Nonlinearity and wide-band parametric amplification in a (Nb,Ti)N microstrip transmission line

S. Shu^{1,*}, N. Klimovich¹, B. H. Eom², A. D. Beyer², R. Basu Thakur¹, H. G. Leduc², and P. K. Day^{2,†}¹The Division of Physics, Mathematics and Astronomy, California Institute of Technology, Pasadena, California 91125, USA²Jet Propulsion Laboratory, California Institute of Technology, Pasadena, California 91109, USA

(Received 15 February 2021; revised 14 May 2021; accepted 19 May 2021; published 4 June 2021)

The nonlinear response associated with the current dependence of the superconducting kinetic inductance was studied in capacitively shunted NbTiN microstrip transmission lines. It was found that the inductance per unit length of one microstrip line could be changed by up to 20% by applying a dc current, corresponding to a single-pass time delay of 0.7 ns. To investigate nonlinear dissipation, Bragg reflectors were placed on either end of a section of this type of transmission line, creating resonances over a range of frequencies. From the change in the resonance linewidth and amplitude with dc current, the ratio of the reactive to the dissipative response of the line was found to be 788. The low dissipation makes these transmission lines suitable for a number of applications that are microwave- and millimeter-wave band analogs of nonlinear optical processes. As an example, by applying a millimeter-wave pump tone, very-wide-band parametric amplification was observed between about 3 and 34 GHz. Use as a current variable delay line for an on-chip millimeter-wave Fourier transform spectrometer is also considered.

DOI: [10.1103/PhysRevResearch.3.023184](https://doi.org/10.1103/PhysRevResearch.3.023184)

I. INTRODUCTION

The nonlinearity of superconducting kinetic inductance has been explored for a number of device applications including kinetic inductance traveling-wave parametric amplifiers (KI-TWPAs) [1], current sensors [2], magnetometers [3], variable inductors [4], delay lines [5], and even qubits [6]. These nonlinear kinetic inductance devices differ in a number of ways from corresponding devices based on the nonlinear inductance of Josephson junctions. For example, the scale of the nonlinearity for a kinetic inductor is set by the critical current I_c of a wire and is generally larger than I_c for a junction. For some types of devices, such as TWPAs, this larger current scale results in a greater dynamic range, making KI-TWPAs more suitable for high signal levels than the Josephson version of the device. Nonlinear kinetic inductors made from large-gap materials can potentially operate at higher frequency than Josephson devices based on aluminum junctions. The maximum operating frequency is set by the gap frequency of the superconducting material, which is as high as 1.4 THz for NbTiN, compared with 90 GHz for aluminum.

A KI-TWPA was first demonstrated using a NbTiN coplanar waveguide (CPW) [1], and in recent years several CPW design variations have been investigated [7–9]. CPWs have the advantage of a simple, single-layer fabrication process,

but the necessary surrounding ground plane and the unwanted slot-line mode created by a curved CPW make compact geometries difficult to realize, which impacts the achievable gain. CPW is also less suitable for millimeter-wave applications due to radiation at bends. In contrast, microstrip lines can be tightly meandered across a chip with little effect on performance, and superconducting microstrip lines with thin, deposited dielectrics have been used at millimeter and submillimeter wavelengths. On the other hand, the use of a deposited dielectric film potentially introduces two-level system loss [10,11].

In this paper, we present results on NbTiN microstrip transmission lines that use a hydrogenated amorphous silicon (aSi) dielectric, which has shown loss tangents on the order of 10^{-5} [12–14]. The high dielectric constant of silicon and the use of a series of capacitive stub sections allows the characteristic impedance to be adjusted to 50 Ω , while the propagation velocity is reduced to less than 0.01c, allowing very compact devices. We characterize the loss and nonlinearity in sections of this transmission line and demonstrate its use as the basis for a variable delay line for an on-chip Fourier transform spectrometer and as a very-wide-band parametric amplifier.

II. NONLINEAR KINETIC INDUCTANCE

A supercurrent modifies the density of states of a superconductor and lowers the energy gap, resulting in an increase in the kinetic inductance [15]. Due to this current dependence, the inductance per unit length of a superconducting transmission line can be expanded as

$$\mathcal{L}(I) = \mathcal{L}_0[1 + (I/I_*)^2 + (I/I'_*)^4 + \dots], \quad (1)$$

where I_* and I'_* depend on the superconducting material and geometry. As the change in the phase velocity, $v_{ph} \sim 1/\sqrt{\mathcal{L}C}$,

*shiboshu@caltech.edu

†peter.k.day@jpl.nasa.gov

FIG. 1. The dispersion curve around the band-gap frequency of the stub-loaded NbTiN microstrip line with modulation length $p = 38 \mu\text{m}$, mean stub length $l = 3 \mu\text{m}$, and stub length modulation $m = 0.2 \mu\text{m}$. The measurement (blue) was taken with a dc bias current of 0.57 mA and is compared with the result of a circuit model (black), where the penetration depth has been adjusted to match the 38-GHz band-gap feature created by the periodic modulation. The upper inset shows the layers that form the transmission line. The lower inset shows the arrangement of the microstrip stubs. The width of all of the lines is $0.25 \mu\text{m}$.

where $\mathcal{C}$ is the capacitance per unit length, varies to first order as I^2 , this nonlinear behavior is equivalent to the Kerr effect in optical materials, in which the refractive index is intensity dependent. When substituted into the wave equation

$$\frac{\partial^2 I}{\partial z^2} - \frac{\partial}{\partial t} \left[\mathcal{L}(I) \mathcal{C} \frac{\partial I}{\partial t} \right] = 0, \quad (2)$$

the I^2 term in the expansion mixes tones at four different frequencies, which is known as four-wave mixing. Three-wave mixing (3WM) processes may also be supported by the transmission line by applying a dc current in addition to the ac tones. In that case, $\mathcal{L}(I)$ develops a term proportional to I . Either three- or four-wave mixing can produce parametric amplification. In the case of three-wave mixing, a signal photon with frequency ω_s stimulates the conversion of a single pump photon into signal and idler photons, and the frequencies $\omega_{p,s,i}$, where the subscripts refer to pump, signal, and idler tones, are related by $\omega_p = \omega_s + \omega_i$.

III. NbTiN MICROSTRIP LINE

The geometry of the microstrip transmission lines is described in the insets of Fig. 1. Both versions of the transmission lines that were studied use a 35-nm NbTiN conductor layer that is deposited first on a high-resistivity silicon substrate and patterned to form the microstrip conductor. An amorphous silicon layer is deposited next and serves as the microstrip dielectric with dielectric constant of 10.3. A 200-nm NbTiN layer deposited last forms the ground plane of the inverted microstrip structure. To maximize the nonlinear response (minimize I_*), a relatively narrow conductor

linewidth of 250 nm is used. In order to maintain a characteristic impedance of 50Ω for convenient matching to external circuitry, a series of microstrip open stubs is connected to the transmission line. At the operating frequency, the stubs are much shorter than a quarter wavelength and approximate shunt capacitances. As a result, the final microstrip line has both a large capacitance and a large inductance per unit length, which reduces the phase velocity so that $v_{ph} \sim 1/\sqrt{\mathcal{L}\mathcal{C}} < 0.01c$. The slow-wave nature of the transmission line effectively increases the electrical length and results in a compact device. For example, this design can shorten the device length from 2 m in Ref. [7] to 0.1 m by a factor of 20.

An estimate of the nonlinear current scale I_* can be made by equating the kinetic energy $\mathcal{L}_{kin} I_*^2/2$, where $\mathcal{L}_{kin}$ is the kinetic inductance per unit length of the microstrip conductor, with the condensation energy $N_0 \Delta^2 w t/2$, where N_0 is the density of states at the Fermi level and w and t are the width and thickness of the conductor. For $t \ll \lambda_L$, where λ_L is the London penetration depth, and $w \ll \lambda^2/t$, the current is nearly uniform throughout the conductor cross section, and $\mathcal{L}_{kin} = \mu_0 \lambda_L^2 / w t$. The nonlinear current scale can then be expressed as

$$I_* = w t \kappa_* \sqrt{\frac{N_0 \Delta^2}{\mu_0 \lambda_L^2}}, \quad (3)$$

where κ_* is of order 1. A more rigorous analysis based on the Usadel theory [15,16] suggests $\kappa_* = 1.37$.

I_* is reduced for materials with high normal-state resistivity, which implies large λ_L . The NbTiN films used for this study were produced by reactive sputtering and have resistivity in the range of $200 \mu\Omega \text{ cm}$. For a film thickness of 35 nm, we obtain a surface inductance $L_s \approx 7 \text{ pH}$. The relatively high critical temperature, $T_c \sim 12.5 \text{ K}$ for a 35-nm film, allows for convenient testing with minimal loss at 1 K.

In a traveling-wave structure, the efficiency of a nonlinear process, such as parametric amplification, is controlled by the degree to which the waves involved maintain a specific phase relation. The frequency range over which phase matching can be achieved determines the bandwidth of the process. The dispersion of the transmission medium is a main factor in phase matching. Superconducting transmission lines operated well below the gap frequency have very little intrinsic material dispersion, which can result in several nonlinear processes being simultaneously phase matched. For a particular application, the phases of waves corresponding to unwanted nonlinear processes, e.g., harmonic generation in the case of parametric amplification, should ideally be mismatched so that those processes are suppressed. Phase matching can be controlled by adding geometrical dispersion to the transmission line structure, a process referred to as “dispersion engineering”. The open stubs of the transmission line structure shown in Fig. 1 have a dispersive effect because they add a frequency-dependent admittance per unit length proportional to $\tan(\omega l / v_{ph})$, where l is the finger length and v_{ph} is the propagation velocity on the microstrip line forming the stub. The dispersion of the transmission line may be further modified by adding a periodic modulation, as was first implemented for a TWPA in Ref. [1]. A similar approach has been applied in Josephson-junction-array-based TWPAs

FIG. 2. (a) The schematic structure of the on-chip interferometer device. (b) The calculated S_{21} of the device without loss. (c) The measured S_{21} at 1 K after baseline correction. The insets show the Lorentzian fit of the resonators.

[17]. As shown in the lower inset of Fig. 1, the length of the stubs is sinusoidally varied with period p producing a band gap at frequency $v/2p$, where v is the propagation velocity on the stub-loaded microstrip structure with average stub length l . The dispersion is modified near the band-gap frequency. The amplitude of the sine-wave modulation sets the frequency width of the band gap and also the degree to which the dispersion deviates near the band-gap edges.

IV. LOSS MEASUREMENT USING AN ON-CHIP INTERFEROMETER

A. On-chip interferometer design

To measure the loss in the transmission line, we designed an on-chip Fabry-Pérot interferometer at 8.3 GHz. The device consists of a stub-loaded transmission line of length $L = 93$ mm and two Bragg reflectors, located at the two ends

of the transmission line, as shown in Fig. 2. The stub length in the reflector sections has a sine-wave variation starting from $l = 26$ μm with an amplitude of $m = 12$ μm and a period of $p = 140$ μm , and the transmission line between the reflectors has a uniform stub length of $l = 26$ μm . The transmission line is arranged in a meandering pattern across a 2.5×26 -mm chip. The spacing between the stubs is 2 μm for the whole device, and the aSi dielectric thickness is 190 nm. The reflector has a length of eight periods of the stub length modulation, which produces an incomplete stop band around 8.3 GHz. To decrease the impedance mismatch between the reflectors and the transmission line near the stop band, there are sections of two periods of length over which the modulation amplitude is tapered in and out at the input and output of each reflector. The device acts as an etalon for frequencies within the stop band, resulting in transmission peaks at frequencies $\omega_n = n\pi v/L$. From the measured frequency spacing (Fig. 2), we find that $v = 0.0077c$ for this device for $I_{dc} = 0$. The transmission of

the etalon may be expressed as

$$S21 = \frac{t^2 e^{-\gamma L}}{1 - r^2 e^{-2\gamma L}}, \quad (4)$$

where t and r are the frequency-dependent Bragg reflector transmission and reflection amplitudes, $\gamma = \alpha + i\beta$ is the propagation constant on the internal transmission line section, and $\beta = \omega/v$. Near the resonance frequencies, the transmission peaks are nearly Lorentzian and

$$S21(\omega_n + \Delta\omega) \approx \frac{Q_r}{Q_c} \frac{t^2}{1 - r^2} \frac{e^{-\gamma L}}{1 - 2iQ_r \Delta\omega/\omega_n}, \quad (5)$$

where Q_r is the full width at half maximum of the resonance and $Q_c = (\omega_n L/v)r^2/(1 - r^2)$ is a measure of losses to the external circuit. Internal losses are represented by $Q_i = \beta/2\alpha$, and the quality factors are related by

$$Q_r^{-1} = Q_c^{-1} + Q_i^{-1}. \quad (6)$$

Neglecting loss in the Bragg reflectors ($r^2 + t^2 = 1$) and a phase factor related to the phases of r and t , and normalizing the transmission to that of a single pass through the internal transmission line section, the transmission becomes

$$S21 \approx \frac{Q_r}{Q_c} \frac{1}{1 - 2iQ_r \Delta\omega/\omega_n}. \quad (7)$$

B. Loss measurement

The microwave transmission of the device was measured at 1 K with a series of dc bias currents. The calculated and measured $S21$ with $I_{dc} = 0$ are shown in Fig. 2. The calculation was made by cascading the ABCD matrices of the microstrip sections making up the device. The measured $S21$ is normalized by subtracting a linear-in-decibels baseline found by fitting $S21$ at frequencies above and below the stop band.

Resonances were fit to a Lorentzian function to extract the quality factors Q_r in both calculation and measurement. Losses were not included in the calculation, so in that case $Q_r = Q_c$. To find the experimental Q_i from the measured Q_r , two methods are used. In the “circuit model method,” the Q_c derived from the $S21$ calculation is used along with Eq. (6). The second method uses the resonance height in the measured, baseline-normalized $S21$ to calculate Q_i . According to Eqs. (6) and (7),

$$Q_i = \frac{Q_r}{1 - |S21_{\max}|}, \quad (8)$$

where $|S21_{\max}|$ is the maximum $|S21|$ of each resonance. Because the Q_c are higher in the center of the stop band, the central resonances are more sensitive to Q_i . The attenuation factors and the total one-pass loss of the intermediate transmission line section are plotted in Fig. 3 using the data from resonances near the stop band center.

The attenuation calculated using the circuit model method has a large variation and decreases as frequency increases, suggesting some discrepancy between the actual and designed Q_c values. The loss found with the baseline method has less variation and increases slightly with frequency. An increase in loss with frequency is expected from the increase in the

FIG. 3. (a) The frequency-dependent attenuation factor without dc bias. (b) The bias-current-dependent attenuation factor. The device loss is calculated assuming a one-way parametric amplifier using this transmission line with the same length, 93 mm. (c) The frequency shift of a single resonance by increasing the dc current. The fitted scale of the nonlinearity is $I_* = 3$ mA.

phase length. Around 8.4 GHz the two methods give consistent results, $\alpha = 3.7$ dB/m with $I_{dc} = 0$. The one-pass loss of this 93-mm transmission line, which corresponds to 333 wavelengths, is 0.35 dB and $Q_i = 2.8 \times 10^4$. The loss is higher than was found in resonator measurements using the same dielectric material where $Q_i > 10^5$ is observed and resonance frequency shift versus temperature measurements imply a single photon loss factor of 10^{-5} [14]. It is possible

that the extra loss comes from fabrication defects along the length of the device, which is much longer than the resonators that were studied.

The transmission was also measured as a function of dc current injected through the length of the transmission line up to 0.8 mA. Past that current, the line became resistive. The same two methods are used for calculating the attenuation factors, shown in Fig. 3. The stop band and resonance frequencies shift to lower frequency when the current is increased, so the resonance closest to 8.4 GHz at each measurement current was used to avoid mixing in the frequency dependence of the loss. Both methods give consistent results at $I_{dc} < 0.6$ mA. When $I_{dc} > 0.6$ mA, the circuit model method gives lower values, most likely because the 8.4-GHz resonance moves to the edge of the stop band, where that method produced less stable results.

The Q_i decreases by a factor of 2.3 from 2.8×10^4 at zero dc current to 1.2×10^4 with $I_{dc} = 0.8$ mA, while the device loss increases by a factor of 3.4 from 0.35 to 0.84 dB. The device loss increase includes a contribution from the increase in the electrical length of the transmission line. We define the ratio of the reactive to the dissipative response, $R = \Delta\beta/2\Delta\alpha$. Therefore we have

$$\frac{1}{Q_i} = \frac{2\alpha}{\beta} = \frac{1}{Q_i(0)} + \frac{1}{R} \frac{\Delta\beta}{\beta}, \quad (9)$$

where $\beta = \omega/v$ and $v = 0.0077c[1 - 0.5(I/I_*)^2 - 0.5(I/I'_*)^4]$. The values of I_* and I'_* are obtained from fitting $\Delta f/f$ (Fig. 3). Fitting the resonance height method loss data to Eq. (9) yields $R = 788$.

The origin of the decrease in Q_i with dc current is not clear. Using the Usadel equations and Nam's equations [16,18–20], we calculated the conductivity quality factor $Q_i = \sigma_2/\sigma_1 > 10^8$ at $I_{dc} = 0.8$ mA, so the measured Q_i is not limited by the current-induced change in the density of states. The Q_i is also relatively insensitive to temperature at 1 K, so microwave heating of the transmission line should have negligible effect. It is possible that the loss happens locally due to the fabrication nonuniformity. In a separate measurement, the degradation of Q_i is also negligible under parallel magnetic field up to 40 mT by putting the device inside a solenoid. It has been shown that the Q_i of a 250-nm-width NbTiN resonator decreases from 10^5 to 10^4 with perpendicular magnetic field from 0 to 150 mT [21]. In our measurement, no external magnetic field was applied, and the magnetic field generated by the dc current is 0.84 mT, suggesting that loss in the microstrip conductor strip is negligible. However, the NbTiN ground plane covers the whole chip, and the electromagnetic field extends to a wider area than the strip width. Degradation of Q_i from 10^5 to 10^3 with a perpendicular magnetic field up to 3 mT has been reported [22], so it is also possible that our Q_i is limited by itinerant vortices and the increased current creates more vortices in the ground plane. As the magnetic field is quite small, the related $\Delta f/f$ is at the level of 10^{-3} , which is small compared with the current-induced one.

As a feature of phase-sensitive amplifiers, parametric amplifiers are able to squeeze the quantum fluctuation [23]. Therefore we further investigate how the asymmetric loss

FIG. 4. The upper panel shows the layout of the multiband KI-TWPA, which is arranged in a meandering path with a total length of 21 mm. The zoom-in shows the modulation of capacitor fingers, which introduce the frequency band gap. The chip is 0.6×4.6 mm and sits on a circuit board in a holder. A lid (not shown) forms a channel around the chip with a cutoff frequency above the operating band.

impacts the squeezing performance of a KI-TWPA in the 3WM regime. Using the measured $Q_i = 2.8 \times 10^4$ and assuming that Q_i is constant at all frequencies, a small amount of loss asymmetry still exists in nondegenerate amplification, due to the different electrical lengths for signal and idler frequencies. Assuming signal at 6 GHz and idler at 8 GHz, the loss gives a maximum squeezing level of 25 dB. The loss asymmetry sets the optimal length of the amplifier to be ~ 50 mm. The calculation details are discussed in Appendix A.

V. A MULTIBAND PARAMETRIC AMPLIFIER

A. Device design

A second stub-loaded microstrip line that was studied was designed for use as a wide-band KI-TWPA. The device has a layer setup similar to that of the microstrip line in Sec. III, except that the amorphous silicon dielectric layer thickness is decreased to 60 nm and the substrate thickness is decreased to 100 μm. The NbTiN thicknesses were nominally the same as those of the earlier device; however, a different sputtering target was used, and the deposition rate was different. The stubs have an average length of $l = 3$ μm, a modulation amplitude of $m = 0.2$ μm, and a modulation period of $p = 38.25$ μm, producing a band gap at 38.5 GHz (as shown in Fig. 6). The transmission line is arranged in a meandering pattern with a total length of $L = 21$ mm across a 4.6×0.6 -mm chip, shown in Fig. 4.

With this compact design, this device can be further integrated with other on-chip circuits to make a multipixel design for astronomical observations.

FIG. 5. Gain of the parametric amplifier gain at 1 K with a -29.3 -dBm pump tone at 38.8 GHz and $I_{dc} = 0.75$ mA. The gray line is the measured result and shows a large ripple. The blue curve is the data averaged in decibels over frequency. The orange curve is the result of the coupled-mode calculation. The inset shows ripple over a narrow frequency range.

B. Gain measurements

The gain measurements were made at 1 K. A dc current was injected through the transmission line along with a microwave pump tone to permit parametric amplification via 3WM. This process results in the formation of an idler tone, with pump, signal, and idler frequencies related by $\omega_i = \omega_p - \omega_s$. The gain of the amplifier is maximized when the dispersion of the transmission line is tuned to provide a phase mismatch, characterized by $\Delta k = \beta_p - \beta_s - \beta_i$, with wave vectors $\beta_n = \beta(\omega_n)$, between the tones that compensates the nonlinear phase difference due to self-modulation and cross-phase modulation [9]:

$$\Delta k = -\frac{I_p^2}{8I_\dagger^2}(\beta_p - 2\beta_s - 2\beta_i), \quad (10)$$

where $I_\dagger^2 = I_*^2 + I_{dc}^2$.

The gain of the amplifier with a dc bias and pump tone applied is shown in Fig. 5. Here, the gain is defined as the ratio of the output power with the pump on to that with the pump off. Based on the results of Sec. IV, a small frequency-dependent loss of approximately 0.01 dB/GHz is expected through the device, which is not included in the reported gain. The device produces gain over a wide bandwidth from 3 to 34 GHz with a pump frequency of 38.8 GHz, which lies just above the band gap at the bias current $I_{dc} = 0.75$ mA. A calculated gain curve derived from integrating coupled-mode equations is shown in Fig. 5 for comparison with the measured result. We found that including nonlinear processes in the calculation in addition to the 3WM gain process improved the match to the experimental result. The other processes included were second- and third-harmonic generation and four-wave mixing (4WM) parametric amplification. These additional processes can be expected to occur because the tones at the frequencies involved are not strongly mismatched. The coupled-mode equations used for the calculated gain profile are explained in

FIG. 6. S_{21} of the multiband KI-TWPA measured at 1 K at different dc current biases. The transmission is normalized by subtracting a linear baseline determined by fitting the data outside the band gap. VNA freq., vector network analyzer frequency.

Appendix B. This method uses the dispersion curve for the stub-shunted, modulated transmission line, calculated using the circuit model of the device, as an input. The difference between the calculation and measurement suggests that some parameters may deviate from our model. The 1-dB compression occurs above -57 dBm with 18-dB gain, measured using a similar device.

VI. MILLIMETER-WAVE CURRENT VARIABLE DELAY LINE

A. Current-dependent transmission

In this section, we characterize the same device without the pump signal for use as a millimeter-wave current variable phase delay element. With only a dc current applied to the transmission line there is no amplification, but the propagation velocity changes as a result of the current-induced change in kinetic inductance. We measure the propagation velocity and inductance change by tracking the frequency of the band gap that is created by the $38\text{-}\mu\text{m}$ stub length modulation. The millimeter-wave transmission through the device near the band-gap frequency is shown in Fig. 6 for a series of dc currents.

The band-gap frequencies ν_{gap} were determined by fitting to an asymmetric boxcar function. The change in kinetic inductance was extracted using

$$\left(\frac{\nu_{\text{gap}}(I = 0)}{\nu_{\text{gap}}(I > 0)} \right)^2 = \frac{\mathcal{L}(I_{dc})}{\mathcal{L}_0}. \quad (11)$$

By comparing the measured band-gap frequency at $I_{dc} = 0$ with the circuit model of the transmission line structure, the penetration depth of the NbTiN conductor layer was determined to be 380 nm, and the ratio of the kinetic inductance to the total inductance of the transmission line was estimated to be approximately 0.99 . Neglecting the magnetic inductance, a fit to the data yields

$$\frac{\mathcal{L}(I_{dc})}{\mathcal{L}_0} = 1 + \left(\frac{I_{dc}}{4.3 \text{ mA}} \right)^2 + \left(\frac{I_{dc}}{2.8 \text{ mA}} \right)^4. \quad (12)$$

The largest current that could be applied before the transmission line switched to the normal state was 1.5 mA. At

FIG. 7. The shift in band gap is converted to the change in inductance using Eq. (11). The fitting result is shown in Eq. (12); this is the left y axis. The right y axis is the equivalent current-controlled delay $\Delta\tau$ in transmission. The inset shows S21 with respect to I_{dc} and frequency ν , derived from Fig. 6.

that current, the fractional inductance change $\delta\mathcal{L}/\mathcal{L}_0$ is 20%, which is close to the maximum theoretical value.

B. Phase delay and on-chip interferometer

From the designed modulation length and the band-gap frequency measured at $I_{dc} = 0$, the zero-current propagation velocity is found to be $v_0 = 0.0098c$. The change in the time delay $\Delta\tau = L(\sqrt{\mathcal{L}(I_{dc})\mathcal{C}} - \sqrt{\mathcal{L}_0\mathcal{C}})$ through the transmission line reached a maximum of ~ 0.7 ns at the highest current (Fig. 7). A possible use of this device would be as a controllable delay line in a compact superconducting on-chip Fourier transform spectrometer (SOFTS), operating in the millimeter band [5]. The principle is that a signal is split and fed into two identical current-biased transmission lines. The recombined outputs are terminated on a power detector. When the current is varied in either line, a relative phase delay is introduced, and the power detector produces an interferogram as a function of I_{dc} . Microwave circuit design allows for construction of both Mach-Zehnder and Michelson interferometers on chip for (sub)millimeter science and other applications. The frequency resolution will be $1/\Delta\tau_{\max} \sim 1.4$ or 0.7 GHz for single- and two-pass circuit designs. Standard optomechanical Fourier transform spectrometers (FTSs) with gigahertz resolution are meter scale, whereas using this concept allows for a millimeter-scale SOFTS [24]. The physical path length and resolution could be increased while still keeping the area small compared with a pixel in a millimeter-wave imaging array. Locating an array of SOFTSs within the individual feeds of the focal plane, an integral field unit could be realized.

VII. CONCLUSION

We have explored the nonlinear kinetic inductance and dissipation of NbTiN microstrip transmission line structures that use a series of microstrip stubs to achieve a $50\text{-}\Omega$ characteristic impedance. Dispersion engineering is realized by changing the length of stubs periodically. At 8.4 GHz, the device loss of 0.35 dB has been measured through a 93-mm length of the

microstrip line, corresponding to 333 wavelengths, using an on-chip Fabry-Pérot interferometer. The low dissipation and large nonlinear current response of these transmission lines make them suitable for realizing traveling-wave devices based on nonlinear conversion processes, such as parametric amplifiers. The low intrinsic dispersion of the superconducting transmission line and the ability to tune dispersion by varying the geometry allow for wide-band operation of such devices. As an example, we demonstrate a $7\text{--}26\text{-GHz}$ broadband parametric amplifier with gain larger than 15 dB using three-wave mixing. The magnitude of the current response also makes these transmission lines suitable as current variable delay elements that operate through the millimeter-wave band.

ACKNOWLEDGMENTS

The authors thank Songyuan Zhao and Eduard Driessen for help in calculations of the Usadel equation. The research was carried out at the Jet Propulsion Laboratory, California Institute of Technology, under a contract with the National Aeronautics and Space Administration (80NM0018D0004).

APPENDIX A: SQUEEZING OF QUANTUM FLUCTUATIONS INCLUDING LOSS

A TWPA with a small amount of loss may function as a quantum limited amplifier as long as the loss can be overcome by the gain of the device and the dissipation of the pump tone does not cause excessive heating. The effect of loss may have more impact on the ability of the TWPA to generate two-mode squeezing. Here, we use the results of Ref. [25] to estimate the amount of squeezing that might be possible with a microstrip TWPA with the loss that was measured in Sec. IV. We assume that the loss is distributed along the length of the TWPA and that the 3WM gain process is phase matched, so Eq. (10) is satisfied. To include the asymmetric loss, we define $\alpha_S = \bar{\alpha} - \epsilon$ and $\alpha_I = \bar{\alpha} + \epsilon$, where $\bar{\alpha}$ is the average attenuation factor and ϵ is the asymmetry. In the low-asymmetry limit, the variance of the squeezed quadrature is given by [25]

$$S \approx \frac{1}{2(\bar{\alpha} + 2)} \left(\bar{\alpha} + \frac{e^{-\bar{\alpha}L}}{2G_{\text{eff}}} \right) + G_{\text{eff}} \frac{\epsilon^2 e^{-\bar{\alpha}L}}{4(2 - \bar{\alpha})}, \quad (\text{A1})$$

where

$$G_{\text{eff}} = \frac{1}{4} e^{2L\Delta\phi} \sqrt{1 + (\epsilon/2)^2} \quad (\text{A2})$$

and $\Delta\phi = k_p I_p I_{dc} / 4I_c^2$. S decreases from the zero-point value of 0.5 as the gain increases. For symmetric loss ($\epsilon = 0$) the second term in Eq. (A1) is zero, and the variance saturates at $S = \bar{\alpha}/2(\bar{\alpha} + 2)$ in the limit of high gain.

In the case that Q_i is the same for both signal and idler, the loss asymmetry is simply due to the electrical length difference. Using the measured $Q_i = 2.8 \times 10^4$, we have $\bar{\alpha} = 0.34/m$, and $\epsilon = 0.05$ for a signal at 6 GHz and idler at 8 GHz. We take $I_{dc}/I_* = 0.16$ and $I_p/I_* = 0.037$, similar to the values used in the multiband gain calculation in Sec. VB. With perfect phase matching, we have $\Delta\phi = 66.85/m$. The variance S is calculated in three cases (Fig. 8). Without loss, the squeezing is proportional to the gain. For symmetric loss, the squeezing saturates with length, as the rate of noise generation

FIG. 8. The variance of the squeezed quadrature in three cases: lossless, symmetric loss, and asymmetric loss. The dashed line shows $S = 0.5$ without squeezing.

balances the squeezing rate. With asymmetric loss, mixing of noise from the amplified quadrature into the squeezed quadrature results in a maximum of the squeezing level at an optimal length [25]. For the parameters chosen above, the maximum of squeezing is 25 dB at

$$L_{\text{opt}} \approx \frac{1}{2\Delta\phi} \ln \frac{\Delta\phi}{2\epsilon} = 54 \text{ mm}. \quad (\text{A3})$$

APPENDIX B: GENERALIZED COUPLED-MODE EQUATIONS FOR 3WM AND 4WM PROCESSES

As in the standard treatment of waves interacting in non-linear optical media, we express the total current in terms of a number of frequency components,

$$I = \frac{1}{2} \left(\sum_{n=1}^m A_n(z) e^{i(\beta_n z - \omega_n t)} + \text{c.c.} \right), \quad (\text{B1})$$

where the slowly varying complex mode amplitudes A_n satisfy

$$\left| \frac{d^2 A_n}{dz^2} \right| \ll \left| \beta_n \frac{dA_n}{dz} \right|. \quad (\text{B2})$$

The number of frequency components included in Eq. (B1) is chosen to describe the particular process or processes under

investigation. For example, a description of parametric amplification in a 3WM medium includes at least three frequencies in the sum: the pump, the signal, and the idler at ω_p , ω_s , and ω_i . Additional frequencies should be included if they correspond to energetically allowed processes for which one of the input tones has significant power and they are not strongly phase mismatched. In the case of 3WM parametric amplification, second-harmonic generation may play a significant role, requiring the inclusion of $2\omega_p$. The evolution of the mode amplitudes A_n inside the transmission line is then found by substituting Eq. (B1) into Eq. (2) and matching terms with the same time dependence, resulting in coupled-mode equations for the evolution of the A_n . For an arbitrary set of 3WM and 4WM processes (procs.) connecting m frequency components, these equations can be written compactly as

$$\frac{dA_n}{dz} = \text{3WM procs.} + \text{4WM procs.}, \quad (\text{B3})$$

where

$$\begin{aligned} \text{3WM procs.} = & \frac{i\omega_n I_{\text{dc}}}{8I_{\text{c}}^2 \bar{c}} \sum_{\substack{0 \leq q_1 \dots m, p_1 \dots m \leq 2 \\ \sum_j p_j + \sum_j q_j = 2}} \frac{2! e^{i\beta_n z}}{p_1! \dots p_m! q_1! \dots q_m!} \\ & \times \delta \left(\sum_j p_j \omega_j - \sum_j q_j \omega_j - \omega_n \right) \\ & \times \prod_{1 \leq j \leq N} (A_j e^{-i\beta_j z})^{p_j} \prod_{1 \leq j \leq N} (A_j^* e^{i\beta_j z})^{q_j} \quad (\text{B4}) \end{aligned}$$

and

$$\begin{aligned} \text{4WM procs.} = & \frac{i\omega_n}{24I_{\text{c}}^2 \bar{c}} \sum_{\substack{0 \leq q_1 \dots m, p_1 \dots m \leq 3 \\ \sum_j p_j + \sum_j q_j = 3}} \frac{3! e^{i\beta_n z}}{p_1! \dots p_m! q_1! \dots q_m!} \\ & \times \delta \left(\sum_j p_j \omega_j - \sum_j q_j \omega_j - \omega_n \right) \\ & \times \prod_{1 \leq j \leq N} (A_j e^{-i\beta_j z})^{p_j} \prod_{1 \leq j \leq N} (A_j^* e^{i\beta_j z})^{q_j}. \quad (\text{B5}) \end{aligned}$$

The sums in the above equations are over energetically allowed processes and can be inventoried in terms of combinations of integers p_j and q_j . In each equation for dA_n/dz , the p_j can be thought of as the numbers of input photons with frequencies ω_j for a particular process that results in an ω_n output photon. The q_j are the numbers of additional output photons at frequencies ω_j for that process.

- [1] B. H. Eom, P. K. Day, H. G. Leduc, and J. Zmuidzinas, A wide-band, low-noise superconducting amplifier with high dynamic range, *Nat. Phys.* **8**, 623 (2012).
- [2] A. Kher, P. Day, B. H. Eom, J. Zmuidzinas, and H. Leduc, Kinetic inductance parametric up-converter, *J. Low Temp. Phys.* **184**, 480 (2016).
- [3] J. Luomahaara, V. Vesterinen, L. Grönberg, and J. Hassel, Kinetic inductance magnetometer, *Nat. Commun.* **5**, 4872 (2014).
- [4] A. J. Annunziata, D. F. Santavicca, L. Frunzio, G. Catelani, M. J. Rooks, A. Frydman, and D. E. Prober, Tunable superconducting nanoinductors, *Nanotechnology* **21**, 445202 (2010).

- [5] R. B. Thakur, N. Klimovich, P. Day, E. Shirokoff, P. Mauskopf, F. Faramarzi, and P. Barry, Superconducting on-chip Fourier transform spectrometer, *J. Low Temp. Phys.* **200**, 342 (2020).
- [6] F. Faramarzi, P. Day, J. Glasby, S. Sympkens, M. Colangelo, R. Chamberlin, M. Mirhosseini, K. Schmidt, K. K. Berggren, and P. Mauskopf, Initial design of a W-band superconducting kinetic inductance qubit, *IEEE Trans. Appl. Superc.* **31**, 1800605 (2021).
- [7] M. R. Vissers, R. P. Erickson, H.-S. Ku, L. Vale, X. Wu, G. Hilton, and D. P. Pappas, Low-noise kinetic inductance

- traveling-wave amplifier using three-wave mixing, *Appl. Phys. Lett.* **108**, 012601 (2016).
- [8] S. Chaudhuri, D. Li, K. Irwin, C. Bockstiegel, J. Hubmayr, J. Ullom, M. Vissers, and J. Gao, Broadband parametric amplifiers based on nonlinear kinetic inductance artificial transmission lines, *Appl. Phys. Lett.* **110**, 152601 (2017).
- [9] M. Malnou, M. Vissers, J. Wheeler, J. Aumentado, J. Hubmayr, J. Ullom, and J. Gao, Three-wave mixing kinetic inductance traveling-wave amplifier with near-quantum-limited noise performance, *PRX Quantum* **2**, 010302 (2021).
- [10] W. Shan, Y. Sekimoto, and T. Noguchi, Parametric amplification in a superconducting microstrip transmission line, *IEEE Trans. Appl. Supercond.* **26**, 1801209 (2016).
- [11] S. Goldstein, N. Kirsh, E. Svetitsky, Y. Zamir, O. Hachmo, C. E. M. de Oliveira, and N. Katz, Four wave-mixing in a microstrip kinetic inductance travelling wave parametric amplifier, *Appl. Phys. Lett.* **116**, 152602 (2020).
- [12] A. D. O'Connell, M. Ansmann, R. C. Bialczak, M. Hofheinz, N. Katz, E. Lucero, C. McKenney, M. Neeley, H. Wang, E. M. Weig, A. N. Cleland, and J. M. Martinis, Microwave dielectric loss at single photon energies and millikelvin temperatures, *Appl. Phys. Lett.* **92**, 112903 (2008).
- [13] B. T. Buijtenorp, J. Bueno, D. J. Thoen, V. Murugesan, P. M. Sberna, J. J. Baselmans, S. Vollebregt, and A. Endo, Characterization of low-loss hydrogenated amorphous silicon films for superconducting resonators, *Proc. SPIE*, 114532X (2020).
- [14] S. Golwala, A. Beyer, D. Datta, F. Defrance, J. Sayers, B. H. Eom, and P. Day, RF loss tangent and two-level-system noise of amorphous silicon and crystalline silicon dielectrics for mm/submm/FIR astronomy applications, in *18th International Workshop on Low Temperature Detectors* (Springer, New York, 2019), poster 357, <https://agenda.infn.it/event/15448/contributions/95858/attachments/65827/80272/180716GolwalaLTD18Dielectric.pdf>.
- [15] A. Anthore, H. Pothier, and D. Esteve, Density of States in a Superconductor Carrying a Supercurrent, *Phys. Rev. Lett.* **90**, 127001 (2003).
- [16] S. Zhao, S. Withington, D. J. Goldie, and C. N. Thomas, Nonlinear properties of supercurrent-carrying single- and multi-layer thin-film superconductors, *J. Low Temp. Phys.* **199**, 34 (2020).
- [17] L. Planat, A. Ranadive, R. Dassonneville, J. Puertas Martínez, S. Léger, C. Naud, O. Buisson, W. Hasch-Guichard, D. M. Basko, and N. Roch, Photonic-Crystal Josephson Traveling-Wave Parametric Amplifier, *Phys. Rev. X* **10**, 021021 (2020).
- [18] K. D. Usadel, Generalized Diffusion Equation for Superconducting Alloys, *Phys. Rev. Lett.* **25**, 507 (1970).
- [19] S. B. Nam, Theory of electromagnetic properties of strong-coupling and impure superconductors. II, *Phys. Rev.* **156**, 487 (1967).
- [20] E. F. C. Driessen, P. C. J. J. Coumou, R. R. Tromp, P. J. De Visser, and T. M. Klapwijk, Strongly Disordered TiN and NbTiN *s*-Wave Superconductors Probed by Microwave Electrodynamics, *Phys. Rev. Lett.* **109**, 107003 (2012).
- [21] N. Samkharadze, A. Bruno, P. Scarlino, G. Zheng, D. P. DiVincenzo, L. DiCarlo, and L. M. K. Vandersypen, High-Kinetic-Inductance Superconducting Nanowire Resonators for Circuit QED in a Magnetic Field, *Phys. Rev. Appl.* **5**, 044004 (2016).
- [22] J. G. Kroll, F. Borsoi, K. L. van der Enden, W. Uilhoorn, D. de Jong, M. Quintero-Pérez, D. J. van Woerkom, A. Bruno, S. R. Plissard, D. Car, E. P. A. M. Bakkers, M. C. Cassidy, and L. P. Kouwenhoven, Magnetic-Field-Resilient Superconducting Coplanar-Waveguide Resonators for Hybrid Circuit Quantum Electrodynamics Experiments, *Phys. Rev. Appl.* **11**, 064053 (2019).
- [23] M. Castellanos-Beltrán, K. Irwin, G. Hilton, L. Vale, and K. Lehnert, Amplification and squeezing of quantum noise with a tunable Josephson metamaterial, *Nat. Phys.* **4**, 929 (2008).
- [24] Z. Pan, M. Liu, R. B. Thakur, B. A. Benson, D. J. Fixsen, H. Goksu, E. Rath, and S. S. Meyer, Compact millimeter-wavelength Fourier-transform spectrometer, *Appl. Opt.* **58**, 6257 (2019).
- [25] M. Houde, L. C. G. Govia, and A. A. Clerk, Loss Asymmetries in Quantum Traveling-Wave Parametric Amplifiers, *Phys. Rev. Appl.* **12**, 034054 (2019).