Reply to “Comment on ‘Kosterlitz-Thouless-type caging-uncaging transition in a quasi-one-dimensional hard disk system’”

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There is indeed a finite-size issue in computer simulations of the correlation decay in a quasi-one-dimensional system of hard disks. The correlation function obtained from the exact canonical NVT partition function shows a decay which is much slower and more complex than the exponential one, presented in the Comment.

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We agree with the Comment [1] that a power-law correlation decay in a quasi-one-dimensional (q1D) hard disk (HD) system at a linear density $\rho = 1.111\,11$ [2] is peculiar for a system of 400 HDs as we found it to go over into a nonalgebraic decay in larger systems of 1000 and 2000 HDs. However, the correlation function obtained analytically from its general definition making use of the exact canonical NVT partition function (PF) derived in Ref. [3], shows that, for this and higher density $\rho = 1.140\,25$, the decay is much slower and more complex than the exponential decay presented in the Comment. The exponential correlation decay presented in the Comment is the corollary of the transfer matrix property ascribed to the isobaric NPT PF. However, it is shown in the Appendix to Ref. [3] that the correct NPT PF of a q1D HD system does not have the transfer matrix property. The

authors of the Comment admit that the result obtained in Ref. [3] is correct, but, at the same time, they state that a potential problem on the generic transfer matrix derivation under the NPT ensemble is negligible in the thermodynamic limit. We cannot agree with this message, claimed without proof or justification. Our theoretical results confirm that the canonical PF [3] differs from the NPT PF based on the transfer matrix property and the correlation behavior in a q1D HD system might be surprisingly complex. The details of our theoretical and simulation results will be presented elsewhere.

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[1] Y. Hu and P. Charboneau, *Phys. Rev. Res.* **3**, 038001 (2021).

[2] A. Huerta, T. M. Bryk, V. M. Pergamenschchik, and A. D. Trokhymchuk, *Phys. Rev. Res.* **2**, 033351 (2020).

[3] V. M. Pergamenschchik, *J. Chem. Phys.* **153**, 144111 (2020).