Bandgap-assisted quantum control of topological edge states in a cavity

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Quantum matter with exotic topological order has potential applications in quantum computation. However, in present experiments, the manipulations on topological states are still challenging. We here propose an architecture for optical control of topological matter. We consider a topological superconducting qubit array with a Su-Schrieffer-Heeger (SSH) Hamiltonian which couples to a microwave cavity. Based on parity properties of the topological qubit array, we propose an optical spectroscopy method to observe the topological phase transition, i.e., edge-to-bulk transition. This new method can be achieved by designing cavity-qubit couplings. A main purpose of this work is to understand how the topological phase transition affects light-matter interaction. We find that the topological bandgap plays an essential role on this issue. In the topological phase, the resonant vacuum Rabi splitting of degenerate edge states coupling to the cavity field is protected from those of bulk states by the bandgap. In the dispersive regime, the cavity induced coupling between edge states is dominant over couplings between edge and bulk states, due to the topological bandgap. As a result, quantum interference between topological edge states occurs and enables single-photon transport through boundaries of the topological qubit array. Our work may pave a way for topological quantum state engineering.

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Introduction. Characterization of topological matter is a crucial issue in condensed matter physics [1]. A hallmark of topological phases is the existence of topological invariants, e.g., Chern number and Zak phase, defined on energy bands of the systems [2–4]. According to edge-bulk correspondence, topological states emerge in the bandgaps and give rise to many novel transport phenomena [5,6]. Due to their insensitivity to local decoherence, topological states have prospective applications in quantum information processing. In particular, zero-dimensional edge states, e.g., Majorana bound states, are candidates to realize topological quantum computation [7–9] and have been observed experimentally in a range of materials, including semiconductor nanowires [10–13], ferromagnetic atomic chains [14], and iron-based superconductors [15]. However, the manipulations of edge states are rather challenging, for which reason topological materials with large bandgaps are explored [16–18].

Cavity quantum electrodynamics (QED), in which quantized electromagnetic fields are strongly coupled to an atomic system, was originally used for studying fundamentals of atomic physics and quantum optics [19]. With superb control of quantum states, cavity QED is now applied to quantum information processing, in which the cavity field is proposed

for manipulating, measuring, or transferring quantum states of atomic systems [20,21]. Circuit QED, in which a microwave transmission line resonator acting as a cavity is coupled to superconducting quantum circuit, is an extension of cavity QED [22,23]. The on-chip circuit QED system is not only a good platform for studying fundamental physics in the microwave regime [24], but also a very promising candidate for realizing quantum computation and simulations [25–42]. In particular, one-dimensional (1D) qubit arrays have been used to explore many-body localization [38,39], Mott insulators of photons [41], and correlated quantum walks [42]. Moreover, superconducting qubit systems also offer hope to simulate topological matter [43–47].

In this work, we study the interaction between a microwave cavity and a topological superconducting qubit array, described by the Su-Schrieffer-Heeger (SSH) model [48], which has been experimentally realized [49–51]. Differently from the electronic transport detections of Majorana fermions [13,52–54], the cavity spectroscopy method we study here unveils the edge states and topological phase transition with proper cavity-qubit couplings. We pinpoint the role of topological bandgap in quantum manipulation of edge states, especially for small qubit arrays.

Spectroscopic characterization of a topological qubit array by a cavity. As schematically shown in Fig. 1(a), we study that a 1D topological qubit array [55], with SSH interactions, is placed inside a cavity. Considering rapid progress and flexible chip designs of superconducting quantum circuits, we here assume that the SSH array with N unit cells, formed by $2N$ superconducting qubits [44], e.g., Xmon qubits [35,38], is coupled to a microwave transmission line resonator, as shown

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FIG. 1. (a) Schematic diagram for the SSH qubit array with dimerized couplings t_1 and t_2 placed inside a cavity. Red and blue balls represent A and B qubits, respectively. (b) Design of (a) with superconducting qubit circuits where the couplings t_1 and t_2 are tunable. The microwave transmission line resonator acting as a cavity is coupled to the qubits. (c) The first unit cell with tunable qubit-qubit coupling via a Josephson junction. Φ_{ext} is an external magnetic flux bias that controls the coupling (see Supplemental Material [64]). (d) Reflection spectrum of the qubit array with eight qubits. The frequencies of qubits and driving field are respectively ω_0 and ω_l ; $\omega_{\pm} = \omega_0 \pm 2t_0$. The reflection at $\varphi = 0.25\pi$ is shown in the right panel. Here we consider cavity-qubit couplings $\mathbf{g} = g_0(-1, 1, 1, 1, -1, 1, 1, 1)$ with $g_0/2\pi = 5$ MHz. Other parameters are $\omega_0/2\pi = 6$ GHz, $t_0/2\pi = 100$ MHz, $\kappa/2\pi = 10$ MHz, $\gamma_{iA} = \gamma_{iB} = 20 \times 2\pi$ kHz. The white-dashed curves represent the energy spectrum of the qubit array. (e) Cavity mediated couplings between qubits, denoted by the orange lines, in the dispersive regime.

in Fig. 1(b). The Hamiltonian of the whole system is

$$H/\hbar = \omega_c \hat{a}^\dagger \hat{a} + \sum_{i=1, \mu=A,B}^{i=N} (\omega_0 \sigma_{i\mu}^+ \sigma_{i\mu}^- + g_{i\mu} \sigma_{i\mu}^+ \hat{a} + g_{i\mu}^* \hat{a}^\dagger \sigma_{i\mu}^-) + \sum_{i=1}^{i=N} (t_1 \sigma_{iA}^+ \sigma_{iB}^- + t_2 \sigma_{i+1A}^+ \sigma_{iB}^- + \text{H.c.}), \quad (1)$$

where ω_c and ω_0 are the frequencies of the cavity and qubits, respectively. The parameter $g_{i\mu}$ denotes the coupling strength of the cavity to the qubit μ in the i th unit cell. The operators of qubits A and B at the i th unit cell are $\sigma_{iA}^+ = |A_i\rangle\langle\alpha_i|$ and $\sigma_{iB}^+ = |B_i\rangle\langle\beta_i|$ with the ground (excited) states $|\alpha_i\rangle$ ($|A_i\rangle$) and $|\beta_i\rangle$ ($|B_i\rangle$), respectively. The second line in Eq. (1) represents the SSH interaction Hamiltonian with tunable coupling strengths t_1 and t_2 , which could be implemented in different ways in superconducting qubit circuits [38,39,56–63]. We here assume that controllable coupling between qubits is realized via a Josephson junction, which is biased by an external magnetic flux [62,63], as shown in Fig. 1(c). The coupling strengths are $t_1 = t_0(1 - \cos \varphi)$ and $t_2 = t_0(1 + \cos \varphi)$ with a tunable parameter φ [64]. Note that the topological phase transition takes place at $\varphi = \pi/2$ ($t_1 = t_2$). The cases for $t_1 < t_2$ and $t_1 > t_2$ correspond to topological and nontopological phases, respectively.

To measure the topological qubit array, we assume that a weak probe field with the strength η and the frequency $\omega_l = \omega_c$ is applied to the qubit array via the cavity. Thus, the dynamics of the reduced density matrix ρ of the whole system can be described by the master equation

$$\dot{\rho} = -\frac{i}{\hbar}[H + i\hbar\eta(\hat{a}^\dagger e^{-i\omega_l t} - \hat{a}e^{i\omega_l t}), \rho] + \kappa \mathcal{D}[\hat{a}]\rho + \sum_{i=1, \mu=A,B}^{i=N} \gamma_{i\mu} \mathcal{D}[\sigma_{i\mu}^-]\rho. \quad (2)$$

Here, κ is the decay rate of the cavity, and γ_{iA} and γ_{iB} are the decay rates of the qubits A and B at the i th unit cell, respectively. The dissipation superoperator is defined as $\mathcal{D}[\hat{O}]\rho = \hat{O}\rho\hat{O}^\dagger - \frac{1}{2}\{\hat{O}^\dagger\hat{O}, \rho\}$. As shown in Fig. 1(d), the topological phase transition can be observed from the reflection of the probe field with special couplings between qubits and the cavity, which can be realized via controllable couplers [64–66]. The reflection spectrum is obtained by solving the master equation in Eq. (2) with H given in Eq. (1). Topological phases have recently been demonstrated in superconducting qubit circuits [50,67–74]. However, the operations on topological states have not been implemented. Below we show that the topological bandgap is helpful for manipulations of edge states.

Vacuum Rabi splitting between the cavity and edge modes.

We consider the single-excitation subspace consisting of $|\mathcal{A}_i\rangle = \sigma_{iA}^+|G\rangle$ and $|\mathcal{B}_i\rangle = \sigma_{iB}^+|G\rangle$ with $|G\rangle$ being the ground state of the qubit array. We rewrite the states $|\mathcal{A}_i\rangle$ and $|\mathcal{B}_i\rangle$ via eigenstates $|\Psi_j\rangle$ in the single-excitation subspace of the qubit array [47], i.e., $|\mathcal{A}_i\rangle = \sum_{j=1}^{2N} \xi_{2i-1,j} |\Psi_j\rangle$ and $|\mathcal{B}_i\rangle = \sum_{j=1}^{2N} \xi_{2i,j} |\Psi_j\rangle$. Here, $j = 1, \dots, 2N$ is the label of the j th eigenstate from the lowest to highest energies. Then, the Hamiltonian in Eq. (1) can be rewritten as [64]

$$\tilde{H}/\hbar = \sum_{j=1}^{2N} \omega_j \Psi_j^+ \Psi_j^- + \omega_c \hat{a}^\dagger \hat{a} + \sum_{j=1}^{2N} (\tilde{\xi}_j \Psi_j^+ \hat{a} + \text{H.c.}), \quad (3)$$

with $\Psi_j^+ = |\Psi_j\rangle\langle G|$, and the eigenfrequency ω_j corresponding to the eigenstate $|\Psi_j\rangle$. For homogeneous cavity-qubit couplings, i.e., $g_{i\mu} \equiv g_0$, the coupling strength between the cavity and the j th eigenmode is $\tilde{\xi}_j = \xi_j g_0$ with coupling coefficient $\xi_j = \sum_i (\xi_{2i-1,j} + \xi_{2i,j})$. The analytical expressions for ξ_j can be found in Ref. [64]. Hereafter, we call Ψ_j^+ bulk or edge modes when $|\Psi_j\rangle$ are bulk or edge eigenstates. In Fig. 2(a), we show $|\xi_j|$ for the qubit array size $2N = 36$. The bulk modes have different couplings to the cavity because of their parities of wave functions. The odd-parity bulk states, i.e., $\xi_{2i-1,j} = -\xi_{2N+2-2i,j}$ and $\xi_{2i,j} = -\xi_{2N+1-2i,j}$, have zero coupling to the cavity. However, the even-parity bulk states, i.e., $\xi_{2i-1,j} = \xi_{2N+2-2i,j}$ and $\xi_{2i,j} = \xi_{2N+1-2i,j}$, are coupled to the cavity [64]. Two edge states have equal coupling strengths to the cavity, i.e., $\xi_{18} = \xi_{19}$.

In Fig. 2(b), we show the energy splitting produced by the cavity-qubit couplings. We assume that the qubit frequency is $\omega_0 = 2\pi \times 6$ GHz. The anticrossing near the driving frequency $\omega_l = 2\pi \times 6$ GHz represents the Rabi splitting due to the resonant interaction between the cavity and edge modes. We also study the disorder effect and find that the Rabi

FIG. 2. (a) The coupling strength between the cavity and eigenmodes of the SSH qubit array with $N = 18$ unit cells (i.e., 36 qubits) for homogeneous cavity-qubit couplings. The number j is the index of eigenmodes, and the middle two points (black dots) with $j = 18, 19$ are edge states. The bulk states with odd (blue dots) and even (red dots) numbers have zero and nonzero couplings to the cavity, respectively. The inset represents the coupling strengths for $j < 18$. (b) Vacuum Rabi splitting between edge states and the cavity for $N = 18$ unit cells. The lower anticrossing shows the coupling between the cavity and the edge states. The upper one is the coupling of the bulk states to the cavity. (c) The coupling strengths ξ_N and ξ_{N+1} , denoted by $\xi_{N/N+1}$, between two edge modes and the cavity with different numbers N of unit cells. Here we consider $\varphi = \pi/5$. Other parameters are the same as in Fig. 1(d).

splitting of edge states is robust to disorder [64]. If the frequency of the cavity is at resonance for the transitions from the ground to bulk states with high energies, a large anticrossing, as shown in upper part of Fig. 2(b), is produced around $\omega_l = 2\pi \times 6.2$ GHz. The topological bandgap of the SSH Hamiltonian protects the Rabi splitting of edge states. In Fig. 2(c), ξ_N and ξ_{N+1} , i.e., the coupling coefficients between the cavity and edge modes, are plotted versus the unit cell number N . When the qubit array is small, e.g., $N \leq 14$, the edge states overlap with each other and form hybridized edge states with odd and even parities. The edge state with odd parity decouples from the cavity. With the increase of the unit cell number, two edge states are far separated from each other. The localized edge states lose parity, thus they have the same coupling strength to the cavity.

We study the relation between the coupling coefficient ξ_N (ξ_{N+1}) and φ in Fig. 3. For example, when the qubit array has $N = 6$ unit cells, the coupling strengths are described by the black-solid and blue-dashed curves. When φ is small, the edge states are unhybridized and have the same couplings to the cavity. However, the increase of φ leads to hybridized edge states with even and odd parities. We find that the hybridized regime becomes smaller with the increase of the system size, e.g., $N = 18$ (green-solid and blue-dash-dotted curves) and $N = 78$ (red-solid and orange-dotted curves) as we show here. We also find that in the topological phase (i.e., $0 \leq \varphi < \pi/2$), the hybridized edge state with even parity has the coupling strength $\tilde{\xi}_+ = \sqrt{2 \cos \varphi} g_0$, which is independent of system size [64]. The couplings for separated edge states

FIG. 3. The couplings between the cavity and edge states (black-solid and blue-dashed curves) for the qubit array with $N = 6$ unit cells. We also present the coupling strengths for $N = 18$ ($N = 78$) by the green-solid and blue-dash-dotted (red-solid and orange-dotted) curves. The inset shows the coefficient ϵ_{2N} of the collective coupling $\tilde{\xi}_j = \epsilon_j \sqrt{2N} g_0$ with $j = 2N$.

are $\tilde{\xi}_L = \tilde{\xi}_R = \sqrt{\cos \varphi} g_0$. Because the coupling strength of $2N$ noninteracting qubits to the cavity is $\sqrt{2N} g_0$ [75], we here assume that the coupling strength of $2N$ interacting qubits to the cavity is $\tilde{\xi}_j = \epsilon_j \sqrt{2N} g_0$ with the rescaling factor ϵ_j given in Eq. (S29) of the Supplemental Material [64]. As shown in the inset of Fig. 3, the rescaling factor ϵ_{2N} is tuned by φ .

Topological-bandgap-protected coupling between two edge modes. When the cavity is far detuned from qubits, i.e., $g_0 \ll \Delta_0$ (let $\Delta_0 = \omega_0 - \omega_c$), virtual-photon-mediated interactions among qubits g_0^2/Δ_0 can be obtained [39,61], as shown in Fig. 1(e). In terms of eigenmodes of the qubit array, the effective coupling strengths between j th and k th eigenmodes are

$$J_{jk} = \frac{\tilde{\xi}_j \tilde{\xi}_k}{2} \left(\frac{1}{\Delta_j} + \frac{1}{\Delta_k} \right), \quad j, k \in [1, \dots, 2N], \quad (4)$$

with $\Delta_{j/k} = \omega_{j/k} - \omega_c$, which depends on system size $2N$ for the coupling between bulk modes, or between bulk and edge modes, due to the size-dependent cavity-bulk coupling. However, the edge-mode coupling is independent of the size, i.e.,

$$J = \cos \varphi \frac{g_0^2}{\Delta_0}, \quad (5)$$

which is protected by the topological bandgap, as schematically shown in Fig. 4(a). We note that the energy splitting induced by hybridization of edge states is assumed to be negligibly small when Eq. (5) is derived. In Figs. 4(b) and 4(c), we show the excitation dynamics of the left-edge qubit (qubit A in the first unit cell is excited initially) in the topological phase with $\varphi = 0.1\pi$ and 0.3π , respectively. Figures 4(b) and 4(c) clearly show the population exchange between two edge states produced by the edge-mode coupling. In fact, the finite topological bandgap makes the effective couplings between edge modes different from Eq. (5) [64]. In Figs. 4(d) and 4(e) with $\varphi = 0.5\pi$ and 0.9π , the excitation propagates through the array and is bounded by the boundaries. In the nontopological phase, excitation propagates along the qubit array with low velocity [see Fig. 4(e)], which is yielded by the smooth energy bands with large gap.

FIG. 4. (a) Edge-state coupling protected by the topological bandgap. (b)–(e) Excitation dynamics in the cavity-coupled SSH qubit array for $\varphi = 0.1\pi, 0.3\pi, 0.5\pi$, and 0.9π . Horizontal and vertical axes denote qubits and time, respectively. The virtual-photon-mediated interactions among qubits are assumed to be $0.1g_0$. The number of unit cells is $N = 6$, and other parameters are the same as in Fig. 1(d).

Quantum interference induced by topological state coupling. As schematically shown in Fig. 5(a), we further consider that the left-edge qubit A_1 is coupled to a waveguide, in which a probe field passes through. The left-edge qubit mainly contributes to the left edge state. Then the left edge state can be driven by fields passing through the waveguide. The single-photon transmission amplitude can be given as [64]

$$t = \frac{(i\Delta_p - \frac{\gamma_L}{2})(i\Delta_p - \frac{\gamma_R}{2}) + J^2}{(i\Delta_p - \frac{\gamma_L + \Gamma_L}{2})(i\Delta_p - \frac{\gamma_R}{2}) + J^2}, \quad (6)$$

and the susceptibility $\chi = -i(t - 1)/t$ is

$$\chi = \frac{\Gamma_L(\Delta_p + i\frac{\gamma_R}{2})}{2J^2 - 2(\Delta_p + i\frac{\gamma_L}{2})(\Delta_p + i\frac{\gamma_R}{2})}, \quad (7)$$

where Δ_p is the detuning between the probe field and the left edge state. As schematically shown in Fig. 5(b), the parameters γ_L and γ_R are the decay rates for left and right edge states, and Γ_L comes from the coupling between the left edge qubit and the waveguide.

The transmission of the probe field as a function of the detuning Δ_p is shown in Fig. 5(c) with $J = 0$ and $0.035\Gamma_L$. When there is no coupling between edge states, the transmission vanishes at the resonance. However, when there is a coupling between two edge states, a transparency window for the probe field appears. This can be further confirmed by the susceptibility, which is plotted as a function of the detuning Δ_p in Fig. 5(d) in the parameter regime $J \ll \Gamma_L$. This transparency window, in which the distance between two peaks is less than $2J$, is from the quantum interference as shown in Fig. 5(e), which is similar to electromagnetically induced transparency [76]. However, in the parameter regime $J > \Gamma_L$, the transparency window, in which the distance between two peaks equals to $2J$, is from the strong-coupling-induced energy splitting, which is similar to Autler-Townes splitting [77].

Discussions and conclusions. In summary, we study cavity control of topological edge states in SSH qubit arrays. We show that the couplings between cavity and edge modes are protected by the topological bandgap, and topological phase

FIG. 5. (a) Coupling between a waveguide and the left-edge qubit in the array. (b) Superatom model of (a). Here γ_L and γ_R are respective decays of left and right edge states, and Γ_L is the waveguide induced decay rate of the left edge state. (c) Transmission of probe light for edge-state coupling $J = 0$ ($J = 0.035\Gamma_L$) is represented by the red-dashed (blue-solid) curve. (d) Real (solid) and imaginary (dashed) parts of the susceptibility. In both (c) and (d), the red and blue curves are for $J = 0$ and $J = 0.035\Gamma_L$, respectively. (e) The imaginary part of susceptibility can be decomposed into two Lorentzian peaks. In these three figures, we consider $\gamma_L = 0.15\Gamma_L$, $\gamma_R = 5 \times 10^{-4}\Gamma_L$.

transitions can be probed via the reflection spectrum of the probe field through the cavity. Due to the bandgap, Rabi splitting of edge modes can be observed. When the cavity is largely detuned from the edge modes, long-range coupling between two edge states can be realized. It results in quantum interference for emissions from two edge states when a qubit at the edge of the array is coupled to a waveguide. Meanwhile, we find that topological properties can also be detected by the cavity even for a small system, where the edge states are hybridized.

We also discuss experimental feasibility via a superconducting qubit array coupled to a microwave transmission line resonator [44]. The tunable couplings between qubits [62] or between qubits and the resonator [65] make our proposal more experimentally accessible. We also analyze the effects of disorder and decay on the results. We find that both the Rabi splitting and excitation dynamics produced by edge-state coupling are robust to the disorder [64]. We also find that with current coherence time $20\text{--}40\ \mu\text{s}$ in 1D arrays [41,42], the Rabi splitting, excitation dynamics, and quantum interference induced by the edge-state coupling should be observed. We mention that our approach can also be applied to other topological quantum systems. Our study here might have potential applications in quantum information and quantum optics.

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