Determination of spin-orbit scattering lifetime at the interface of $\text{LaAlO}_3/\text{SrTiO}_3$ from the superconducting upper critical fields

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The intrinsic mechanism of the spin-orbit coupling at the $\text{LaAlO}_3/\text{SrTiO}_3$ interface remains a debatable issue. Rashba-type spin-orbit coupling is an appealing candidate that has been demonstrated by several magnetotransport results. On the other hand, the atomic spin-orbit coupling was also shown to play an important role, particularly when the Fermi level is close to the Lifshitz point. In this paper, we focus on the measurements of the anisotropic superconducting upper critical fields in gated $\text{LaAlO}_3/\text{SrTiO}_3$ devices. By rigorous fittings of the H_{c2} - T curves using both the Werthamer-Helfand-Hohenberg theory and Klemm-Luther-Beasley model, an upper limit of the spin-orbit scattering lifetime can be determined in the two-dimensional limit of the superconducting state. We found that the extracted spin-orbit scattering lifetime monotonically increases with the effective transport lifetime that spans over two orders of magnitude in the regime with sheet density close to and higher than that at the Lifshitz point. Those results suggest the dominant role of Elliott-Yafet type spin relaxation. The comparison to the weak localization fittings on magnetoconductance reveals a striking difference. Such a model dependence infers an electronic phase separation at the $\text{LaAlO}_3/\text{SrTiO}_3$ interface, where the superconductivity originates from a selective d -subband occupancy.

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I. INTRODUCTION

The relativistic spin-orbit coupling (SOC) is a fundamental physical quantity, which turns out to be an important basis for some newly discovered phenomena, such as topological-protected electronic phases [1–3] and unconventional superconductivity in noncentrosymmetric systems [4,5]. Nevertheless, the extraction of the spin-orbit coupling strength in experiment has been always challenging due to the lack of a direct probe of such relativistic coupling effect. In a two-dimensional disordered system, SOC can largely influence the weak localization (WL) effect, where the transverse weak-field magnetoresistance (MR) can go from a negative MR to a positive MR as the SOC strength increases [6,7]. This provides a practical method to extract the SOC parameters from the fittings to the magnetoconductance (MC) data [8,9]. On the other hand, in a thin superconductor, the observations of finite zero temperature magnetic susceptibility [10] and also large upper critical fields with parallel magnetic fields are attributed to the presence of the SOC. Werthamer, Helfand, and Hohenberg (WHH) performed theoretical calculations of upper critical field H_{c2} in a type-II superconductor [11], incorporating the spin-paramagnetic and spin-orbit scattering

(SOS) terms. In a dirty limit, the spin-orbit scattering parameter can be extracted from an implicit equation with complex digamma functions, relating the H_{c2} and superconducting transition temperature T_c . An advancement was put forward by Klemm, Luther, and Beasley (KLB) [12], where they further extended a similar concept to a type-II and layered superconductor. In a dirty limit with strong SOC, a remarkably simple relation between zero temperature $H_{c2}(T = 0 \text{ K})$ and the SOS lifetime τ_{SO} can be obtained when taking the two-dimensional (2D) limit of zero interlayer coupling strength.

In the quasi-two-dimensional electron liquid (q2DEL) at the $\text{LaAlO}_3/\text{SrTiO}_3$ (LAO/STO) (100) interface, the superconducting phase can be driven into the normal state by electric gating [13,14]. The phase diagram with respect to the sheet density exhibits a superconducting dome [15], similar to high- T_c superconducting cuprates, where the optimum T_c occurs at a Fermi level close to the Lifshitz point. The exact electronic condition for the highest T_c remains debatable [16,17]. Below the Lifshitz point, most of the electrons sit in the lowest d_{xy} orbital, and WL fittings to the MC have been reported [18] to reveal the presence of the Rashba-type SOC in the *underdoped* regime. The situation becomes more complicated when trying to perform similar fittings to the MC for the *overdoped* regime with a Fermi level above the Lifshitz point, where electrons start to fill in multiple d orbitals. While the WL model can still apply and give averaged SOC parameters over multiple bands, other mechanisms, such as two-band magnetotransport [19] and magnetic ordering [20–22], can also significantly affect the MC behavior and thus jeopardize the fittings by the WL model alone. In this work, we focus on the issue of the intrinsic spin-relaxation mechanism at

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the LAO/STO (100) interface in the *optimum-doped* and *overdoped* regimes. From rigorous superconducting upper critical field measurement and analysis, an upper limit for the SOS lifetime can be readily determined based on both the modified WHH theory and KLB model in superconducting states. Comparing to the magnetoconductance data fittings on the same sample in normal state with weak localization model, a striking model dependence on the determination of SOS lifetime was uncovered, which supports the intriguing electronic phase separation at the LAO/STO interface.

II. EXPERIMENTAL METHOD AND RESULTS

Sample preparation and magnetotransport data. An oxide molecular beam epitaxy (MBE) system with a base pressure better than 7×10^{-11} torr was used to grow ultrathin epitaxial LAO films on atomically flat TiO_2 -terminated (100) STO substrates [23] (see Appendix A for detailed sample growth conditions). Magnetotransport measurements on several samples with LAO thickness of ten unit cells (uc) were performed with sample temperature down to about 50 mK using an Oxford dilution refrigerator system. The thin-film contact electrodes were fabricated with a standard Hall-bar geometry. The upper and lower panels of Fig. 1 show the Hall mobility (μ_H) and the sheet density (n_{2D}), respectively, as a function of inverse sheet resistance ($1/R_{\square}$) from the magnetotransport data at $T = 0.8$ K. Sample A shows higher $R_{\square}$ that can be tuned from 34 k Ω at $V_g = -20$ V to 2.5 k Ω at $V_g = +50$ V, covering the left-panel regime in Fig. 1. The sheet densities of n_{LH} and n_{HH} , shown in the lower panel of Fig. 1, are obtained by linear fittings to the Hall resistance data in the low field regime ($0 \leq \mu_0 H \leq 1$ T) and high field regime ($9 \leq \mu_0 H \leq 10$ T), respectively. Both n_{LH} and n_{HH} gradually increase with increasing V_g going from a sheet density of $n_{2D} \approx 2 \times 10^{13}$ to $\approx 4 \times 10^{13} \text{ cm}^{-2}$ as V_g varies from -20 V to $+50$ V, where the corresponding μ_H changes by more than eightfold from $\mu_H \approx 8.0$ to $65.0 \text{ cm}^2/\text{V s}$. We note that n_{LH} and n_{HH} values are very close to each other for $V_g \geq 0$, and a minor difference between n_{LH} and n_{HH} becomes more apparent for negative V_g values. On the other hand, $R_{\square}$ in sample B is smaller than that in sample A by nearly two orders of magnitude, going from 0.9 k Ω at $V_g = -20$ V to 0.11 k Ω at $V_g = +50$ V. A nonlinear Hall resistance versus fields behavior was observed for all the applied V_g values, and the difference between n_{LH} and n_{HH} grows larger as V_g increases. A rough and single-band estimate of the Hall mobility using n_{LH} values gives $\mu_H \approx 300 \text{ cm}^2/\text{V s}$ at $V_g = -20$ V, and it increases by more than 10-fold to a value of $\approx 3170 \text{ cm}^2/\text{V s}$ at $V_g = +50$ V. As illustrated in the inset cartoons of the upper panel in Fig. 1, the Fermi level in sample A likely falls below the Lifshitz point, while that in sample B lies above the Lifshitz point. Therefore, the combination of the transport data from samples A and B with electrical gating covers the intriguing regime near the Lifshitz point, where the sheet resistance and the corresponding Hall mobility can vary by more than two orders of magnitude. It thus provides a wonderful platform to study the interface superconductivity and spin-relaxation mechanism of the q2DEL at LAO/STO (100) interface. We like to point out that such a difference in sample A and sample B with nearly the same growth condition can be attributed to

FIG. 1. Sheet density n_{2D} (lower panel) and the corresponding Hall mobility μ_H (upper panel) versus $1/R_{\square}$ determined from the Hall measurements at $T = 0.8$ K. The green solid squares (n_{LH}) and blue open squares (n_{HH}) represent the data obtained from Hall slopes in the low field and high field regimes, respectively. The data in the left-panel region come from sample A with a Fermi level below the Lifshitz point. A large difference in n_{LH} and n_{HH} is found in sample B, which grows larger with increasing bottom gate voltage V_g . This indicates a multiband conduction in sample B with a Fermi level above the Lifshitz point as illustrated in the inset cartoon.

a minute difference in the stoichiometry in the grown LAO films as reported by several authors [23,24].

Electric field tuning of the interface superconductivity. Figure 2(a) shows the $R_{\square}$ - T curves at several different bottom gate voltages ranging from $V_g = +50$ V to -20 V, where upper panel and lower panel share the same color codes and are the results for sample A and sample B, respectively. The zero field superconducting transition temperature T_c is defined as the temperature at which the sheet resistance drops by half of the normal state value. In sample A, the T_c is ≈ 0.334 K at $V_g = 0$, and it only decreases slightly as the magnitude of V_g increases, giving a T_c of ≈ 0.325 K and ≈ 0.306 K at $V_g = +50$ V and -20 V, respectively. The corresponding sheet resistance at $T = 0.8$ K monotonically varies from $R_{\square} \approx 2.5$ k Ω at $V_g = +50$ V to $R_{\square} \approx 34.1$ k Ω at $V_g = -20$ V, as shown in the upper panel of Fig. 2(b). We note that, for $V_g < 0$, $R_{\square}$ undergoes a superconducting transition with a finite residual value of 0.6 k Ω at $V_g = -10$ V that grows bigger at larger negative V_g values. This can be attributed to the percolative electronic inhomogeneity [25] arising from significant interface scattering by negative V_g bias, where an

FIG. 2. (a) $R_{\square}$ versus T at different V_g values ranging from +50 V to −20 V for sample A (upper panel) and sample B (lower panel), sharing the same color codes. The corresponding normal state sheet resistance (open squares) and zero-field superconducting transition temperature T_c (solid triangles) are shown in (b). With the V_g tuning, sample A and sample B cover the *optimum-doped* regime and *overdoped* regime in the superconducting phase diagram.

apparent resistance tail with a finite residual resistance was observed (see Appendix B). It is not likely that the KLB and modified WHH models can apply on such an inhomogeneous superconducting regime, which thus should be excluded from the discussion. On the other hand, the superconducting transition in sample B can be much more effectively suppressed by applying bottom gate voltage, where the superconducting transition temperature gradually marches down from $T_c \approx 0.334$ K to 0.168 K as V_g goes from −20 V to +40 V. At $V_g = +50$ V, no superconductivity was observed down to $T = 0.1$ K as demonstrated in the lower panel of Fig. 2(b). The normal state sheet resistance at $T = 0.8$ K goes from 0.9 $k\Omega$ at $V_g = -20$ V to 0.11 $k\Omega$ at $V_g = +50$ V. The overall behavior agrees with the dome-shaped superconducting phase diagram reported previously [13,15], and the combination of results from samples A and B covers the *optimum-doped* and *overdoped* regimes.

Anisotropic superconducting upper critical fields. The normalized resistance R/R_n - T curves for magnetic fields applied parallel and perpendicular to the LAO/STO interface were shown in the upper and lower panels of Fig. 3(a), respectively. The superconducting transition is fully suppressed by a perpendicular field as small as 0.4 T, while the onset of resistive transition remains observable up to 2.5 T in a parallel field. Using the same criteria, the upper critical field at certain temperatures were defined as the field and temperature values that the sheet resistance drops by half of the normal state value. The extracted parallel-field upper critical field $H_{c2,\parallel}/H_p$ and perpendicular-field upper critical field $H_{c2,\perp}/H_p$ versus reduced temperature of T/T_c at different V_g 's are summarized in the upper and lower panels of Fig. 3(b). The Pauli limiting

field H_p equals $\frac{1}{\sqrt{2}} \frac{\Delta(0)}{\mu_B} \approx 1.84 \times T_c$, where $\Delta(0)$ and μ_B are the zero-temperature superconducting gap and Bohr magneton, respectively, and its V_g dependence is shown in the inset of the lower panel in Fig. 3(b). For $V_g < +20$ V, the $H_{c2,\parallel}/H_p$ vs T/T_c curves follow $\sqrt{1 - T/T_c}$ dependence, and the lack of temperature dependence on $H_{c2,\parallel}$ for $T \approx T_c$ is a common feature for a thin superconductor without orbital limiting effect [26]. In addition, $H_{c2,\parallel}$ is well exceeding the H_p and can be as large as $2.5 \times H_p$ for $V_g = -20$ V at $T/T_c \approx 0.6$, indicating the presence of a strong spin-orbit scattering effect at the LAO/STO interface. On the contrary, for $V_g = +40$ V, both $H_{c2,\parallel}$ and $H_{c2,\perp}$ show T -linear dependence with similar magnitude well below the H_p , suggesting a regular orbital-limiting superconducting state. We do notice a minor deviation from the T -linear dependency of $H_{c2,\perp}$ for $V_g < 0$ V that is likely associated with possible multiband characteristics [27].

III. DISCUSSIONS

Modified WHH theory and KLB model. Based on the WHH theory for a bulk type-II superconductor in a dirty limit, the temperature dependent upper critical fields can be obtained from the following equation with complex digamma functions of $\psi(x)$'s:

$$\ln t + \left[\frac{1}{2} + \frac{i\lambda_{so}}{4\gamma} \right] \psi \left(\frac{1}{2} + \frac{\bar{h} + \frac{1}{2}\lambda_{so} + i\gamma}{2t} \right) + \left[\frac{1}{2} - \frac{i\lambda_{so}}{4\gamma} \right] \times \psi \left(\frac{1}{2} + \frac{\bar{h} + \frac{1}{2}\lambda_{so} - i\gamma}{2t} \right) - \psi \left(\frac{1}{2} \right) = 0, \quad (1)$$

FIG. 3. (a) Upper and lower panels are the normalized sheet resistance R/R_n versus T at $V_g = 0$ and difference fields applied parallel and perpendicular, respectively, to the LAO/STO interface in sample B. A summary of the corresponding parallel ($H_{c2,\parallel}$) and perpendicular ($H_{c2,\perp}$) upper critical fields, normalized to the Pauli limiting field H_p , versus reduced superconducting transition temperature (T/T_c) at different V_g values are shown in the upper and lower panel in (b), respectively. The inset figure of the lower panel in (b) shows the Pauli-limiting fields H_p as a function of V_g . $H_{c2,\parallel}$ values are observed to well exceed the H_p , indicating the presence of a strong spin-orbit scattering at the superconducting LAO/STO interface.

where $t = T/T_c$ is the reduced temperature, $\bar{h} = eD\mu_0 H_{c2}/\pi k_B T_c$ is the reduced critical field, D is the diffusion constant, and μ_0 is the vacuum permeability. $\alpha = \mu_B/eD$ and $\lambda_{SO} = 2\hbar/3\pi k_B T_c \tau_{SO}$ are the two fitting parameters, representing the strength of the Pauli paramagnetic coupling and the spin-orbit scattering, respectively, with $\gamma \equiv \sqrt{(\alpha\bar{h})^2 - \lambda_{SO}^2}/4$. In the modified WHH theory with a finite effective thickness d of the superconductor [28], one can replace the reduced critical field $\bar{h}$ in Eq. (1) by

$$\bar{h}(\theta) = \frac{D}{2\pi k_B T_c} \left(2e\mu_0 H_{c2} |\cos \theta| + \frac{1}{3\hbar} [de\mu_0 H_{c2} \sin \theta]^2 \right), \quad (2)$$

where θ is the angle between the applied magnetic field and the normal of the superconducting layer. In the following discussions, we denote $H_{c2,\parallel}$ and $H_{c2,\perp}$ as the upper critical fields for $\theta = \pi/2$ and 0, respectively. As demonstrated in Fig. 4(a), the H_{c2} - T curve for $\alpha = 0$ sets an upper bound in the H_{c2} - T phase diagram. When turning on the Pauli paramagnetic coupling to the Cooper pairs, the H_{c2} - T curve with a given α and $\lambda_{SO} = 0$ produces the lower bound instead. By further switching on the SOC strength (nonzero λ_{SO}), the pair breaking due to field-induced spin polarization is suppressed. H_{c2} is thus enhanced, but it shall not exceed the thermodynamic limit set by the upper bound even when λ_{SO} goes infinite. In principle, both the α and λ_{SO} parameters can be determined by fitting to the temperature dependent $H_{c2}(T)$

curve. However, in the case of a strong SOC with $\lambda_{SO} \gg 1$, the extraction of λ_{SO} using modified WHH theory requires experimental H_{c2} data at sufficient low temperatures close to 0 K, which can be challenging for low T_c superconducting systems, such as the superconductivity at the LAO/STO interface. Therefore, a different scheme for the determination of the SOC parameter is needed.

We first examine the modified WHH theory in the limit of $T \approx T_c$. It can be shown that Eq. (1) reduces to $\bar{h}(\theta) = 4/\pi^2(1 - T/T_c)$, where it reproduces the well-known Ginzburg-Landau (GL) theory for upper critical fields [26] (see Appendix C). The SOC effect becomes irrelevant in both modified WHH theory with $T \approx T_c$ and GL theory because of vanishing H_{c2} , and spins are not polarized. For perpendicular upper critical field ($\theta = 0$), the slope of $H_{c2,\perp}$ - T curve near T_c can be derived to give $[-d(\mu_0 H_{c2,\perp})/dT]|_{T=T_c} = 4k_B/\pi De$. The diffusion constant D and thus the Pauli paramagnetic term $\alpha = \mu_B/eD$ can be determined and illustrated in Fig. 4(b) using $H_{c2,\perp} - T$ curve at $V_g = 0$ in sample B as an example. We note that a larger α value gives rise to a bigger decrease in superconducting transition temperature by external magnetic field as expected for a stronger Pauli-paramagnetism coupled superconductor. In a simplified isotropic 2D case with single band, the diffusion constant relates to the sheet resistance by $1/[D \times (m^*/m_e)] = (3e^2 m_e / 2\pi \hbar^2) R_{\square}$. Figure 4(c) plots the extracted $1/[D \times (m^*/m_e)]$ as a function of the corresponding sheet resistance $R_{\square}$ at $T = 0.8$ K in normal state, where we

FIG. 4. (a) Universal curves of reduced upper critical field h^* versus reduced temperature $t \equiv T/T_c$ based on the WHH model with different α and λ_{SO} values. (b) The determination of diffusion constant D from the slope of $H_{c2,\perp}$ - T curve at $T = T_c$. (c) The extracted $1/[D \times (m^*/m_e)]$ as a function of the corresponding sheet resistance $R_\square$ at $T = 0.8$ K. We used $m^*/m_e = 1.2$ and 4.8 for sample A and sample B, respectively. The $1/[D_B \times (m^*/m_e)]$ in sample B is practically linear with $R_\square$, giving a slope of about $8.2 \text{ s/m}^2\Omega$.

used an effective mass ratio of $m^*/m_e = 1.2$ and 4.8 [29] for samples A and B, respectively. For sample B with a higher sheet density, a linear dependence between $1/D$ and $R_\square$ is observed as expected, giving a slope of about $8.2 \text{ s/m}^2\Omega$ that falls in the same order of magnitude as the slope of $3e^2 m_e / 2\pi \hbar^2 \approx 1.0 \text{ s/m}^2\Omega$ for a single band and isotropic 2D case. The nearly eightfold larger in the slope can be accounted for by the multiband effect and also a likely larger effective mass than the one we used in sample B. On the contrary, $1/D$ is weakly dependent on $R_\square$ in sample A with a lower sheet density regardless of nearly an order of magnitude variation in $R_\square$, which indicates a surprisingly weak correlation of the superconductivity to the normal state effective transport lifetime.

We now turn to discuss the other limiting case of $T \approx 0$ and a strong SOC of $\lambda_{SO} \gg 1$. The modified WHH theory gives a simple relation between the parallel upper critical fields ($H_{c2,\parallel}$) and spin-orbit scattering lifetime (τ_{SO}) of

$$\mu_0 H_{c2,\parallel}(T=0) = \sqrt{\frac{\pi \hbar k_B T_c}{\Gamma [3\mu_B^2 \tau_{SO} + D(de)^2/3]}}, \quad (3)$$

where $\Gamma = \frac{\pi k_B T_c}{\Delta(0)} \approx 1.78$ (see Appendix C). In a 2D limit with $d = 0$, we note that Eq. (3) reduces to the remarkable concise form of

$$\mu_0 H_{c2,\parallel}(T=0) = \sqrt{\frac{\pi \hbar k_B T_c}{3\Gamma \mu_B^2 \tau_{SO}}} = 0.245 \sqrt{\frac{\hbar/\tau_{SO}}{k_B T_c}} \mu_0 H_p. \quad (4)$$

We used the Pauli limiting field $\mu_0 H_p = \frac{1}{\sqrt{2}} \frac{\Delta(0)}{\mu_B} \approx 1.86 \times T_c$ (Tesla), and a sizable SOC strength results in an enhanced parallel upper critical field that well exceeds the Pauli limiting field H_p . Therefore, spin-orbit scattering lifetime τ_{SO} can be calculated using Eq. (4) once $\mu_0 H_{c2,\parallel}(T=0)$ is properly determined. We remark that Eq. (4) can be quite general, and it was first revealed in the KLB model [12], considering a layered superconductor in a 2D limit (see Appendix C). More recently, it was shown that the exact same formula can be derived from a model based on the superconductivity with a Rashba spin-split band in a dirty limit [30].

Here, we proposed a practical scheme for the determination of τ_{SO} from $\mu_0 H_{c2,\parallel}(T=0)$. First, the α parameter is extracted from the slope of $H_{c2,\perp}$ - T curve near T_c as shown in Fig. 4(b). Secondly, from the $H_{c2,\parallel}$ versus T curves, the experimental data can be extrapolated to $T = 0$ K via modified WHH theory as demonstrated in Fig. 5(a), giving an upper and lower bound for the $H_{c2,\parallel}(T=0)$ values. The resulting $H_{c2,\parallel}(T=0)$ from the midpoint of the upper and lower bounds with uncertainty $\delta(H_{c2,\parallel})$ thus allows us to determine the SOS lifetime τ_{SO} using Eqs. (3) and (4), where the effective thickness d of the superconducting layer is roughly estimated using the GL equation [26,31].

Determination of τ_{SO} from superconducting upper critical fields. Figure 5(b) shows the extracted α values for samples A (α_A) and B (α_B) as a function of the corresponding $R_\square$ at $T = 0.8$ K, where we used $\alpha = \pi \mu_B / 4k_B \times (-d\mu_0 H_{c2,\perp}/dT)|_{T=T_c} = 0.527 \times (-d\mu_0 H_{c2,\perp}/dT)|_{T=T_c}$. As shown in Fig. 5(b), the Pauli-paramagnetic coupling

FIG. 5. (a) Scheme for the determination of zero temperature parallel upper critical field $\mu_0 H_{c2,\parallel}(0$ K) with an uncertainty of $\delta[\mu_0 H_{c2,\parallel}(0$ K)] based on the WHH theory. (b) The extracted α values in sample A (α_A) and sample B (α_B) are plotted as a function of the corresponding $R_{\square}$. The upper panel of (c) shows the extracted spin-orbit scattering lifetime τ_{SO} (solid squares) in the 2D limit and the corresponding ratio of $E_{SO}/k_B T_c$ versus $1/R_{\square}$. τ_{SO} monotonically increases with $1/R_{\square}$, where the strong SOC criteria of $E_{SO} > k_B T_c$ is well satisfied. The lower panel of (c) shows the T_c (down triangles) and E_{SO} (up triangles) versus $1/R_{\square}$.

strength (α) monotonically increases with increasing $R_{\square}$ as expected in sample B. On the contrary, a much weaker $R_{\square}$ dependence on α in sample A with more than one order of magnitude variation in $R_{\square}$ is surprising, inferring an unusual superconducting state to be discussed later. The upper panel of Fig. 5(c) plots the extracted SOS lifetime τ_{SO} in the 2D limit [Eq. (4)] and also the corresponding ratio of SOC energy ($E_{SO} = \hbar/\tau_{SO}$) to $k_B T_c$ vs $1/R_{\square}$. τ_{SO} equals ≈ 0.58 ps at $1/R_{\square} = 2.93 \times 10^{-5} \Omega^{-1}$, and it monotonically increases to ≈ 0.82 ps at $1/R_{\square} = 3.99 \times 10^{-4} \Omega^{-1}$ in high- $R_{\square}$ regime (sample A). In low- $R_{\square}$ regime (sample B), τ_{SO} in the 2D limit monotonically increases from ≈ 0.89 ps at $1/R_{\square} = 1.12 \times 10^{-3} \Omega^{-1}$ to 165.6 ps at $1/R_{\square} = 7.57 \times 10^{-3} \Omega^{-1}$. The ratio of $E_{SO}/k_B T_c$ is well above 100 for $1/R_{\square} < 2 \times 10^{-3} \Omega^{-1}$, but it decreases more rapidly in sample B with increasing $1/R_{\square}$, giving a ratio of 1.7 at $1/R_{\square} = 7.57 \times 10^{-3} \Omega^{-1}$. The criteria for strong SOC in Eq. (4) with $E_{SO}/k_B T_c > 1$ is thus justified. The zero-field superconducting transition temperature T_c (down triangle) as a function of $1/R_{\square}$ is shown in the lower panel of Fig. 5(c). The maximum T_c equals 0.34 K at about $1/R_{\square} = 2.1 \times 10^{-4} \Omega^{-1}$, and it decreases more rapidly as $1/R_{\square}$ increases above $10^{-3} \Omega^{-1}$, which shows resemblance to the *overdoped* regime of a superconducting dome. In high- $R_{\square}$ regime, T_c varies slightly with $1/R_{\square}$, and the corresponding

E_{SO} gradually decreases from 7.1 meV to 5.1 meV as $1/R_{\square}$ increases. For low- $R_{\square}$ regime ($1/R_{\square} > 6 \times 10^{-4} \Omega^{-1}$), the E_{SO} shows a steeper drop in magnitude from ≈ 4.6 meV with $T_c \approx 0.33$ K to nearly 0 meV with $T_c \approx 0.17$ K as $1/R_{\square}$ increases.

The superconducting coherence length (ξ_{GL}) and the effective superconducting layer thickness (d_{GL}) can be estimated from the anisotropy of the upper critical fields based on Ginzburg-Landau theory [31] in the absence of the SOC. Figure 6(a) shows the resulting ξ_{GL} and d_{GL} as a function of $1/R_{\square}$. In sample A, ξ_{GL} and d_{GL} is about 38 nanometers (nm) and 12 nm, respectively, which is nearly independent with V_g tuning. In sample B, larger variations were observed, and both ξ_{GL} and d_{GL} increase with increasing V_g , giving $\xi_{GL} \approx d_{GL} \approx 90$ nm at $V_g = +40$ V. The 2D limit criteria of $d_{GL} < \xi_{GL}$ is thus well satisfied in both sample A and sample B, except for $V_g \geq +40$ V in sample B. In the bottom gate geometry, the application of a positive V_g not only increases the electron density at the interface but also pulls the electrons away from the interface, thus broadening the confining potential at the interface. The reduction of the interface scattering thus gives rise to the further enhancement in the mobility. On the contrary, significant interface scattering and thus decline in mobility can occur when applying negative V_g values. This scenario is in accord with the growth in d_{GL} with increasing

FIG. 6. (a) Superconducting coherence length ξ_{GL} and thickness d_{GL} determined from the GL theory without SOC. The 2D limit criteria of $d_{GL} < \xi_{GL}$ is satisfied for both sample A and sample B, except for $V_g \geq +40$ V in sample B. (b) The extracted τ_{SO} versus the corresponding $1/R_{\square}$ values with finite effective superconducting layer thickness d . The 2D limiting case of $d = 0$ sets an upper bound for the τ_{SO} . (c) The magnetoconductance, normalized to the quantum conductance of $e^2/\pi h$, with fields perpendicular to the LAO/STO interface in sample A, where the long dashed lines are the fitting curves based on the WL model with SOC. (d) The comparison of the τ_{SO} values determined from the WL model (solid up triangles) and from combined KLB and modified WHH models (solid circles). A striking difference on the τ_d dependence is observed, suggesting different types of spin relaxation processes. (e) An illustration of the layer dependent splitting of Ti d subbands near the LAO/STO interface. The energy splitting (ΔE) grows larger when moving closer to the interface.

V_g as demonstrated in Fig. 6(a). Nevertheless, we like to point out that the application of Eq. (4) to a realistic sample with a finite thickness gives an upper (a lower) limit of the τ_{SO} (E_{SO}) value. This is demonstrated in Fig. 6(b) by taking the effective thickness $d = f \times d_{GL}$ in Eq. (3) with $f = 0.5, 0.8$, and 0.9 . As f decreases, the τ_{SO} approaches the upper limit set by the 2D case of $d = 0$. The intrinsic thickness of the superconducting layer in the LAO/STO interface is still a debatable issue, but it is known [22] that d_{GL} overestimates the real thickness for the superconducting layer. For LAO/STO samples with low $n_{2D} \approx 3 \times 10^{13} \text{ cm}^{-2}$, the effective thickness for the conducting layer was measured by a scanning probe technique giving an effective thickness of $d \leq 7 \text{ nm}$ [32], which corresponds to $f \leq 0.6$ in our analysis. On the other hand, hard x-ray photoelectron spectroscopy measurement gives an effective thickness for the electron gas to be less than 4 nm [33]. We note that the intrinsic superconducting layer thickness can only be equivalent to or less than the conducting layer thickness. Those results support the validity of our discussions on τ_{SO} in the 2D limit.

Comparison to the results from weak localization fittings to magnetoconductance data. We also carried out detailed WL fittings to the MC data at $T = 0.8 \text{ K}$ in sample A, well above the superconducting transition temperature (see Appendix D). Excellent fittings (dashed lines), based on quantum correction formula to the WL with SOC, to the MC data with fields applied perpendicular to the LAO/STO interface can be obtained as demonstrated in Fig. 6(c). The τ_{SO} values at different

V_g values can then be determined from the fitting parameter of spin-orbit field H_{SO} , which are shown as solid up triangles in Fig. 6(d). The corresponding effective Drude scattering lifetime τ_d was calculated from sheet density and Hall mobility by the Hall measurements. The extracted τ_{SO} from WL fitting (up triangles) was found to be inversely proportional to the τ_d , which suggests a Dyakonov-Perel type spin relaxation due to Rashba SOC and is consistent with a previous paper [18]. The τ_{SO} values extracted from KLB and modified WHH models [solid circles in Fig. 6(d)] give a similar order of magnitude of about 0.5 – 1 ps , but its τ_d dependency is the opposite to that extracted from the WL model, inferring an Elliott-Yafet type spin relaxation due to intrinsic SOC instead. We note that similar WL fittings on sample B turn out to be difficult due to a significant contribution from the two-band effect on the MC (see Appendix E).

Model dependence on the determination of τ_{SO} . The striking difference in the behavior of τ_{SO} versus τ_d shown in Fig. 6(d) is remarkable. In addition, for $\tau_d < 0.03 \text{ ps}$, the τ_{SO} values from WL fittings appear to exceed the upper limit set by the modified WHH theory in the 2D limit. Those results may lead to several intriguing questions. The extraction of τ_{SO} from the MC data well above superconducting transition temperature is based on the quantum correction to the WL effect with SOC on normal electrons. On the other hand, the determination of τ_{SO} from $H_{c2,\parallel}$ based on the KLB and modified WHH models considers the breaking of Cooper pairs by spin-orbit scattering effect, which is fundamentally different from the

conditions for the WL model. The question about whether the spin-orbit scattering acts differently on Cooper pairs and normal electrons may require further investigations. This can be a reminiscence to the known paraconductivity [34,35] phenomena, where the excess conductivity near the superconducting transition results from the difference in the impurity scattering by fluctuating local Cooper pairs as compared to that by normal electrons. Another possible effect may derive from the band renormalization associated with the coupling to bosonic modes, which has been observed in several high- T_c superconducting cuprates [36,37]. In STO-based systems, a special longitudinal optical phonon mode with an energy scale of about 100 meV has been recently discovered [38,39] and is believed to be crucial for the appearance of the superconductivity. It gives rise to the electronic replica bands and also polaronic states with enhanced effective masses, which may alter the SOC property of the system. On the other hand, we also remark that such a model dependence in the determination of τ_{SO} may infer that the Cooper pairs and normal electrons are occupying different interface conduction channels. Such an electronic phase separation at the LAO/STO interface has been reported previously to arise from selective occupancy of interface subbands [40]. More recently, magnetotransport data on LAO/STO using different substrate crystalline orientations reveal a robustness of superconductivity for the heavy-mass electrons from d_{xz} and d_{yz} orbitals [41], while the light-mass electrons from d_{xy} have higher mobility and thus dominate the normal state charge transport behavior instead. In addition, several microwave resonator measurements [42,43] also revealed that the superfluid density at LAO/STO interface is more than an order of magnitude lower than the electron density. Those results are in accord with our observation of the unusual weak $R_{\square}$ dependence on the Pauli-paramagnetic coupling strength in sample A as shown in Fig. 5(b). As illustrated in Fig. 6(e), the charge transfer results in a confining electron density profile, covering a few nm away from the LAO/STO interface. The energy splitting (ΔE) of the Ti d subbands is at maximum for the first STO layer from the interface, and it decreases with increasing number of STO layers away from the interface. Therefore, the multiple d subbands [44,45] involved for the superconductivity near the interface may result in a large atomic SOC term [46] and also an additional intersubband SOC term [47,48], giving rise to different types of spin relaxation as compared to the Rashba SOC term.

IV. CONCLUDING REMARKS

In conclusion, high quality LAO(10-uc) epitaxial films on STO (100) substrates are grown by oxide MBE technique, exhibiting interface superconductivity. For two LAO(10-uc)/STO (100) samples we studied, the sheet density and Hall mobility can be largely tuned by bottom gate voltages. The corresponding variation in the sheet resistance spans more than two orders of magnitude and covers the *optimum-doped* and *overdoped* regimes in the superconducting phase diagram, which makes it an ideal platform to study the effective Drude scattering lifetime dependent properties. A practical scheme based on the combination of the KLB and modified WHH models with a finite thickness is proposed, and an upper

limit for τ_{SO} is determined from zero temperature parallel upper critical fields [$H_{c2,\parallel}(T = 0 \text{ K})$], which is applicable to other quasi-2D superconducting systems. The resulting τ_{SO} values for both *optimum-doped* and *overdoped* regimes were found to monotonically increase with increasing effective Drude scattering lifetime, suggesting the importance of the Elliott-Yafet type spin relaxation. In particular, a striking difference in the τ_{SO} versus τ_d behavior was uncovered while comparing to the τ_{SO} extracted from WL fittings in sample A (*optimum-doped*) well above the superconducting transition temperature. In addition, it also exceeds the upper limit obtained from the upper critical field analysis in the 2D limit. Such a model dependence on the determination of τ_{SO} in difference temperature regimes calls for further investigations, and it may also support the possible existence of the electronic phase separation at the LAO/STO interface, where d_{xz}/d_{yz} subband occupancies play a more important role for the superconducting phase. We remark that a similar orbital selectivity for Cooper pairing was recently discovered in the iron-based superconductor of FeSe [49,50].

All the supporting data are included in the main text. The raw data and other related data for this paper can be requested from one of the authors (W.-L.L.).

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APPENDIX A: GROWTH OF LaAlO_3 ON TiO_2 -TERMINATED SrTiO_3 SUBSTRATE

During the LAO film growth, a highly concentrated ozone was supplied at a pressure of 1×10^{-6} torr as an oxidation agent. Figure 7(a) shows the *in situ* reflective high energy electron diffraction (RHEED) intensity oscillations of 00 streaks for 10-uc LAO growth at a sample temperature of 800 °C, where 10 periods of oscillations can be clearly observed both in sample A and sample B. The electron beam was along the [011] azimuth direction, and the resulting RHEED patterns with streak-like features are shown in Fig. 7(b), indicating smooth growth of the ultrathin LAO films on the STO substrates. We also remark that the extracted interatomic spacing on the surface of LAO films from the first diffraction peak location is nearly the same as that in the STO substrate, where the corresponding line profiles before and after the LAO growth are demonstrated in Fig. 7(c). After the growth, LAO films were then *ex situ* postannealed in pure oxygen with an ambient pressure at 400 °C for 8 h to minimize the oxygen-vacancy problem.

FIG. 7. (a) *In situ* RHEED intensity oscillations during the growth of 10-uc LAO on STO (100) substrate. (b) The resulting RHEED patterns for sample A and sample B with the corresponding intensity line profile shown in (c).

APPENDIX B: RESISTANCE TAILS AND RESIDUAL RESISTANCE IN SAMPLE A WITH NEGATIVE V_g BIAS

In sample A, the Hall resistivity is practically linear with the magnetic field up to 10 T with a sheet density of about 3×10^{13} per cm^2 at $V_g = +50$ V. While depleting the electrons at the LAO/STO interface by reducing V_g to negative values, we found that the resistance does not drop completely to zero down to $T = 100$ mK, and the $R - T$ curve becomes flattened at lower temperatures with a residual resistance that grows larger as the magnitude of negative V_g increases, giving a residual R/R_n at $T = 100$ mK as large as 0.06 for $V_g = -20$ V. Such resistance tails at low temperatures may suggest a percolative inhomogeneous superconducting phase [51], where local superconducting regions were connected via the normal conducting channel at the LAO/STO interface. We argue that such inhomogeneous superconductivity is likely to occur for low sheet density at a negative V_g bias. It has been shown that a negative bottom gate voltage not only decreases the sheet density but also distorts the charge confining potential to be closer to the LAO/STO interface [14], which in turn significantly decreases the Hall mobility due to the enhanced interface scattering. On the contrary, the residual R/R_n for sample B in the superconducting phase are all well below 10^{-3} . (See Fig. 8.)

APPENDIX C: DETERMINATION OF $H_{c2,\parallel}(T = 0 \text{ K})$ AND ITS UNCERTAINTY BASED ON MODIFIED WHH MODEL

According to the WHH model, the relation between reduced critical field and reduced temperature ($\bar{h} - t$) can be obtained implicitly from Eq. (4) shown in the main text with complex digamma functions, where α and λ_{SO} are the two independent parameters. By further normalizing the reduced critical field h to the slope of $(-d\bar{h}/dt|_{t=1})$, it gives a universal $h^* - T$ curve with $h^* \equiv \bar{h}/(-d\bar{h}/dt|_{t=1}) = (\pi^2/4)\bar{h}$. In Fig. 9, we plot the $h^* - T$ curves with different α and λ_{SO} values. Several important features can be clearly identified. First, the $h^* - T$ curve for $\alpha = 0$ clearly gives the upper bound in the $h^* - T$ phase diagram, which is independent

of λ_{SO} values. This is because spin flips due to spin-orbit scatterings become irrelevant to the breaking of Cooper pairs if electron spin does not get polarized by an external magnetic field. Secondly, with a growing α value, the $h^* - T$ curve gradually bends down to a lower h^* value in small t , which indicates an increased contribution from Pauli paramagnetism to the Cooper pair breaking. At last, when turning on the SOC λ_{SO} with the same α value, the $h^* - T$ curve moves closer to the upper-bound curve with $\alpha = 0$, inferring the enhanced upper critical fields due to the SOC effect. It is also clear to see that the determination of the λ_{SO} parameter by the WHH model alone can be quite challenging, where experimental data points, sufficiently close to $t = 0$, are required to give a reasonable estimation of the λ_{SO} . This can be very difficult for low T_c superconducting systems, such as LAO/STO. Therefore, the combination of KLB and WHH we developed can provide a great schematic approach to determine the SOC parameters for the interface superconductivity of LAO/STO,

FIG. 8. (a) Normalized sheet resistance (R/R_n) versus T in sample A at different V_g values ranging from -20 V to $+50$ V. (b) The residual R/R_n values at $T = 100$ mK as a function of the corresponding V_g values.

FIG. 9. Universal $h^* - t$ curves with various α and λ_{SO} parameters based on the WHH model.

which should be applicable to other 2D superconducting systems. Nevertheless, for exotic superconducting systems, such as FFLO states due to a large spin exchange interaction, the $H_{c2} - T$ behavior can be quite different at the low temperature regime [4,5], which cannot be described simply by the WHH model. The schematic approach of combined KLB and WHH models would not be suitable.

As an example, the upper, middle, and lower panels of Fig. 10(a) show the $H_{c2,\parallel}$ versus t for sample B with $V_g = +40$ V, $+20$ V, and 0 V, respectively. The black curves are the upper-bound curves with $\alpha = 0$, while the red curves are the lower-bound curves with α values extracted from the perpendicular upper critical fields via $\alpha = \pi \mu_B / 4k_B \times (-d\mu_0 H_{c2} / dT)|_{T=T_c}$. On the other hand, for $V_g = -10$ V and -20 V in sample B shown in Fig. 10(b), we note that the lower-bound curves (red curves) deviated significantly from the experimental data points, thus giving a huge uncertainty for the determination of $H_{c2,\parallel}(T = 0$ K). In order to give a more reasonable estimation, a suitable minimum $\lambda_{SO} = 1.0$, corresponding to a $\tau_{SO} \approx 5$ ps that is more than five times longer than the extracted τ_{SO} from $H_{c2,\parallel}(T = 0$ K), was used as lower-bound curves shown as blue lines in Fig. 10(b). We

FIG. 10. $\mu_0 H_{c2,\parallel}$ versus t in sample B for different V_g values. The black curves (upper bound) and red curves (lower bound) are from the WHH model with $\alpha = 0$ and $(\alpha, \lambda_{SO}) = [\pi \mu_B / 4k_B \times (-d\mu_0 H_{c2} / dT)|_{T=T_c}, 0]$, respectively. The blue lines in (b) are more reasonable estimations for the lower bound with $(\alpha, \lambda_{SO}) = [\pi \mu_B / 4k_B \times (-d\mu_0 H_{c2} / dT)|_{T=T_c}, 1.0]$. The dashed lines are from the GL model, which gives rise to the overestimation of $H_{c2,\parallel}(T = 0$ K).

FIG. 11. $\mu_0 H_{c2,||}$ versus t in sample A for different V_g values. The black curves (upper bound) and red curves (lower bound) are from the WHH model with $\alpha = 0$ and $(\alpha, \lambda_{SO}) = [\pi \mu_B / 4k_B \times (-dH_{c2}/dT)|_{T=T_c}, 0]$, respectively.

remark that the simple extrapolations based on the GL model largely overestimated the $H_{c2,||}(T = 0 \text{ K})$ values, which were shown as the dashed lines in Fig. 10. The amount of overestimation of $H_{c2,||}(T = 0 \text{ K})$ can be as large as 40%, giving rise to a nearly twofold underestimation (overestimation) of τ_{SO} (E_{SO}). The GL equation can be expressed as

$$\mu_0 H_{c2,||} = \frac{\sqrt{3}\Phi_0}{\pi d\xi(0)} \sqrt{1 - \frac{T}{T_c}}, \quad (\text{C1})$$

where Φ_0 and $\xi(0)$ are the fluxoid and zero temperature coherence length, respectively, and d is the thickness of the superconducting layer. However, the lack of spin-orbit coupling effect in the GL theory that is also only valid for temperature sufficiently close to T_c can thus give rise to a large uncertainty on the extrapolated $\mu_0 H_{c2,||}(T = 0)$ value.

Figure 11 shows similar extrapolations of $H_{c2,||} - t$ curves at different V_g values in sample A based on modified WHH models. We remark that the extrapolated $H_{c2,||}(T = 0 \text{ K})$ values in both samples A and B are clearly discernible for different V_g values, making it possible to study the V_g dependence of $H_{c2,||}(T = 0 \text{ K})$ and thus the τ_d dependence of τ_{SO} .

1. Modified WHH model in the 2D limit with a strong SOC

In order to distinguish the orbital and spin components, the modified WHH equation can be rewritten in the following concise form [52] in the nature units:

$$\ln t = \psi\left(\frac{1}{2}\right) - \frac{\alpha_+}{2\gamma_a} \psi\left(\frac{1}{2} + \frac{\epsilon + 2\alpha_-}{4\pi T}\right) + \frac{\alpha_-}{2\gamma_a} \psi\left(\frac{1}{2} + \frac{\epsilon + 2\alpha_+}{4\pi T}\right), \quad (\text{C2})$$

where $\alpha_{\pm} = \beta \pm \gamma_a$, $\beta = 1/3\tau_{SO}$, and $\gamma_a = (\beta^2 - I^2)^{1/2}$. $I \equiv \mu_B B_c$ and $\epsilon(\theta) = D[2eB_c \cos\theta + \frac{1}{3}(deB_c \sin\theta)^2]$. For $T \approx T_c$, both I and ϵ are small, and Eq. (C2) becomes $(1 - \frac{T}{T_c}) \approx \psi'(\frac{1}{2}) \frac{\epsilon}{4\pi T_c}$. We then obtained an expression of

$$\epsilon = \frac{8T_c}{\pi} \left(1 - \frac{T}{T_c}\right), \quad (\text{C3})$$

where we used $\psi'(\frac{1}{2}) = \pi^2/2$. The above formula well reproduces the well-known GL equation shown in Eq. (3) of the main text, which is only valid for $T \approx T_c$. In the other extreme case of $T \approx 0$, the argument of the digamma function ψ diverges, and $\psi(z) \approx \ln(z) - \frac{1}{2z}$. By further taking the 2D

FIG. 12. (a) Magnetoconductance data, normalized to the quantum conductance of $e^2/\pi h$, with applied perpendicular fields at different V_g ranging from -20 V to $+50$ V in sample A. The dashed lines are fitting curves based on the weak localization model with SOC. (b) The magnetoconductance data with fields applied parallel to the LAO/STO interface. (c) Extracted spin-orbit fields H_{SO} and inelastic fields H_i as a function of $1/R_{\square}$ from the weak localization model. (d) The spin-orbit scattering lifetime τ_{SO} (solid triangles) and the $1/H_{SO}$ (open triangles) versus $1/\tau_d$.

limit with d going to zero, Eq. (C2) reduces to

$$0 = \psi\left(\frac{1}{2}\right) - \frac{\alpha_+}{2\gamma_a} \ln\left(\frac{2\alpha_-}{2\pi T_c}\right) + \frac{\alpha_-}{2\gamma_a} \ln\left(\frac{\alpha_+}{2\pi T_c}\right). \quad (\text{C4})$$

From Eq. (C4), it is now clear that the upper critical field is governed by the spin component only, and it is consistent with Eq. (37) in Ref. [12] of the main text (the KLB model) in the same limiting case. Considering a strong SOC with a large β and $\beta \gg I$, $\gamma_a \approx \beta(1 - \frac{I^2}{2\beta^2})$, Eq. (C4) can then be expressed as $0 = \psi(\frac{1}{2}) - \ln(\frac{I^2}{4\pi T_c \beta})$. Finally, the parallel upper critical field at $T = 0$ K can be obtained and described by the following simple form of

$$[\mu_B B_{c2,\parallel}(T = 0 \text{ K})]^2 = \left(\frac{\pi T_c}{3\tau_{SO}\Gamma}\right), \quad (\text{C5})$$

where we used $e^{\psi(\frac{1}{2})} = \frac{1}{4\Gamma}$ and $\Gamma = 1.78$. We note that Eq. (C5) well recovers Eq. (4) in the main text.

2. KLB model in the 2D limit with a strong SOC

The upper critical field $H_{c2}(\theta, T)$ in a layered superconductor was calculated by Klemm, Luther, and Beasley with the inclusion of the spin paramagnetism and SOS in the pair breaking formula, where θ is the angle between the applied

magnetic field and the direction perpendicular to the normal of the superconducting layer. In the dirty limit with a strong spin-orbit coupling, the solution of parallel upper critical field at $T = 0$ K, $H_{c2}(\pi/2, 0)$ can be obtained from the following equation in the nature units ($\hbar = c = k_B = 1$) of

$$f(h_0) + \left[\frac{3}{2} \frac{\tau_{SO}\mu_B^2}{e^2 s^2 D}\right] h_0^2 = \frac{\Delta(0)}{T_c r}, \quad (\text{C6})$$

where $h_0 = esH_{c2}(\pi/2, 0)\sqrt{\frac{2D}{rT_c}}$ is the reduced parallel critical field at zero temperature, s is a characteristic length along the perpendicular direction of the layer, and $f(h_0)$ is a dimensionless function giving a value between 0 and 1. D and r are the diffusion constant and the interlayer coupling strength parameter, respectively. It is noteworthy that the second term on the left hand side of Eq. (C6) signifies the contribution from the Pauli paramagnetism and SOS, which would become dominant as long as the interlayer coupling strength is sufficiently small. In a two-dimensional limit with r approaching zero, the zero temperature parallel critical field $\mu_0 H_{c2,\parallel}(T = 0)$ can be readily deduced from Eq. (C6) and expressed in a remarkably simple form as $\mu_0 H_{c2,\parallel}(T = 0) = \sqrt{\frac{\pi \hbar k_B T}{3\Gamma \mu_B^2 \tau_{SO}}}$.

FIG. 13. Magnetoconductance data, normalized to the quantum conductance of $e^2/\pi h$, with magnetic field applied perpendicular and parallel to the LAO/STO interface are shown in (a) and (b), respectively.

APPENDIX D: WEAK LOCALIZATION FITTINGS TO THE MAGNETOCONDUCTANCE (MC) DATA

According to the calculation by Punnoose [53], the quantum correction to the MC due to weak localization effect with the inclusion of SOC can be expressed as

$$\frac{\Delta\sigma(H)}{e^2/\pi h} = 2\bar{\psi}\left(\frac{H}{H_i + H_{SO}}\right) + \bar{\psi}\left(\frac{H}{H_i + 2H_{SO}}\right) - \bar{\psi}\left(\frac{H}{H_i}\right), \quad (D1)$$

where inelastic field H_i and spin-orbit field H_{SO} are the only two fitting parameters and $e^2/\pi h$ is the quantum conductance unit. $\bar{\psi}(x)$ function is defined as $\bar{\psi}(x) \equiv \ln(x) + \psi(\frac{1}{2} + \frac{1}{x})$ and $\psi(x)$ is the digamma function. It is useful to note that $\bar{\psi}(x) \rightarrow 0$ as $x \rightarrow 0$, and $\bar{\psi}(x)$ monotonically increases with increasing x values. We note that more general results with inclusion of both SOC and Zeeman splitting effects were carried out by Maekawa and Fukuyama (MF) [6], and it can be shown that, by setting the Zeeman splitting term to zero, the MF formula reduces to the Punnoose formula shown in Eq. (D1). The MF model also predicted a negative MC in weak fields applied parallel to the interface if only nonmagnetic impurity scatterings are considered. We, therefore, restricted our weak localization fittings on the MC data to the low field regime up to about 2 T when using Eq. (D1).

Figure 12 shows the MC data, normalized to the quantum conductance, with a magnetic field applied perpendicular to the LAO/STO interface. The weak localization fittings using Eq. (D1) are shown as dashed lines in the figure, which gives excellent agreement with the experimental data. The MC with fields parallel to the LAO/STO interface also gives a negative

MC as expected by the MF model. The extracted inelastic field H_i and spin-orbit field H_{SO} as a function of $1/R_{\square}$ are shown in Fig. 12(c), where the corresponding spin-orbit lifetime τ_{SO} can be extracted. The resulting τ_{SO} was found to be proportional to $1/\tau_d$ as shown in Fig. 12(d), inferring a Dyakonov-Perel type spin relaxation due to the Rashba-type SOC, which is fully consistent with several earlier reports on determination of τ_{SO} from the weak localization model. On the other hand, the weak localization fittings to the MC data in sample B, shown in Fig. 13(a), turn out to be very challenging and difficult. The normalized MC gives huge negative MC with more than two orders of magnitude larger than the quantum conductance, suggesting a significant contribution to the MC due to other effects, such as the two-band effect. It is, therefore, difficult to extract the weak localization contribution to MC, typically of the order of quantum conductance, from a huge background from two-band effects. On the other hand, the MC with fields parallel to the LAO/STO interface also indicates a positive MC, which indicates the weak localization effect may not play an important role to the observed MC in sample B.

APPENDIX E: TWO BAND MODEL FITTINGS TO MAGNETOTRANSPORT DATA IN SAMPLE B

Figure 14(a) shows the Hall conductivity (σ_{xy}) and magnetoconductivity (σ_{xx}) data in sample B at $T = 0.8$ K for different V_g values. The long dashed lines in Fig. 14(a) are the curves obtained from the fitting parameters based on the two band model with four fitting parameters of (n_1, μ_1) and (n_2, μ_2) , where n_1 (n_2) and μ_1 (μ_2) are the sheet

FIG. 14. (a) Upper panel and lower panel show the field dependent Hall conductivity (σ_{xy}) and magnetoconductivity (σ_{xx}) in sample B, respectively. The dashed lines are the fitting curves based on the two band model. The electron transport lifetimes of τ_{d1} and τ_{d2} are calculated from the Hall mobilities extracted from the two band model fittings. The transport lifetime in sample A and sample B as a function of the corresponding $1/R_{\square}$ is shown in (b).

density and Hall mobility for band 1(2), respectively. By using effective mass $m^* = m_e$ and $6 \times m_e$ for band 1 and band 2, respectively, the corresponding electron transport lifetime for band 1 (τ_{d1}) and for band 2 (τ_{d2}) can be determined.

Figure 14(b) plots the transport lifetime τ_d as a function of the corresponding $1/R_{\square}$ in sample A and sample B. Apparent monotonic and linear dependence between τ_d and $1/R_{\square}$ is demonstrated.

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