Optimization of N_2^+ lasing through population depletion in the $X^2\Sigma_g^+$ state using elliptically modulated ultrashort intense laser fields

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We generate unidirectional coherent radiations at 391 and 428 nm corresponding to the $B^2\Sigma_u^+-X^2\Sigma_g^+$ (0,0) and (0,1) emission of N_2^+ by irradiating neutral N_2 with an elliptically modulated near-IR ultrashort laser field generated using a quarter-wave plate (QWP) with the different orders, and reveal that the lasing intensity at 391 nm becomes maximum at a specific orientation of the fast axis of QWP with respect to the polarization direction of the input laser field. We reproduce this observation by numerical calculations and show that the lasing intensity can be optimized by adjusting the temporal separation between the two polarization components of the elliptically modulated laser pulse and their relative amplitudes so that the inverted population between the $B^2\Sigma_u^+$ and $X^2\Sigma_g^+$ states can be achieved efficiently and the population in the $X^2\Sigma_g^+$ ($v'' = 0$) level is further depleted in the latter part of the laser pulse by the $A^2\Pi_u-X^2\Sigma_g^+$ transition.

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When intense near-infrared ultrashort laser pulses are focused in air, unidirectional and coherent light at specific wavelengths in the visible and ultraviolet wavelength regions can be generated. This phenomenon, called “air lasing,” has attracted researchers in the past two decades not only in the research field of remote sensing but also in the research field of atomic, molecular, and optical physics [1–6]. Among the air lasing, the lasing at 391 and 428 nm, corresponding, respectively, to the $B^2\Sigma_u^+-X^2\Sigma_g^+$ ($v', v'' = (0, 0)$ and $(0, 1)$) emission lines of N_2^+ , has attracted special interest because the mechanism of the efficient preparation of N_2^+ in the $B^2\Sigma_u^+$ state, leading to the lasing action, has been mysterious [3,7–20].

In 2015, by introducing an idea that ion species created in an intense laser field are suddenly exposed to an intense electric field of light, which is called a sudden turn-on model, we showed that the electronically excited $B^2\Sigma_u^+$ state can efficiently be populated so that the population is inverted between the $B^2\Sigma_u^+$ state and the $X^2\Sigma_g^+$ state, leading to the lasing at 391 nm [12] and that the $B^2\Sigma_u^+-X^2\Sigma_g^+$ lasing intensity can be enhanced by the post-ionization coupling

between the $A^2\Pi_u$ state and the $X^2\Sigma_g^+$ state, through which the population in the $X^2\Sigma_g^+$ state of N_2^+ is depleted by the population transfer to the $A^2\Pi_u$ state [12]. The involvement of the $A^2\Pi_u$ state in the air lasing was also pointed out by Yao *et al.* [13].

The creation of population inversion in a two-level system interacting with a sudden turn-on laser pulse was later examined by the quasistationary Floquet theory [21], and the efficient creation of the population inversion between the $B^2\Sigma_u^+$ state and the $X^2\Sigma_g^+$ state of N_2^+ was also interpreted well by the quasistationary Floquet theory on the basis of the $B^2\Sigma_u^+-X^2\Sigma_g^+-A^2\Pi_u$ three-state coupling scheme [22].

More recently, it was demonstrated that the intensity of N_2^+ lasing at 391 nm can be increased by two orders of magnitude when the polarization state within an ultrashort laser pulse is modulated so that the $A^2\Pi_u-X^2\Sigma_g^+$ coupling in the latter part of the laser pulse is enhanced [18]. Furthermore, using a pair of few-cycle laser pulses whose polarization directions are perpendicular to each other, it was shown that the oscillations with the periods corresponding to the $A^2\Pi_u-X^2\Sigma_g^+$ rovibronic transitions appear in the intensity of the $B^2\Sigma_u^+-X^2\Sigma_g^+$ lasing at 391 nm [20].

Indeed, these experimental results of the N_2^+ lasing are consistent with the $B^2\Sigma_u^+-X^2\Sigma_g^+-A^2\Pi_u$ three-state coupling model composed of two crucial stages: (i) the preparation of the inverted population between the $B^2\Sigma_u^+$ state and the $X^2\Sigma_g^+$ state of N_2^+ created at the most intense part of the laser pulse through the sudden turn-on mechanism and (ii) the depletion of the $X^2\Sigma_g^+$ state population by the $A^2\Pi_u-X^2\Sigma_g^+$ coupling in the later stage. Therefore, if we can vary the relative contributions of these two stages by modifying the

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FIG. 1. (a) The preparation of the population inversion between the $B^2\Sigma_u^+$ and $X^2\Sigma_g^+$ states of N_2^+ by the E_y component of the elliptically modulated laser pulse and the coupling between the $A^2\Pi_u$ and $X^2\Sigma_g^+$ states by the E_x component. (b) The amplitude of the E_x and E_y components of the laser electric field when $n = 10$ and $\theta = 20^\circ$, where the x and y directions correspond to the directions perpendicular and parallel to the fast axis (ordinary light axis) of the quarter-wave plate.

temporal variation of an ultrashort pulsed laser field, we are able to find a clue for interpreting the polarization dependence of air lasing, on which no consensus has been reached so far [14–17], based on the three-state coupling scheme shown in Fig. 1(a).

In the present study, we modulate the polarization state of a pump laser pulse using multiple-order quarter-wave plates [see Fig. 1(b)] so that we can optimize the relative contributions of the two stages to maximize the intensity of the $B^2\Sigma_u^+ - X^2\Sigma_g^+$ lasing, which is induced by a probe laser pulse acting as the external seeding, and confirm that the $B^2\Sigma_u^+ - X^2\Sigma_g^+ - A^2\Pi_u$ three-state coupling model gives us a solid guideline to control of the $B^2\Sigma_u^+ - X^2\Sigma_g^+$ lasing of N_2^+ . Furthermore, on the basis of the three-state coupling model, we interpret the origin of the significantly different ellipticity dependences of the two lasing lines of N_2^+ at 391 and 428 nm.

We create a pump laser pulse by making a vertically polarized pulse pass through a quarter-wave plate (QWP) with the order of n . We can modulate the ellipticity of the pump laser pulse by changing the angle θ between the polarization direction of the linearly polarized input laser pulse and the fast axis of QWP as well as by changing n ($= 0, 3, 7$, and 10), whose electric field can be expressed as

$$E_n(\theta, t) = E_0 \cos(\theta) g\left(t - \frac{T_n}{2}\right) \mathbf{e}_o + E_0 \sin(\theta) g\left(t + \frac{T_n}{2}\right) \mathbf{e}_e, \quad (1)$$

where E_0 is the amplitude of the input laser pulse, and $\mathbf{e}_o$ and $\mathbf{e}_e$ are the unit vectors parallel (the ordinary light) and perpendicular (the extraordinary light) to the fast axis of the n -order QWP, respectively, and $T_n = \frac{2\pi(n+1/4)}{\omega}$ is the delay time of the extraordinary light with respect to the ordinary light induced by the QWP having the order n , and $g(t) = e^{-2\ln 2(t^2/\tau^2)} \sin(\omega t + \varphi)$ is the Gaussian function representing the carrier envelope having the FWHM τ , the carrier frequency ω , and the carrier envelope phase φ . As an example, the amplitude as well as the E_x and E_y components of the electric field produced by a tenth-order QWP with $\theta = 20^\circ$ is shown in Fig. 1(b).

In order to secure the external seeding scheme and avoid the self-seeding, we attenuate the pulse energy of linearly polarized near-IR femtosecond laser pulses (40 fs, 800 nm), generated by a Ti:sapphire femtosecond laser (Spectra Physics, Spitfire ACE), to 0.7 mJ/pulse using a half-wave plate and a polarizer so that no N_2^+ lasing signal can be observed when the external seed pulses are blocked. The details of the pump-probe scheme can be seen in [18]. Briefly, the linearly polarized external seed pulses are generated in a β -barium borate crystal and the pump and the seed pulses are collinearly combined and focused into a chamber filled with a pure N_2 gas at 10 mbar. We adjust the delay time between the pump pulse and the seed pulse to obtain the most intense lasing signal for the respective cases of the QWPs having the different orders. The generated UV light is measured by a spectrometer (Andor, Shamrock) equipped with an ICCD camera (Andor iStar).

When linearly polarized pump pulses (800 nm) and linearly polarized seed pulses (400 nm) are adopted, the spectrum of the forwardly propagating lasing emission of N_2^+ at the $B^2\Sigma_u^+ - X^2\Sigma_g^+(0, 0)$ transition is recorded as shown in Fig. 2(a), in which the P branch at ~ 391 nm and the R branch at ~ 389 nm can be identified. No lasing lines can be seen when either the pump pulses or the seed pulses are blocked as plotted in Fig. 2(a) with the dotted line (seed only) and the dashed line (pump only).

As shown in Fig. 2(b), in which the lasing intensity at 391 nm is plotted as a function of the angle θ , the intensities of the lasing signals at 391 nm increase first, and then decrease as the absolute value of θ increases from 0 to 45° for all four QWP cases. It can also be seen in this figure that the maximum lasing intensity at 391 nm increases monotonically as the order of the QWP increases from $n = 0$ to 10 so that the lasing intensity at $n = 10$ becomes about three times as large as that at $n = 0$, while the angle θ at which the lasing intensity becomes maximum moves from $\theta \sim 12^\circ$ to $\theta \sim 18^\circ$.

The results obtained for the forwardly propagating lasing emission of N_2^+ at the $B^2\Sigma_u^+ - X^2\Sigma_g^+(0, 1)$ transition at 428 nm are shown in Figs. 2(d) and 2(e). It can be seen in Fig. 2(e) that the lasing intensity at 428 nm decreases monotonically as the absolute value of θ increases from 0 to 45° for all four QWP cases with the maximum intensity position being at $\theta \sim 0^\circ$, which exhibits a marked difference from the θ dependences of the lasing intensity at 391 nm shown in Fig. 2(b).

Because the lasing lines at 391 and 428 nm share the same upper vibrational level, $B^2\Sigma_u^+(v' = 0)$, the difference in the ellipticity dependences of the lasing intensities shown in Figs. 2(b) and 2(e) can be ascribed to the difference in the variations of the population in the two lowest vibrational levels, $v'' = 0$ and 1, in the $X^2\Sigma_g^+$ state associated with the change in θ .

In order to examine the variation in the population in the upper $B^2\Sigma_u^+(v' = 0)$ level of N_2^+ as a function of θ , we record the fluorescence intensities at 391 and 428 nm using the QWPs with the measurement scheme described in [18]. The resultant fluorescence intensities at 391 and 428 nm are shown, respectively, in Figs. 2(c) and 2(f) as a function of θ for the four QWP cases.

FIG. 2. (a) The forwardly propagating externally seeded lasing spectrum of the $B^2\Sigma_u^+(v'=0)-X^2\Sigma_g^+(v''=0)$ emission of N_2^+ induced by linear-polarized pump pulses (blue solid line). The spectrum obtained only with the pump pulse (red dotted line) and that obtained only with the seed pulse (pink dashed line) are also shown. The lasing intensity (b) and the fluorescence intensity (c) at 391 nm as a function of θ with $n = 0$ (purple triangle), 3 (violet asterisk), 7 (red dot), and 10 (blue square). The lasing spectrum (d), the lasing intensity at 428 nm (e), the fluorescence intensity at 428 nm (f) for the $B^2\Sigma_u^+(v'=0)-X^2\Sigma_g^+(v''=1)$ emission of N_2^+ . For $n = 0$, the angles $\theta = 0^\circ$ and $\pm 45^\circ$ correspond to the ellipticities of $\varepsilon = 0$ and 1.

It can be seen in Figs. 2(c) and 2(f) that the fluorescence intensities at 391 and 428 nm decrease as the absolute value of θ increases from $\theta = 0^\circ$ to 45° and that their θ dependences are almost exactly the same in all four QWP cases. Therefore, the difference in the θ dependences of the lasing intensities at 391 and 428 nm shown in Figs. 2(b) and 2(e) can be ascribed securely to the difference in the variations in the population in the two vibrational levels, $v'' = 0$ and 1, of the $X^2\Sigma_g^+$ state induced by the elliptically modulated intense laser fields.

In order to clarify the characteristic intensity variation of the lasing line at 391 nm shown in Fig. 2(b), we perform numerical simulations of N_2^+ interacting with the elliptically modulated laser field and examine the variations of the populations in the two lowest vibrational levels ($v'' = 0$ and 1) of the ground $X^2\Sigma_g^+$ state. We solve the time-dependent Schrödinger equation for the vibronic wave functions of N_2^+ , which start interacting with the laser pulse just after its creation, and calculate the populations in the vibrational states in the $X^2\Sigma_g^+$ state, the $B^2\Sigma_u^+$ state, and those in the two degenerate $A^2\Pi_u$ states in a similar manner as in [12] and [23]. Rotational motion of N_2^+ is neglected in the numerical simulations.

In Figs. 3(a) and 3(b), we show examples of the time-dependent populations at the two QWP angles of $\theta = 0^\circ$ and 15° obtained when $n = 0$. In the simulations, we employ laser field parameters: $I = 3.2 \times 10^{14}$ W/cm², $\lambda = 800$ nm, and $\Delta t = 40$ fs, which mimic the laser parameters in the experiment. We assume that N_2^+ , which is created in the $X^2\Sigma_g^+$ state at the peak of the laser pulse, starts interacting with the latter half of the elliptically modulated laser pulse. We also assume that the N-N molecular axis of N_2^+ is parallel to the fast axis of the QWP (the y axis), considering that the ionization probability becomes the largest when the N-N axis is parallel to the laser polarization direction [24].

As shown in Figs. 3(a) and 3(b), the population inversion between the $B^2\Sigma_u^+(v'=0)$ and $X^2\Sigma_g^+(v''=0)$ states is achieved in both cases of $\theta = 15^\circ$ and $\theta = 0^\circ$ and the extent of the population inversion is much larger when $\theta = 15^\circ$ than when $\theta = 0^\circ$. Indeed, if we denote the final population in the vibrational state v in the electronic state α as $p[\alpha(v)]$, the population differences, $\Delta p(391 \text{ nm}) = p[B(0)] - p[X(0)]$, are $\Delta p(391 \text{ nm}) = 0.37$ at $\theta = 15^\circ$ and $\Delta p(391 \text{ nm}) = 0.09$ at $\theta = 0^\circ$. The major difference between the two cases is the existence of the $E_x(t)$ component in the case of Fig. 3(b).

FIG. 3. (a),(b) The time-dependent population in the $X^2\Sigma_g^+(v''=0,1)$, $B^2\Sigma_u^+(v'=0)$, and $A^2\Pi_u$ states when $n=0$ and $\theta=0^\circ$ (a) and when $n=0$ and $\theta=15^\circ$ (b), respectively. The ionization $N_2 \rightarrow N_2^+(X^2\Sigma_g^+)$ occurs at the peak of the field located at $t = -T_{n=0}/2 = -0.3$ fs. (c),(d) The final population differences $\Delta p(391 \text{ nm})$ (c) and $\Delta p(428 \text{ nm})$ (d) for $n=0$ and 10 as a function of θ . The ellipticity, $\varepsilon = |\tan \theta|$, for $n=0$ is indicated on the upper horizontal axis in (c) and (d). Because the alignment angle of the N-N axis is not averaged, the observed variation of the peak position of the lasing intensity for the different n values in Fig. 2(b) cannot be reproduced. In addition, the slower intensity drops in (d) than in Fig. 2(e) is ascribed to the fact that the decrease in the ionization probability associated with the increase in the ellipticity is not included in the simulation.

This means that the delayed x component of the electric field, $E_x(t)$, efficiently transfers the population in the $X^2\Sigma_g^+(v''=0)$ level to the $A^2\Pi_u$ state through the $A^2\Pi_u$ - $X^2\Sigma_g^+$ coupling, resulting in the enhancement of the population inversion between the $B^2\Sigma_u^+(v'=0)$ level and the $X^2\Sigma_g^+(v''=0)$ level, which leads to more intense lasing emission at 391 nm.

On the other hand, as can be seen in Figs. 3(a) and 3(b), the population in the $X^2\Sigma_g^+(v''=1)$ level of N_2^+ is very small throughout the period of the interaction with the laser field in both cases of $\theta=15^\circ$ and $\theta=0^\circ$. The small population in the $X^2\Sigma_g^+(v''=1)$ level of N_2^+ at $t=0$ reflects the small Franck-Condon overlap of the vibrational wave functions (0.08 [25]) between the vibrational ground $X^1\Sigma_g^+$ state of neutral N_2 and the $X^2\Sigma_g^+(v''=1)$ level of N_2^+ . Because the population is very small from the beginning, the population in the $X^2\Sigma_g^+(v''=1)$ state cannot be further reduced significantly by the coupling with the $A^2\Pi_u$ state through the $E_x(t)$ component of the laser electric field, and consequently, the extent of the population inversion between the $B^2\Sigma_u^+(v'=0)$ level and the $X^2\Sigma_g^+(v''=1)$ level is not affected by the $A^2\Pi_u$ - $X^2\Sigma_g^+$ coupling in both cases of $\theta=15^\circ$ and $\theta=0^\circ$.

FIG. 4. The intensity of the E_x component of the laser pulse (40 fs, 800 nm) integrated over the range of $0 < t < \infty$ as a function of θ for the four QWP cases.

We examine the final population difference between the population in the $B^2\Sigma_u^+(v'=0)$ level and that in the $X^2\Sigma_g^+(v''=0)$ level, $\Delta p(391 \text{ nm})$, and the final population difference between the $B^2\Sigma_u^+(v'=0)$ level and that in the $X^2\Sigma_g^+(v''=1)$ level, $\Delta p(428 \text{ nm}) = p[B(0)] - p[X(1)]$, as a function of the angle θ . As shown in Fig. 3(c), $\Delta p(391 \text{ nm})$ exhibits a local minimum at $\theta=0^\circ$ and a maximum at $|\theta| \approx 15^\circ$ for both cases of $n=0$ and 10. On the other hand, as shown in Fig. 3(d), $\Delta p(428 \text{ nm})$ exhibits a maximum at $\theta=0^\circ$ and decreases monotonically as $|\theta|$ increases. The dependences shown in Figs. 3(c) and 3(d) for $\Delta p(391 \text{ nm})$ and $\Delta p(428 \text{ nm})$ are in remarkably good agreement, respectively, with the θ dependences of the experimental lasing intensities at 391 and 428 nm shown in Figs. 2(b) and 2(e).

In the case of the lasing line at 391 nm, the increase in its intensity as θ increases from $\theta=0^\circ$ can be ascribed to the increase in the intensity of the $E_x(t)$ component, which efficiently transfers the population of the $X^2\Sigma_g^+(v''=0)$ to the vibrational levels of the $A^2\Pi_u$ state through the $X^2\Sigma_g^+$ - $A^2\Pi_u$ coupling after N_2^+ is generated upon the ionization.

On the other hand, the enhancement of the lasing intensity at 391 nm for the larger n shown experimentally in Fig. 2(b) and theoretically in Fig. 3(c) can be attributed to the delayed $E_x(t)$ component inducing the $X^2\Sigma_g^+$ - $A^2\Pi_u$ coupling after N_2^+ is prepared. As the order of the QWP, n , increases, the $E_x(t)$ component is delayed more with respect to the $E_y(t)$ component, resulting in the larger contribution of the $E_x(t)$ component after the creation of N_2^+ upon the ionization induced by the $E_y(t)$ component.

Indeed, it is shown clearly in Fig. 4 that the integrated intensity of the E_x component of the pulsed laser field after $t=0$ increases monotonically for all the cases with the different values of n ($n=0, 3, 7$, and 10), and that its magnitude becomes larger as n increases. Therefore, the key factor for the enhancement of the population inversion between $B^2\Sigma_u^+(v'=0)$ and $X^2\Sigma_g^+(v''=0)$ is the $E_x(t)$ component of the laser field interacting with N_2^+ in the latter part of the laser field.

However, when θ increases further toward $\theta=45^\circ$, the elliptically polarized laser field will become closer to a circularly polarized laser field, which means that the magnitude of the $E_y(t)$ component becomes smaller, and consequently, the ionization probability decreases so that the population inversion between the $B^2\Sigma_u^+(v'=0)$ and the $X^2\Sigma_g^+(v''=0)$ could not be achieved efficiently even by the sudden turn-on pulse.

In the case of the lasing at 428 nm, the efficiency of the creation of the inverted population between the $B^2\Sigma_u^+$ ($v' = 0$) level and the $X^2\Sigma_g^+$ ($v'' = 1$) level achieved by the $E_y(t)$ component of the laser field plays a dominant role because the population in the $X^2\Sigma_g^+$ ($v'' = 1$) level is so small and could not be depleted further by the $E_x(t)$ component in the latter half of the laser pulse. Therefore, the monotonic decrease of the $\Delta p(428 \text{ nm})$ shown in Fig. 3(d) as a function of the absolute value of θ simply represents the decrease in the amplitude of the $E_y(t)$ component, resulting in the less efficient creation of the inverted population between the $B^2\Sigma_u^+$ ($v' = 0$) level and the $X^2\Sigma_g^+$ ($v'' = 1$) level.

In summary, we have investigated both experimentally and theoretically the $B^2\Sigma_u^+ - X^2\Sigma_g^+$ (0,0) and (0,1) lasing intensities of N_2^+ at 391 and 428 nm induced by an elliptically modulated laser field. On the basis of the dependence of the lasing intensities on the angle θ as well as on the order, n , we

have revealed that (i) the lasing intensities can be maximized by optimizing the temporal separation between the E_x and E_y components and their relative amplitudes, (ii) the optimization of the N_2^+ lasing can be interpreted well based on the post-ionization coupling model, and (iii) the significantly different ellipticity dependence of the lasing intensity at 428 nm from the lasing intensity at 391 nm can be ascribed to the fact that the population in the $X^2\Sigma_g^+$ ($v'' = 1$) level prepared by the E_y component of the laser field is significantly small.

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- [1] Q. Luo, W. Liu, and S. L. Chin, *Appl. Phys. B* **76**, 337 (2003).
 - [2] A. Dogariu, J. B. Michael, M. O. Scully, and R. B. Miles, *Science* **331**, 442 (2011).
 - [3] J. Yao, B. Zeng, H. Xu, G. Li, W. Chu, J. Ni, H. Zhang, S. L. Chin, Y. Cheng, and Z. Xu, *Phys. Rev. A* **84**, 051802(R) (2011).
 - [4] D. Kartashov, S. Ališauskas, G. Andriukaitis, A. Pugžlys, M. Shneider, A. Zheltikov, S. L. Chin, and A. Baltuška, *Phys. Rev. A* **86**, 033831 (2012).
 - [5] D. Kartashov, S. Ališauskas, A. Baltuška, A. Schmitt-Sody, W. Roach, and P. Polynkin, *Phys. Rev. A* **88**, 041805(R) (2013).
 - [6] A. Laurain, M. Scheller, and P. Polynkin, *Phys. Rev. Lett.* **113**, 253901 (2014).
 - [7] H. Zhang, C. Jing, J. Yao, G. Li, B. Zeng, W. Chu, J. Ni, H. Xie, H. Xu, S. L. Chin, K. Yamanouchi, Y. Cheng, and Z. Xu, *Phys. Rev. X* **3**, 041009 (2013).
 - [8] Y. Liu, Y. Brelet, G. Point, A. Houard, and A. Mysyrowicz, *Opt. Express* **21**, 22791 (2013).
 - [9] T. J. Wang, J. Ju, J. F. Daigle, S. Yuan, R. Li, and S. L. Chin, *Laser Phys. Lett.* **10**, 125401 (2013).
 - [10] J. P. Yao, W. Chu, Z. X. Liu, B. Xu, J. M. Chen, and Y. Cheng, *New J. Phys.* **20**, 033035 (2018).
 - [11] A. Azarm, P. Corkum, and P. Polynkin, *Phys. Rev. A* **96**, 051401(R) (2017).
 - [12] H. Xu, E. Lötstedt, A. Iwasaki, and K. Yamanouchi, *Nat. Commun.* **6**, 8347 (2015).
 - [13] J. Yao, S. Jiang, W. Chu, B. Zeng, C. Wu, R. Lu, Z. Li, H. Xie, G. Li, C. Yu, Z. Wang, H. Jiang, Q. Gong, and Y. Cheng, *Phys. Rev. Lett.* **116**, 143007 (2016).
 - [14] H. Zhang, C. Jing, G. Li, H. Xie, J. Yao, B. Zeng, W. Chu, J. Ni, H. Xu, and Y. Cheng, *Phys. Rev. A* **88**, 063417 (2013).
 - [15] Y. Liu, P. Ding, G. Lambert, A. Houard, V. Tikhonchuk, and A. Mysyrowicz, *Phys. Rev. Lett.* **115**, 133203 (2015).
 - [16] M. Britton, P. Laferriere, D. H. Ko, Z. Li, F. Kong, G. Brown, A. Naumov, C. Zhang, L. Arissian, and P. B. Corkum, *Phys. Rev. Lett.* **120**, 133208 (2018).
 - [17] Y. Wan, B. Xu, J. Yao, J. Chen, Z. Liu, F. Zhang, W. Chu, and Y. Cheng, *J. Opt. Soc. Am. B* **36**, G57 (2019).
 - [18] H. Li, M. Hou, H. Zang, Y. Fu, E. Lötstedt, T. Ando, A. Iwasaki, K. Yamanouchi, and H. Xu, *Phys. Rev. Lett.* **122**, 013202 (2019).
 - [19] H. Li, Q. Song, J. Yao, Z. Liu, J. Chen, B. Xu, K. Lin, J. Qiang, B. He, H. Xu, Y. Cheng, H. Zeng, and J. Wu, *Phys. Rev. A* **99**, 053413 (2019).
 - [20] T. Ando, E. Lötstedt, A. Iwasaki, H. Li, Y. Fu, S. Wang, H. L. Xu, and K. Yamanouchi, *Phys. Rev. Lett.* **123**, 203201 (2019).
 - [21] Y. Zhang, E. Lötstedt, and K. Yamanouchi, *J. Phys. B: At., Mol. Opt. Phys.* **50**, 185603 (2017).
 - [22] Y. Zhang, E. Lötstedt, and K. Yamanouchi, *J. Phys. B: At., Mol. Opt. Phys.* **52**, 055401 (2019).
 - [23] H. Xu, E. Lötstedt, T. Ando, A. Iwasaki, and K. Yamanouchi, *Phys. Rev. A* **96**, 041401(R) (2017).
 - [24] X. M. Tong, Z. X. Zhao, and C. D. Lin, *Phys. Rev. A* **66**, 033402 (2002).
 - [25] F. R. Gilmore, R. R. Laher, and P. J. Espy, *J. Phys. Chem. Ref. Data* **21**, 1005 (1992).