta}\rangle, \quad (4)$$

where we obtain four Floquet-intrinsic scar states for $\alpha, \beta = \pm$, and the detailed expressions of $|\Phi_{1,\alpha}\rangle$ and $|\Phi_{2,\alpha}\rangle$ are given below. Even at this level, we can list several physically crucial aspects. First, the period in the inset of Fig. 2 is consistent with the value $T = 10\sqrt{2}\pi$ (i.e., $m = 5$). Second, the entanglement entropy is exactly zero from the structure. Third, the above expressions are different from the Lin-Motrunich (LM) eigenstates for the static Hamiltonian $H_1 + H_2$ [33]. The LM eigenstate is given with the matrix product form, which is clearly different from the above form.

We now consider the detailed expression for (4). Although the LM eigenstates are not identical to the Floquet-intrinsic scar state, those are still beneficial in deriving the desired state. We first focus on the left part that consists of the sites from $j = 1$ to $j = N/2 + 1$. Since $[Z_{N/2+1}, H_1] = 0$, we fix the state at $j = N/2 + 1$ to the down state. Then H_1 becomes the PXP model of the system size $N/2 + 1$ with the open boundary condition. In this case, the LM eigenstates of H_1 are given by the following matrix product form:

$$|\Gamma_{\alpha,\beta}\rangle = \sum_{\{\sigma\}} v_{\alpha}^t B^{\sigma_1} C^{\sigma_2} \dots B^{\sigma_{N/2-1}} C^{\sigma_{N/2}} v_{\beta} \\ \times |\sigma_1 \dots \sigma_{N/2}\rangle \otimes |\downarrow\rangle_{N/2+1}, \quad (5)$$

where $v_{\pm} \equiv (1, \pm 1)^t$ and

$$B^{\downarrow} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, \quad B^{\uparrow} = \sqrt{2} \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 1 \end{pmatrix}, \\ C^{\downarrow} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \\ 0 & 0 \end{pmatrix}, \quad C^{\uparrow} = \sqrt{2} \begin{pmatrix} 1 & 0 \\ 0 & 0 \\ -1 & 0 \end{pmatrix}. \quad (6)$$

The eigenenergies are $E = 0$ for $|\Gamma_{\pm,\pm}\rangle$ and $E = \pm\sqrt{2}$ for $|\Gamma_{\pm,\mp}\rangle$. In addition, we make a new wave function, $|\Psi_{1,\alpha}\rangle = (1/\sqrt{2})(|\Gamma_{\alpha,+}\rangle - |\Gamma_{\alpha,-}\rangle)$ for $\alpha = \pm$. Through straightforward calculation, this state turns out to be identical to the following expression:

$$|\Psi_{1,\alpha}\rangle = |\Phi_{1,\alpha}\rangle \otimes |\downarrow\rangle_{N/2} |\downarrow\rangle_{N/2+1}, \\ |\Phi_{1,\alpha}\rangle = \sum_{\{\sigma\}} v_{\alpha}^t B^{\sigma_1} C^{\sigma_2} \dots B^{\sigma_{N/2-1}} w_L |\sigma_1 \dots \sigma_{N/2-1}\rangle, \quad (7)$$

with the new vector $w_L = (\sqrt{2}, 0, 0)^t$. The state (7) is a superposition of the two eigenstates with the energy $E = 0$ and $\sqrt{2}$ and is therefore not the eigenstate of the Hamiltonian

FIG. 3. Schematic of the protocol (9). The consecutive interactions are turned on and off. Blue areas stand for the periods of “turned on.” (a) The case of $n = 2$. The scar state (10) reduces to the all-down state. (b) The case of $n = 4$. The scar state has the structure of singlet states sandwiched by two down states.

H_1 . However, when we consider the unitary time evolution e^{-itH_1} starting from this state, the wave function returns to the initial state with the time $t = \sqrt{2}\pi$, i.e., the two states $|\Psi_{1,\alpha}\rangle$ ($\alpha = \pm$) are the eigenstates of $e^{-iH_1 t_m/2}$.

A similar analysis is performed for the unitary time evolution $e^{-iH_2 t_m/2}$ by considering the site $j = N/2$ to $j = N$. Fixing the state at the site $N/2$ to the down state, we obtain the following wave function:

$$\begin{aligned} |\Psi_{2,\beta}\rangle &= |\downarrow\rangle_{N/2} |\downarrow\rangle_{N/2+1} \otimes |\Phi_{2,\beta}\rangle, \\ |\Phi_{2,\beta}\rangle &= \sum_{\{\sigma\}} w_R^t C^{\sigma_{N/2+2}} B^{\sigma_{N/2+3}} \dots C^{\sigma_N} v_\beta \\ &\quad \times |\sigma_{N/2+2} \sigma_{N/2+3} \dots \sigma_N\rangle, \end{aligned} \quad (8)$$

where $\beta = +, -$, and the vector $w_R = (\sqrt{2}, 0, 0)^t$. The states (8) are the eigenstates for $e^{-iH_2 t_m/2}$. Remarkably, both $|\Phi_{1,\alpha}\rangle$ and $|\Phi_{2,\beta}\rangle$ contain the product states with the down states at the site $N/2$ and $N/2 + 1$, and thus, we can safely merge these states to obtain the desired expression (4). From these derivations, one can see that our Floquet-intrinsic scar states are not the eigenstates of the static Hamiltonian H_j ($j = 1, 2$), but they are the simultaneous eigenstates for unitary operators $e^{-iH_1 t_m/2}$ and $e^{-iH_2 t_m/2}$. One can readily find the scar eigenstates in the periodic boundary condition [53]. The rig-

orous proof of the Floquet-intrinsic scar state by showing the explicit Floquet eigenstate and the underlying physical mechanism are the main results in this paper.

V. FLOQUET-SCAR ENGINEERING

Having understood the underlying mechanism to have scar states, we now demonstrate that other systems which have scar states can be systematically engineered. We emphasize that various systems are systematically constructed. As the second main result in this paper, we present the example of the Floquet-scar engineering below.

We consider the system of size $N = 2n\ell$, where n and ℓ are integers. Then we make a unitary time evolution of each n sites by dividing the static Hamiltonian as follows:

$$H_1 = \sum_{k=0}^{\ell-1} \sum_{j=2nk+1}^{n(2k+1)} h_j, \quad H_2 = \sum_{k=0}^{\ell-1} \sum_{j=n(2k+1)+1}^{n(2k+2)} h_j, \quad (9)$$

where $h_j := P_{j-1} X_j P_{j+1}$ and we impose the periodic or open boundary condition. By following the same procedure as before, regardless of the boundary conditions one can find the exact scar state for the period $T = t_m$:

$$|\text{FS}\rangle = \bigotimes_{k=0}^{2\ell-1} |\downarrow\rangle_{nk+1} \otimes |\tilde{\Phi}_{nk+2, n(k+1)-1}\rangle \otimes |\downarrow\rangle_{n(k+1)}, \quad (10)$$

where $|\tilde{\Phi}_{i,j}\rangle$ is a pure state defined from the site i to j :

$$|\tilde{\Phi}_{i,j}\rangle = \sum_{\{\sigma\}} w_R^t C^{\sigma_i} B^{\sigma_{i+1}} \dots B^{\sigma_j} w_L |\sigma_i \dots \sigma_j\rangle. \quad (11)$$

In the state $|\text{FS}\rangle$, $|\tilde{\Phi}_{i,j}\rangle$ and $|\downarrow\rangle$ appear alternatively. We note that the spins at the edges $j = 1$ and N are both down states for the open boundary condition. The four scar states (4) do not have the down states at the edge. However, by superposing them we have the Floquet-intrinsic scar state, which has $|\downarrow\rangle_1$ and $|\downarrow\rangle_N$. In the same way we make the edge states in (10) the down states.

In Figs. 3(a) and 3(b) the two simplest cases of the protocol (9) and the Floquet-intrinsic scar state (10) are schematically illustrated. The upper figure is the case for $n = 2$, and the lower one is for $n = 4$. Interestingly, the state (10) becomes

FIG. 4. Time evolutions starting from the Floquet-intrinsic scar state and $\mathbb{Z}_2$ state under the protocol (9) for $n = 2$. The vertical axis is the magnetization at site 5, and the horizontal axis is time. The length of the chain is 20. Blue: The initial state is the all-down state, which is the Floquet-intrinsic scar state for $n = 2$. The spin perfectly returns to the down state $|\downarrow_5\rangle$ in every period. Orange: The initial state is the $\mathbb{Z}_2$ state $|\downarrow_1 \uparrow_2 \dots \downarrow_{19} \uparrow_{20}\rangle$. This is not the Floquet scar state, and thus, the amplitude of the oscillation decays in time.

simple: For $n = 2$, it is an all-down state, because the term $|\tilde{\Phi}_{nk+2,n(k+1)-1}\rangle$ vanishes, and for $n = 4$, $|\tilde{\Phi}_{nk+2,n(k+1)-1}\rangle$ is the singlet state. We note that for $n = 2$ the state (10) becomes the Floquet scar state for $T = \sqrt{2}\pi m$ with positive integer m . In Fig. 4 we show the numerical results of the time evolution for the case $n = 2$, which start from the Floquet-intrinsic scar state $|\downarrow\downarrow\cdots\rangle$ (blue) and from the $\mathbb{Z}_2$ state $|\uparrow\downarrow\uparrow\downarrow\cdots\rangle$ (orange). We impose the open boundary condition and take $T = \sqrt{2}\pi$. The Floquet-intrinsic scar state exhibits perfect returns to the initial state. By contrast, the $\mathbb{Z}_2$ state quickly relaxes to a stationary state, whose value of $\langle Z_5 \rangle$ is around $-1/\sqrt{5}$. This is the ensemble average value at infinite temperature; the value is nonzero due to the Rydberg blockade.

VI. SUMMARY AND PERSPECTIVE

In this paper, we discussed the Floquet many-body scar states in driven Hamiltonian systems. Our model consists of the PXP-type interactions without disorder. We exactly demonstrate that the Floquet-intrinsic scar states certainly exist by showing the explicit expressions of the eigenstates. The crucial mechanism of the Floquet-intrinsic scar states discovered here is that the states are simultaneous eigenstates

of different unitary operators, while they are not simultaneous eigenstates for the Hamiltonians. Another important feature is the absence of the conserved quantities in our system. Hence the mechanism presented in this paper should be different from those in previous works where the system has conserved quantities [21]. All the Hamiltonians in this study can be implemented in a chain of Rydberg dressed alkali-metal atoms in principle [32,42,46]. In particular, the protocols depicted in Fig. 3 are the most feasible for experimental realization, because the Floquet-intrinsic scar states reduce to simple states which can be readily prepared in experiments. It is an important future subject of interest to observe the Floquet-intrinsic scar state in experiments.

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- $$|\text{FS}\rangle = |\downarrow\rangle_N |\downarrow\rangle_1 \otimes |\tilde{\Phi}_{2,N/2-1}\rangle \otimes |\downarrow\rangle_{N/2} |\downarrow\rangle_{N/2+1} \otimes |\tilde{\Phi}_{N/2+2,N-1}\rangle.$$
- The entanglement entropy for the subsystem consisting of the sites $i = 1, \dots, N/2$ is zero from the structure.