Generalized overlap quantum state tomography*

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We propose and experimentally demonstrate a quantum state tomography protocol that generalizes and improves upon the Wallentowitz-Vogel-Banaszek-Wódkiewicz point-by-point Wigner function reconstruction. The full density operator of an arbitrary quantum state is efficiently reconstructed in the Fock basis, using semidefinite programming, after interference with a small set of calibrated coherent states. This protocol is resource- and computationally efficient, is robust against noise, does not rely on approximate state displacements, and ensures the physicality of results.

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Introduction. Since a quantum system is fully characterized by its density operator [1], the experimental implementation and investigation of quantum state tomography [2] plays a crucial role in quantum information (QI). While the dimension 2^N of an N -qubit Hilbert space prohibits full quantum state tomography for large values of N , except in the particular case of sparse density operators [3], full state tomography of single, or few, quantum systems can still be realized and is essential to characterizing important resource states, e.g., quantum error correcting codes. Here, we focus on bosonic quantum modes, a.k.a. qumodes, as implemented in general by vibrational eigenmodes of quantum harmonic oscillators and, in particular, by quantum electromagnetic fields as used in continuous-variable (CV) QI [4,5], the latter being particularly well suited to the generation of massively scalable multipartite entanglement, in particular of the universal quantum computing substrates that are cluster states [6–12].

In the CVQI context, the Wigner function [13,14] plays a central role as a quantum state descriptor strictly equivalent to

the density operator ρ :

$$W(q, p) = \frac{1}{\pi} \int_{-\infty}^{\infty} e^{2ipy} \langle q - y | \rho | q + y \rangle dy, \quad (1)$$

where quantum phase space variables q and p are the eigenvalues of the positionlike amplitude quadrature, $\hat{Q} = (\hat{a} + \hat{a}^\dagger)/\sqrt{2}$, and momentumlike phase quadrature, $\hat{P} = i(\hat{a}^\dagger - \hat{a})/\sqrt{2}$, of the electromagnetic field, and where $\hat{a}$ is the boson annihilation operator for a given qumode, typically specified by its wave vector and polarization. The experimental determination of the Wigner function, first proposed and realized by way of interferometric, homodyne quadrature measurements [15], thus constitutes another approach to quantum state tomography. A technical difficulty of the aforementioned optical homodyne (OHD) tomography approach resides in the need for computationally intensive reconstruction procedures, using either the inverse Radon transform (whence the “tomography” moniker) or maximum likelihood (MaxLik) algorithms [16]. Recently Tiunov *et al.* proposed a scheme using machine learning, restricted Boltzmann machine (RBM), which has significant advantages over MaxLik-based OHD [17], yet is not provably efficient for arbitrary quantum states [18].

Such difficulties can be alleviated by replacing field measurements with photon-number ones, using the fact that the Wigner function at the origin of phase space coincides with the expectation value of the photon-number parity operator [19]. This yields an expression of the Wigner function in the Fock basis which is easy to reconstruct, as was first proposed by Wallentowitz-Vogel [20] and Banaszek-Wódkiewicz [21] (WVBW). In this method, a simple phase-space translation, i.e., displacement, of the quantum state to be characterized, followed by parity measurements, allows easy determination of the Wigner function. This was first implemented experimentally on the phononic field of vibration of a single trapped

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FIG. 1. (a) Schematic of the experiment: the field to be measured, of density operator ρ , interferes with a calibrated field in coherent state α at a beamsplitter of field reflectance $r \in \mathbb{R}$ and transmittance $t = (1 - r^2)^{1/2}$. PNRD: photon-number-resolving detector. (b) Principle of generalized overlap tomography exemplified with a two-photon Fock state. (c) Limit case of (b), where a highly unbalanced beamsplitter merely implements a displacement of ρ by $-\beta$.

ion [22], as well as on microwave cavity fields [23,24]. More recently, the coming of age of photon-number-resolving (PNR) detection [25] has opened the door to using the full WVBW method on traveling optical fields with no prior knowledge of the measured quantum state [26,27]. While the WVBW method presents clear advantages in terms of the numerical demands on reconstruction, it requires a phase-space raster scan involving a large number of optical displacements, and the pitch of the raster scan is determined by the specific features of the (unknown) Wigner function to be resolved. Moreover, the best experimental implementation of phase-space displacements is intrinsically lossy [28]. Finally, the method does require, like homodyne tomography, a very high system detection efficiency in order to prevent the quantum decoherence caused by vacuum fluctuation contamination. Additionally, the WVBW protocol mandates a matrix inversion for each experimental data point in order to infer the true photon-number distribution from the measured loss-degraded distribution which could be experimentally demanding for probing the Wigner functions of complicated structures, such as cat states or Gottesman-Kitaev-Preskill (GKP) states [29].

In this Rapid Communication, we present a generalization of the WVBW approach which uses a Wigner function overlap measurement to reconstruct the density operator, rather than the Wigner function, using computationally efficient semidefinite programming. This general method requires considerably less data acquisition, and ensures physical results which are robust against measurement noise. The effect of known system losses can also be entirely deconvoluted from the measured density operator. We present the mathematical derivation of this generalized overlap quantum state tomography and present experimental results for a single-photon Fock state with performance that far exceeds that of the recent WVBW demonstration [27]. Furthermore, we can perform loss compensation in one fell swoop for the entire density matrix ρ , unlike at each experimental data point in the WVBW method. The proposed scheme requires significantly fewer measurements as compared with MaxLik and RBM tomography methods and achieves higher fidelity for all the states considered in Ref. [17].

Theory. We consider the situation depicted in Fig. 1(a): a field with unknown density operator ρ and Wigner function

$W_1(q_1, p_1)$ interferes with a reference field in a coherent state $|\alpha\rangle\langle\alpha|$ of Wigner function $W_2(q_2, p_2)$. We then simply count photon statistics at only one beamsplitter output using the PNR detector; using these, we evaluate the expectation value of the photon-number parity operator, i.e., the value of the origin of the Wigner function of this output mode [30,31]

$$W_1'(0, 0; r, t) = \frac{1}{r^2} \iint W_\rho(q, p) W_{|\alpha\rangle\langle\alpha|}\left(\frac{t}{r}q, \frac{t}{r}p\right) dq dp, \quad (2)$$

as illustrated in Fig. 1(b). Setting $r = t$ in the above formula yields, by virtue of the Wigner function overlap theorem [32], the overlap $\mathcal{O}$ of the unknown ρ with $|\alpha\rangle\langle\alpha|$:

$$\mathcal{O} = \text{Tr}[\rho|\alpha\rangle\langle\alpha|] = \pi W_1'\left(0, 0; \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right). \quad (3)$$

Note that $\mathcal{O}$ is proportional to the Husimi Q function, $Q(\alpha)$, which we sample sparsely [33]. Note also that, in the limit case $t \gg r$, the function $W_2(\frac{t}{r}q, \frac{t}{r}p)$ in Eq. (2) is a contracted Gaussian that tends toward a Dirac delta function $\delta(\sqrt{2}\text{Re}[\beta], \sqrt{2}\text{Im}[\beta])$, where $\beta = r\alpha/t$, thereby yielding $W_1(\sqrt{2}\text{Re}[\beta], \sqrt{2}\text{Im}[\beta])$, i.e., precisely the WVBW tomography protocol, as illustrated in Fig. 1(c). The validity of this limit case is equivalent to the validity of implementing a displacement with an unbalanced beamsplitter. Our overlap approach is free of such considerations.

From here on, we set $r = t$ for simplicity. We provide a treatment of the general case of arbitrary r, t in the Supplemental Material [31]. Even though this would appear to cause an irremediable loss of information (the role of the other output port is examined in the Supplemental Material [31]), we show that ρ can nonetheless be accurately and efficiently retrieved by measuring $\mathcal{O}_j$ for a series of distinct coherent states $|\alpha_j\rangle$. In the Fock basis, we get

$$\mathcal{O}_j = \sum_{n=0}^{n_0} \sum_{m=0}^{n_0} c_{jn}^* c_{jm} \rho_{nm}, \quad (4)$$

where $c_{jm} = \exp(-|\alpha_j|^2/2) \alpha_j^m / \sqrt{m!}$ and the size of the Hilbert space is $n_0 + 1$. Our superconducting transition-edge sensor (TES) has high-efficiency PNR capabilities up to five photons at 1064 nm, leading to our choice of $n_0 = 5$ [25,26].

For $N = (n_0 + 1)^2$ measurements, we can write Eq. (4) in matrix form in N -dimensional Liouville space

$$\mathbf{O} = \mathbf{C}\mathbf{P}, \quad (5)$$

where $\mathbf{O} = (\mathcal{O}_j)_j^T$, $\mathbf{C} = (c_{jn}^* c_{jm})_{j,nm}$, and $\mathbf{P} = (\rho_{nm})_{nm}^T$ [31]. Inverting Eq. (5) yields the unknown Liouville vector $\mathbf{P}$. To do this, we employ semidefinite programming (SDP) to run a convex quadratic optimization algorithm that minimizes the ℓ^2 norm, $\|\mathbf{O} - \mathbf{C}\mathbf{P}\|_2$, subject to physicality constraints in order to extract $\mathbf{P}$. The procedure is computationally efficient and yields a unique solution [34]. Note that $\mathbf{C}$ does not have to be a square matrix, so that the number of measured overlaps (the dimension of $\mathbf{O}$) can be increased for better data statistics.

A crucial point is the impact of inevitable experimental fluctuations on the numerical stability of the solution. Indeed, by nature of its slowly decaying Poissonian coefficients, matrix $\mathbf{C}$ necessarily contains both large and small entries and, therefore, both large and small singular values, which make it ill conditioned [35] and make its inversion extremely sensitive

to experimental fluctuations in the measured photon statistics. In order to suppress these instabilities, we choose to use a Tikhonov regularization procedure [36], formulated as the SDP problem

$$\begin{aligned} & \text{Minimize} \quad \|\mathbf{O} - \mathbf{C}\mathbf{P}\|_2 + \gamma \|\mathbf{P}\|_2 \\ & \text{Subject to} \quad \rho \geq 0, \quad \text{Tr}[\rho] = 1, \end{aligned} \quad (6)$$

where γ is a small regularization parameter set according to the noise level [37] and the optimization remains quadratic convex.

Another crucial point is the effect of decoherence on density matrix reconstruction. Optical losses to the state before and after interference with $|\alpha_j\rangle$, including detector efficiency, can be modeled by inserting a single fictitious beamsplitter of transmittivity η in front of a perfect detector that both leaks light out and couples in vacuum fluctuations as discussed in detail in the Supplemental Material [31]. This leads to the resulting, binomial-law density matrix,

$$\rho'_{nm} = \sum_{k=0}^{\infty} \rho_{n+k, m+k} \binom{n+k}{k}^{1/2} \binom{m+k}{k}^{1/2} (1-\eta)^k \eta^{(n+m)/2}, \quad (7)$$

which can be inverted to recover ρ from ρ' [38]. If we only consider the diagonal elements of ρ , then this inversion is exactly described by correcting a loss-degraded photon-number distribution, as has been experimentally demonstrated for state characterization [39] and the WVBW protocol, but requiring a matrix inversion for each experimental data point [29]. In our general case, we can perform such an inversion in one fell swoop for the whole ρ , in lieu of entrywise as above. The same difficulty arises, though, of high sensitivity of the inversion procedure to small experimental fluctuations in ρ' which can lead to unphysically large or negative diagonal density matrix elements in the reconstructed ρ [31]. Again, we solve this problem by SDP:

$$\begin{aligned} & \text{Minimize} \quad \sum_{i=0}^{N_{\max}} \|\rho^{(i)} - \mathbf{M}^{(i)}(\eta)\rho^{(i)}\|_2 \\ & \text{Subject to} \quad \rho \geq 0, \quad \text{Tr}[\rho] = 1, \quad \text{and} \quad \rho_{nn} \leq \eta^{-n} \rho'_{nn}, \end{aligned} \quad (8)$$

where $\rho^{(i)}$ denotes the i th diagonal of ρ ($i = 0$: main diagonal) and $\mathbf{M}^{(i)}(\eta)$ is the matrix describing the binomial-law loss degradation along each diagonal of ρ . The third constraint stems from the fact that Eq. (7) yields $\rho'_{nn} = \eta^n \rho_{nn} + \epsilon$, where ϵ is positive. If the value of the loss parameter η is known, this loss deconvolution method is very efficient and reliable, as is further detailed in the Supplemental Material where we provide a side-by-side comparison of the improvement over an analytical approach in the presence of noise [31].

The case of mode mismatch deserves a separate mention. In contrast to homodyne detection, nonideal-contrast interference between the coherent-state and signal fields cannot simply be treated as loss. To account for mode mismatch, we consider a multimode detection theory where the coherent state is decomposed into $|\sqrt{\mathcal{M}}\alpha^j\rangle_{\parallel}$, which interferes entirely with the signal, and an orthogonal component, $|\sqrt{1-\mathcal{M}}\alpha^j\rangle_{\perp}$, which interferes with vacuum [40]. Parameter $\mathcal{M}$ is determined by the degree of overlap, and can be calculated from a bright-field visibility measurement [41].

FIG. 2. Left, absolute value of the experimentally reconstructed density matrix elements for the coherent state. Right, associated Wigner function. The black error bars are obtained from the measurement statistics.

Because the PNR detectors used herein are mode insensitive, the total measured signal is represented by the sum of detected modes. This yields a measured photon number distribution that is a convolution of the individual mode distributions [42]. We measured a mode overlap parameter of $\mathcal{M} = 0.83(2)$ [0.86(2)] when performing the tomography of the coherent (Fock) state and deconvolve the Poissonian distribution of the mode-mismatched $|\sqrt{1-\mathcal{M}}\alpha^j\rangle_{\perp}$ from our measurements. It is important to note that the $\mathcal{O}$ values obtained from the expectation of parity are now between ρ and $|\sqrt{\mathcal{M}}\alpha^j\rangle$ for each coherent state probe, and therefore the coefficient matrix $\mathbf{C}$ must be modified accordingly as further detailed in the Supplemental Material [31].

Experimental implementation. The experimental setup was identical to our previous implementation of WVBW tomography [27] and is described in detail in the Supplemental Material [31]. It was based on a very stable CW Nd:YAG laser which provided all coherent states upon phase and amplitude modulations by a piezoelectric-actuated mirror and a homemade RbTiOAsO₄ electro-optic modulator, respectively. The calibrated coherent-state amplitude range was $|\alpha| = 0.138(2)$ to $0.339(3)$, in six steps, directly calibrated using a TES. The coherent-state probe amplitudes were calibrated by comparing the TES photon statistics to that of a Poisson distribution with the signal beam blocked as detailed in our previous implementation [27]. The phase scan was ten discrete steps of $0.58(5)$ radians each. The laser was also resonantly frequency doubled to pump an optical parametric oscillator whose narrowband pair emission provided heralded single-photon Fock states [27]. All data acquisition was computer controlled.

Results: Coherent state. We implemented the generalized overlap tomography protocol for a weak coherent state and a single-photon state. The rationale for measuring a coherent state was to display a phase-dependent, i.e., noncylindrically symmetric structure in phase space. The coherent state $|\beta\rangle$ was chosen $|\beta| = 0.191(3)$, as calibrated by the TES Poissonian photon statistics. For each of the 60 coherent-state probes $|\alpha_j\rangle$, data was acquired for 3 s to obtain $\sim 10^5$ events from which to construct the photon-number probability distributions. The SDP tomography results after correcting for mode mismatch are displayed in Fig. 2.

Examining the magnitude of the density matrix elements, we clearly see that the diagonal and off-diagonal terms of ρ were both successfully reconstructed. The phase and amplitude accuracy is more evident when comparing the associated

FIG. 3. Generalized overlap tomography of a single-photon Fock state. Top row, (a), (b): SDP with mode mismatch corrected but no correction for losses. Bottom row, (c), (d): loss-deconvoluting SDP reconstruction. Left column: direct reconstructions. Right column: reconstructions using phase-averaged measurements. Reconstruction fidelities are 0.85(8) for the bottom left panel and 0.94(6) for the bottom right panel. Inset: Wigner functions calculated from reconstructed density matrices.

Wigner functions plotted in Fig. 2, right, where the red dashed lines delineate the zero-axis values. We achieve a fidelity of $\mathcal{F} = 0.97(2)$ between the reconstructed state, ρ , and the target pure state, $|\beta\rangle$. The slight asymmetry of the Wigner function is imputable to residual phase noise in our measurements (only passive noise cancellation techniques were used for the optical paths).

Single-photon Fock state. The reconstructed density matrices and constructed Wigner functions are shown in Fig. 3. Due to the nature of our heralded source undergoing an overall loss, η , we expect to measure a statistical mixture of the one-photon and vacuum states which has a diagonal density matrix in the Fock basis and rotationally symmetric Wigner function [27,43]. Under this assumption, an average over the optical phases of the coherent probes can be performed, yielding the results on the right column of Fig. 3. It is, of course, also interesting to examine the unaveraged measurements, left column of Fig. 3, in order to assess the quality of our tomographic reconstruction. Despite the effects of experimental noise, visible in the off-diagonal terms, the reconstruction has a fidelity of $\mathcal{F} = 0.94(2)$ with the expected mixture where $\eta = 0.50(1)$, as measured by the heralding ratio as in Ref. [27].

Finally, the performance of the loss deconvolution by SDP is displayed in the bottom row of Fig. 3. Assuming no prior knowledge about the state other than this calibrated measurement loss, the reconstructed loss-compensated state is depicted in the bottom left of Fig. 3, where we achieved a fidelity of $\mathcal{F} = 0.85(8)$ with a single-photon Fock state. While

the phase noise of the LO does not contribute to the overlap measurements with a phase-insensitive state such as heralded single-photon Fock state, the amplitude noise and other experimental fluctuations might be attributed to the ripples seen in the reconstructed Wigner function. Adding the assumption of a phase-invariant state as in Refs. [43–45] and averaging measurements for each of the ten phases before compensating for loss yielded the nearly perfect reconstruction shown in the bottom right panel, where we achieved $\mathcal{F} = 0.94(6)$. It is worth emphasizing that the negativity of the single-photon Wigner function was fully recovered after compensating for loss and mode mismatch (Fig. 3, bottom row), even though the 50% loss level suppressed negativity when no loss deconvolution was performed (Fig. 3, top row).

Finally, it is important to note that the maximum amplitude probe was $|\alpha_{\max}| \simeq 0.34$, which led to a mean photon number detection of $\langle N \rangle \simeq 0.56$, yet our overlap tomography accurately reconstructed the Wigner function at quadrature coordinates beyond q or $p = 3$ (consistent with our truncation of the Hilbert space to $n_o = 5$). This is in stark contrast to the WVBW case of Ref. [27], in which the maximum of the Wigner function, at q or $p = 1$, could not be reached using displacements with $|\alpha_{\max}| \simeq 0.80$ and $\langle N \rangle \simeq 1.64$. Therefore, generalized overlap tomography necessitates PNR detection of significantly lower photon flux while still requiring the detection of only a single field mode.

Conclusion. We proposed and experimentally demonstrated generalized overlap quantum state tomography using

PNR measurements on a single field mode. Our approach (i) makes no prior assumption on the initial state, (ii) exploits numerically efficient, noise-robust SDP that enforces physicality, (iii) requires only a *single* PNR detector, (iv) implements no approximated displacement operations, (v) uses fewer, lower-amplitude probes than point-by-point WVBW tomography (see Supplemental Material [31]), and (vi) compensates for known losses with fewer numerical instabilities.

Our approach is equally valid for other physical systems and can be readily applied in circuit quantum electrodynamics [46,47]. It could also be used to directly measure the purity of a quantum state by measuring the overlap between two copies of the same system, which allows access to the second-order Rényi entropy extensively used to quantify the entanglement

of many-body physical systems [48]. Finally, the proposed scheme can be simply extended to characterize a multimode quantum system by interfering with a multimode set of coherent states followed by measuring the overall parity of the state after the interference.

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