Naturalness versus stringy naturalness with implications for collider and dark matter searches

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The notion of stringy naturalness—that an observable $\mathcal{O}_2$ is more natural than $\mathcal{O}_1$ if more (phenomenologically acceptable) vacua solutions lead to $\mathcal{O}_2$ rather than $\mathcal{O}_1$ —is examined within the context of the Standard model (SM) and various supersymmetric models (SUSY) extensions: Constrained minimal supersymmetric standard model (CMSSM)/minimal supergravity model (mSUGRA), high-scale SUSY, and radiatively driven natural SUSY (RNS). Rather general arguments from string theory suggest a (possibly mild) statistical draw towards vacua with large soft SUSY breaking terms. These vacua must be tempered by an anthropic veto of nonstandard vacua or vacua with too large a value of the weak scale m_{weak} . We argue that the SM, the CMSSM and various high-scale SUSY models are all expected to be relatively rare occurrences within the string theory landscape of vacua. In contrast, models with TeV-scale soft terms but with $m_{\text{weak}} \sim 100$ GeV and consequent light higgsinos (SUSY with radiatively driven naturalness) should be much more common on the landscape. These latter models have a statistical preference for $m_t \simeq 125$ GeV and strongly interacting sparticles beyond current LHC reach. Thus, while conventional naturalness favors sparticles close to the weak scale, stringy naturalness favors sparticles so heavy that electroweak symmetry is barely broken and one is living dangerously close to vacua with charge-or-color breaking minima, no electroweak breaking or pocket universe weak-scale values too far from our measured value. Expectations for how landscape SUSY would manifest itself at collider and dark matter search experiments are then modified compared to usual notions.

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I. INTRODUCTION

Naturalness. The gauge hierarchy problem (GHP) [1]—what stabilizes the weak scale so that it does not blow up to the GUT/Planck scale—is one of the central conundrums of particle physics. Indeed, it provides crucial motivation for the premise that new physics should be lurking in or around the weak scale. Supersymmetric models (SUSY) with weak-scale soft SUSY breaking terms provide an elegant solution to the GHP [2,3] but so far weak-scale sparticles have failed to appear at the CERN Large Hadron Collider (LHC) and WIMPs have failed to appear in direct detection experiments [4]. The latest search limits from LHC Run 2 require gluinos with $m_{\tilde{g}} \gtrsim 2.25$ TeV [5] and top squarks $m_{\tilde{t}_1} \gtrsim 1.1$ TeV [6]. These lower bound search limits stand in sharp contrast to early sparticle mass upper bounds from naturalness that seemingly required $m_{\tilde{g},\tilde{t}_1} \lesssim 0.4$ TeV [7–10].¹ Thus LHC limits seem to imply the soft SUSY breaking scale m_{soft} lies in the multi-TeV rather than the weak-scale range. This then opens

up a Little hierarchy problem (LHP) [12]: why does the weak scale not blow up to the energy scale associated with soft SUSY breaking, i.e., why is $m_{\text{weak}} \ll m_{\text{soft}}$?

Early upper bounds on sparticle masses derived from naturalness were usually computed using the EENZ/BG logarithmic-derivative measure [7,8]: for an observable $\mathcal{O}$, then

$$\Delta_{\text{BG}}(\mathcal{O}) \equiv \max_i \left| \frac{\partial \ln \mathcal{O}}{\partial \ln p_i} \right| = \max_i \left| \frac{p_i}{\mathcal{O}} \frac{\partial \mathcal{O}}{\partial p_i} \right|, \quad (1)$$

where the p_i are fundamental parameters of the underlying theory. For an observable depending linearly on model parameters, $\mathcal{O} = a_1 p_1 + \dots + a_n p_n$, then $\partial \mathcal{O} / \partial p_i = a_i$ and $\Delta_{\text{BG}}(\mathcal{O})$ just picks off the maximal right-hand-side contribution to $\mathcal{O}$ and compares it to $\mathcal{O}$. In the case where one contribution $a_i p_i \gg \mathcal{O}$, then some other contribution(s) will have to be finetuned to large opposite-sign values such as to maintain $\mathcal{O}$ at its measured value. Such finetuning of fundamental parameters seems highly implausible in nature absent some symmetry or parameter selection mechanism. Thus the logarithmic derivative is a measure of [13] *practical naturalness*. An observable $\mathcal{O}$ is natural if all *independent* contributions to $\mathcal{O}$ are comparable to or less than $\mathcal{O}$.

For the case of the LHP, the observable $\mathcal{O}$ is traditionally taken to be m_Z^2 and the p_i are taken to be the MSSM μ term and soft SUSY breaking terms so that naturalness then requires all contributions to m_Z^2 to be comparable to m_Z^2 ; this is the basis for expecting sparticles to occur around the 100-GeV scale. A fundamental issue in computing Δ_{BG} is [14–17]: what is the correct choice to be made for the p_i ? If soft terms

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¹For a recent analysis in the context of EENZ/BG, see, e.g., Ref. [11].

are correlated with one another, as expected in fundamental supergravity/superstring theories, then one gets a very different answer than if one assumes some effective $4-d$ SUSY theory with many independent soft parameters which are introduced to parametrize one's ignorance of their origin.² Alternatively, even if the parameters p_i are independent, it is possible that some selection mechanism is responsible for the values towards which they tend.

It is also common in the literature to apply practical naturalness to the Higgs mass:

$$m_h^2 \simeq m_{H_u}^2(\text{weak}) + \mu^2(\text{weak}) + \text{mixing} + \text{rad. corr.}, \quad (2)$$

where the mixing and radiative corrections are both comparable to m_h^2 . Also, $m_{H_u}^2(\text{weak}) = m_{H_u}^2(\Lambda) + \delta m_{H_u}^2$ where it is common to estimate $\delta m_{H_u}^2$ using its renormalization group equation (RGE) by setting several terms in $dm_{H_u}^2/dt$ (with $t = \ln Q^2$) to zero so as to integrate in a single step:

$$\delta m_{H_u}^2 \sim -\frac{3f_t^2}{8\pi^2}(m_{Q_3}^2 + m_{U_3}^2 + A_t^2) \ln(\Lambda^2/m_{\text{soft}}^2). \quad (3)$$

Taking $\Lambda \sim m_{\text{GUT}}$ and requiring the high-scale measure

$$\Delta_{\text{HS}} \equiv \delta m_{H_u}^2/m_h^2. \quad (4)$$

$\Delta_{\text{HS}} \lesssim 1$ then requires three third generation squarks lighter than 500 GeV [18,19] (now highly excluded by LHC top-squark searches) and small A_t terms (whereas $m_h \simeq 125$ GeV typically requires large mixing and thus multi-TeV values of A_0 [20,21]). The simplifications made in this calculation ignore the fact that $\delta m_{H_u}^2$ is highly dependent on $m_{H_u}^2(\Lambda)$ (which is set to zero in the simplification) [14,16,17]. In fact, the larger one makes $m_{H_u}^2(\Lambda)$, then the larger becomes the canceling correction $\delta m_{H_u}^2$. Thus these terms are *not independent*: one cannot tune $m_{H_u}^2(\Lambda)$ against a large contribution $\delta m_{H_u}^2$. Thus weak-scale top squarks and small A_t are not required by naturalness.

To ameliorate the above naturalness calculational quandaries, a more model independent measure Δ_{EW} was introduced [22,23].³ By minimizing the weak-scale SUSY Higgs potential, including radiative corrections, one may relate the measured value of the Z-boson mass to the various SUSY contributions:

$$\begin{aligned} m_Z^2/2 &= \frac{m_{H_d}^2 + \Sigma_d^d - (m_{H_u}^2 + \Sigma_u^u) \tan^2 \beta}{\tan^2 \beta - 1} - \mu^2 \\ &\simeq -m_{H_u}^2 - \mu^2 - \Sigma_u^u(\tilde{t}_{1,2}). \end{aligned} \quad (5)$$

The measure

$$\Delta_{\text{EW}} = |(\text{max RHS contribution})|/(m_Z^2/2) \quad (6)$$

²For instance, in dilaton-dominated SUSY breaking, one expects $m_0^2 = m_{3/2}^2$ and $m_{1/2} = -A_0 = \sqrt{3}m_{3/2}$. In such a case, it would not make sense to adopt m_0 and $m_{1/2}$ as free, independent parameters.

³A desirable feature of Δ_{EW} is that for a given SUSY spectrum, one obtains exactly the same finetuning measure whether the spectrum is generated from multi- or few parameter theory or at the weak scale (such as pMSSM) or at much higher scales. This model independence is not shared by other measures such as Δ_{BG} or Δ_{HS} .

is then low provided all *weak-scale* contributions to $m_Z^2/2$ are comparable to or less than $m_Z^2/2$. The Σ_u^u and Σ_d^d contain over 40 radiative corrections which are listed in Appendix of Ref. [23]. The conditions for natural SUSY (for, e.g., $\Delta_{\text{EW}} < 30$)⁴ can then be read off from Eq. (6).

(1) The superpotential μ parameter has magnitude not too far from the weak scale, $|\mu| \lesssim 300$ GeV. This implies the existence of light higgsinos $\tilde{\chi}_{1,2}^0$ and $\tilde{\chi}_1^\pm$ with $m(\tilde{\chi}_{1,2}^0, \tilde{\chi}_1^\pm) \sim 100\text{--}300$ GeV.

(2) $m_{H_u}^2$ is radiatively driven from large high-scale values to small negative values at the weak scale (SUSY with radiatively driven naturalness or RNS [22]).

(3) Large cancellations occur in the $\Sigma_u^u(\tilde{t}_{1,2})$ terms for large A_t parameters which then allow for $m_{\tilde{t}_1} \sim 1\text{--}3$ TeV for $\Delta_{\text{EW}} < 30$. The large A_t term also lifts the Higgs mass m_h into the vicinity of 125 GeV. The gluino contributes to the weak scale at two-loop order so its mass can range up to $m_{\tilde{g}} \lesssim 6$ TeV with little cost to naturalness [23–25].

(4) Since first/second generation squarks and sleptons contribute to the weak scale through (mainly canceling) D terms, they can range up to 10–30 TeV at little cost to naturalness (thus helping to alleviate the SUSY flavor and CP problems) [26].

By combining dependent soft terms in Δ_{BG} or by combining the dependent terms $m_{H_u}^2(\Lambda)$ and $\delta m_{H_u}^2$ in Δ_{HS} , then these measures roughly reduce to Δ_{EW} (aside from the radiative contributions $\Sigma_{u,d}^{u,d}$). Since Δ_{EW} is determined by the weak-scale SUSY parameters, then different models which give rise to exactly the same sparticle mass spectrum will have the same finetuning value (model independence). Using the naturalness measure Δ_{EW} , then it has been shown that plenty of SUSY parameter space remains natural even in the face of LHC Run 2 Higgs mass measurements and sparticle mass limits [23].

String theory landscape. Weinberg's anthropic solution to the cosmological constant (CC) [27], along with the emergence of the string theory landscape of vacua [28,29], have presented a challenge to the usual notion of naturalness. In Weinberg's view, in the presence of a vast assortment ($\gg 10^{120}$) of pocket universes which are part of an eternally inflating multiverse, it may not be so surprising to find ourselves in one with $\Lambda_{\text{cc}} \sim 10^{-120} m_p^4$ since if it were much larger, then the expansion rate would be so large that galaxies would not be able to condense, and life as we know it would be unable to emerge. This picture was bolstered by the discovery of the string theory discretuum of flux vacua [28,29] where each metastable minimum of the string theory scalar potential would have a different value of Λ_{cc} , and also different laws of physics and perhaps even different space-time dimensions. The number of metastable minima has been estimated at around 10^{500} [31] although far larger numbers have also been considered. In such a scenario, then it is possible that many of the constants of nature may take on environmentally determined values rather than being determined by fundamental underlying principles.

One example of the challenge to naturalness comes in the form of split supersymmetry (SS) [32]. In SS, one retains the

⁴The onset of finetuning for $\Delta_{\text{EW}} \gtrsim 30$ is visually displayed in Fig. 1 of Ref. [24].

positive features of gauge coupling unification and a WIMP dark matter candidate while eschewing the motivation of naturalness. Then the expected SUSY particle spectrum of SS can contain weak-scale gauginos and higgsinos (which furnish gauge coupling unification and a WIMP dark matter candidate) while allowing most scalars (except the SM-like Higgs doublet) to gain unnatural mass values of perhaps $\tilde{m} \sim 10^3\text{--}10^8$ TeV. Here, one might expect the weak scale to blow up to the $\tilde{m}$ scale, but the anthropic requirement of $m_{\text{weak}} \sim 100$ GeV selects a finely tuned scalar spectrum that would otherwise seem unnatural. In a similar vein, the notion of high-scale SUSY has also been entertained wherein all sparticle masses are large and unnatural, perhaps at the $10^2\text{--}10^3$ TeV scale [33–36].

At first glance, one might expect that the string landscape would make BSM physics nonpredictive since: how are we to determine the metastable vacuum minimum that our universe inhabits out of $\sim 10^{500}$ or more choices? To make progress, Douglas and collaborators [37] have advanced the notion of a *statistical* program for determining BSM physics. In this case, if we can identify statistical trends for the many landscape vacua solutions, then we might be able to determine probabilistically what sort of pocket universe we are likely to live in.

To this end, Douglas has proposed the notion of *stringy naturalness* [37]: the value of an observable $\mathcal{O}_2$ is more natural than a value $\mathcal{O}_1$ if more *phenomenologically viable* vacua lead to $\mathcal{O}_2$ than to $\mathcal{O}_1$.

If we apply this definition to the cosmological constant, then *phenomenologically viable* is interpreted in an anthropic context in that we must veto vacua which would not allow galaxy condensation. Out of the remaining viable vacua, we would expect Λ_{cc} to be nearly as large as anthropically possible since there is more volume in Λ_{cc} space for larger values. Such reasoning allowed Weinberg to predict the value of Λ_{cc} to within a factor of a few of its measured value more than a decade before its value was determined from experiment [27].

Our goal in this paper is to examine Douglas' notion of stringy naturalness and to compare and contrast it to the above conventional notions of naturalness. We will examine what naturalness and what stringy naturalness imply for the SM, the scale of SUSY breaking in SUSY models, and for the magnitude of the weak scale. Our central conclusion from stringy naturalness is: the soft SUSY breaking terms should be as large as possible subject to the constraint that the value of the weak scale in various pocket universes—with the MSSM as the low-energy effective theory—not deviate by more than a factor of a few from its measured value. This is in contrast to the conventional measures which tend to favor smaller soft terms comparable to the weak scale. In fact, stringy naturalness in this form with a relatively mild draw to large soft terms statistically favors a light Higgs mass $m_h \sim 125$ GeV with as yet no sign of sparticles at LHC.

In Sec. II, we present details of how to implement the notion of stringy naturalness including Douglas' notion of a prior distribution of power-law probability increase in soft term values. This is to be combined with a *selection* criteria enforcing that the value of the weak scale in various pocket universes not deviate from our measured value by a factor of a few. The latter notion has been presented in some detail by calculations of Agrawal *et al.* [38]. The landscape selection

of the cosmological constant, as emphasized by Douglas *et al.* [30], operates separately from the soft SUSY breaking term/weak-scale selection criteria.

In Sec. III, we examine first what stringy naturalness implies for the (non-SUSY) standard model (SM). We find the SM—valid up to a cutoff scale Λ_{SM} —to be highly improbable within the landscape for a cutoff $\Lambda_{\text{SM}} \gg m_{\text{weak}}$. In Sec. IV, we argue that the paradigm CMSSM SUSY model should also be relatively rare on the landscape. In Sec. V, we examine the case of the MSSM on the landscape. In this case, there can be significant swaths of model parameters leading to SUSY with radiatively driven naturalness (RNS). In fact, the statistical draw to large soft terms coupled with a weak scale not too far from its measured value, is exactly what is needed for SUSY with radiatively driven naturalness. In Sec. VI, we present arguments as to why unnatural SUSY models such as split SUSY, high-scale SUSY, spread SUSY and minisplit SUSY are likely to be relatively rare occurrences on the landscape. In Sec. VII, we briefly discuss consequences of stringy naturalness for collider and dark matter searches. A summary and our conclusions are presented in Sec. VIII.

The present work is a continuation of a theme started in Ref. [39] where a qualitative picture of landscape SUSY was presented. Probability distributions for Higgs and sparticle masses were derived from landscape considerations in Ref. [40], while in Ref. [13], predictions for landscape SUSY were compared to LHC simplified model searches along with WIMP direct and indirect detection searches. In Ref. [41], similar methods were applied to determination of the Peccei-Quinn scale in SUSY axion models. Some complementary investigations on the likelihood of $N = 1$ SUSY emerging from the heterotic landscape have been performed in Ref. [42].

II. PRIOR DISTRIBUTIONS AND SELECTION CRITERIA FOR LANDSCAPE SUSY

A simple ansatz for the distribution of string vacua in terms of SUSY breaking scales m_{hidden}^2 (where the soft breaking mass scale $m_{\text{soft}} = m_{\text{hidden}}^2/m_P$ and m_P is the reduced Planck mass) is given by

$$dN_{\text{vac}}[m_{\text{hidden}}^2, m_{\text{weak}}, \Lambda_{cc}] = f_{\text{SUSY}}(m_{\text{hidden}}^2) \cdot f_{\text{EWFT}} \cdot f_{cc} \cdot dm_{\text{hidden}}^2. \quad (7)$$

The cosmological constant finetuning penalty is expected to be $f_{cc} \sim \Lambda_{cc}/m^4$ where initial expectations were that m^4 was taken to be m_{hidden}^4 . In the 4d supergravity effective theory, which emerges after string compactification, the cosmological constant is given by

$$\Lambda_{cc} = m_{\text{hidden}}^4 - 3e^{K/m_P^2} |W|^2/m_P^2, \quad (8)$$

where $m_{\text{hidden}}^4 = \sum_i |F_i|^2 + \frac{1}{2} \sum_\alpha D_\alpha^2$ is a mass scale associated with the hidden sector (and usually in SUGRA-mediated models it is assumed $m_{\text{hidden}} \sim 10^{12}$ GeV such that the gravitino gets a mass $m_{3/2} \sim m_{\text{hidden}}^2/m_P$).

A key observation of Susskind [43] and Denef and Douglas [44] (DD) was that W at the minima is distributed uniformly as a complex variable, and the distribution of $e^{K/m_P^2} |W|^2/m_P^2$ is not correlated with the distributions of F_i and D_α . Setting the cosmological constant to nearly zero, then, has no effect on

the distribution of supersymmetry breaking scales. Physically, this can be understood by the fact that the superpotential receives contributions from many sectors of the theory, supersymmetric as well as non-supersymmetric. In this case, the m^4 in f_{cc} should be taken to be m_{string}^4 instead of m_{hidden}^4 , rendering this term inconsequential to how the number of vacua are distributed in terms of m_{soft} .

Another key observation from examining flux vacua in Sec. II B string theory is that the SUSY breaking F_i and D_α terms are likely to be uniformly distributed—in the former case as complex numbers, while in the latter case as real numbers. Then one expects the following distribution of supersymmetry breaking scales

$$f_{\text{SUSY}}(m_{\text{hidden}}^2) \sim (m_{\text{hidden}}^2)^{2n_F + n_D - 1}, \quad (9)$$

where n_F is the number of F -breaking fields and n_D is the number of D -breaking fields in the hidden sector. For just a single F -breaking term, then one expects a *linear* statistical draw towards large soft terms $f_{\text{SUSY}} \sim m_{\text{soft}}^n$, where $n = 2n_F + n_D - 1$ and in this case $n = 1$. For SUSY breaking contributions from multiple hidden sectors, as typically expected in string theory, then n can be much larger, with a consequent stronger pull towards large soft breaking terms.

An initial guess for f_{EWFT} , the (anthropic) finetuning factor, was $m_{\text{weak}}^2/m_{\text{soft}}^2$ which would penalize soft terms which were much bigger than the weak scale. This ansatz fails on several points.

(1) Many soft SUSY breaking choices will land one into charge-or-color breaking (CCB) minima of the EW scalar potential. Such vacua would likely not lead to a livable universe and should be vetoed.

(2) Other choices for soft terms may not even lead to EW symmetry breaking (EWSB). For instance, if $m_{H_u}^2(\Lambda)$ is too large, then it will not be driven negative to trigger spontaneous EWSB. These possibilities also should be vetoed.

(3) In the event of appropriate EWSB minima, then sometimes *larger* high-scale soft terms lead to *more natural* weak-scale soft terms. For instance, if $m_{H_u}^2(\Lambda)$ is large enough that EWSB is *barely broken*, then $|m_{H_u}^2(\text{weak})| \sim m_{\text{weak}}^2$. Likewise, if the trilinear soft breaking term A_t is big enough, then there is large top squark mixing and the $\Sigma_u^u(\tilde{t}_{1,2})$ terms enjoy large cancellations, rendering them $\sim m_{\text{weak}}^2$. The same large A_t values lift the Higgs mass m_h up to the 125-GeV regime.

Here, we will assume a *natural* solution to the SUSY μ problem [45]. A recent possibility is the hybrid CCK or SPM models [46], which are based on a $\mathbb{Z}_{24}^R$ discrete R symmetry which can emerge from compactification of extra dimensions in string theory. The $\mathbb{Z}_{24}^R$ symmetry is strong enough to allow a gravity-safe $U(1)_{PQ}$ symmetry to emerge (which solves the strong CP problem) while also forbidding RPV terms (so that WIMP dark matter is generated). Thus both Peccei-Quinn (PQ) and R -parity conservation (RPC) arise as approximate accidental symmetries similar to the way baryon and lepton number conservation emerge accidentally (and likely approximately) due to the SM gauge symmetries. These hybrid models also solve the SUSY μ problem via a Kim-Nilles [47] operator so that $\mu \sim \lambda_\mu f_a^2/m_P$ and $\mu \sim 100\text{--}200$ GeV (natural) for $f_a \sim 10^{11}$ GeV (the sweet zone for axion dark matter). The $\mathbb{Z}_{24}^R$ symmetry also suppresses dimension-5 proton decay operators [48].

FIG. 1. Value of m_{weak} as predicted when the SM is valid up to energy scale (1) $Q = m_P$, (2) the neutrino see-saw scale, and (3) valid just up to the measured value of $m_{\text{weak}} \sim 100$ GeV. We also show several anthropic bounds on m_{weak} from nuclear physics considerations of Agrawal *et al.* [38].

Once a natural value of $\mu \sim 100\text{--}300$ GeV is obtained, then we may invert the usual usage of Eq. (6) to determine the value of the weak scale in various pocket universes (with MSSM as low-energy effective theory) for a given choice of soft terms. Based on nuclear physics calculations by Agrawal *et al.* [38], a pocket universe value of $m_{\text{weak}}^{\text{PU}}$ which deviates from our measured value by a factor 2–5 is likely to lead to an unlivable universe as we understand it. Weak interactions and fusion processes would be highly suppressed and even complex nuclei could not form. The situation is shown in Fig. 1. We will adopt a conservative value that the weak scale should not deviate by more than a factor four from its measured value. This corresponds to a value of $\Delta_{\text{EW}} \lesssim 30$. Thus, for our final form of f_{EWFT} , we will adopt $f_{\text{EWFT}} = \Theta(30 - \Delta_{\text{EW}})$ while also vetoing CCB or no EWSB vacua.

III. WHY THE SM IS LIKELY A RARE OCCURRENCE IN THE LANDSCAPE

Before using Eq. (7) to evaluate various SUSY models, we will first examine the case of EWSB in the SM. For the SM with

$$V_{\text{SM}} = -\mu_{\text{SM}}^2 \phi^\dagger \phi + \lambda_{\text{SM}} (\phi^\dagger \phi)^2, \quad (10)$$

then the Higgs mass, including quadratic divergent radiative corrections, is found to be

$$m_H^2 \simeq m_H^2(\text{tree}) + \delta m_H^2, \quad (11)$$

FIG. 2. The value of the SM Higgs mass m_H versus SM μ_{SM} parameter for theory cutoff values $\Lambda_{\text{SM}} = 1, 10^2, 10^4, 10^8$, and 10^{13} TeV. The anthropic band is shown in blue.

where $m_H^2(\text{tree}) = 2\mu_{\text{SM}}^2$ and $\delta m_H^2 = \frac{3}{4\pi^2}(-\lambda_t^2 + \frac{g^2}{4} + \frac{g^2}{8\cos^2\theta_W} + \lambda_{\text{SM}})\Lambda_{\text{SM}}^2$, and where Λ_{SM} is the mass scale cutoff beyond which the SM is no longer the appropriate low-energy effective field theory. To gain the measured value of $m_H \simeq 125$ GeV, then for $\Lambda_{\text{SM}} \gg m_H$, we can freely tune $m_H^2(\text{tree})$ to compensate for the large radiative corrections. The situation is shown in Fig. 2 where we show the required value of μ_{SM} needed to gain $m_H = 125$ GeV for various choices of Λ_{SM} . Since nothing in the SM favors any particular value of μ_{SM} , we will assume its value is uniformly distributed (logarithmically over the decades of values) in the landscape. It is plain to see that for $\Lambda_{\text{SM}} = 1$ TeV, then a wide range of values for μ_{SM} leads to a weak scale (typified here by m_H) within the Agrawal band of allowed values. However, for $\Lambda_{\text{SM}} \gg m_{\text{weak}}$, then only a tiny (finetuned) range of μ_{SM} values leads to a viable value for m_{weak} . In this case, stringy naturalness and conventional natural coincide in that an anthropically allowed value of the weak scale requires that the SM be a valid effective field theory only for cutoff value $\Lambda_{\text{SM}} \lesssim 1$ TeV.

IV. WHY CMSSM/MSUGRA IS LIKELY AN INFREQUENT OCCURRENCE IN THE LANDSCAPE

In SUSY models, all the quadratic divergences cancel, leaving only logarithmic divergences whose effects may be captured via renormalization group (RG) equations. Thus SUSY models carry with them a solution to the Big hierarchy problem. The question then is: do they carry with them a Little hierarchy problem?

The CMSSM or mSUGRA model [49] is defined by GUT scale input parameters

$$m_0, m_{1/2}, A_0, \tan\beta, \text{sign}(\mu) \quad (\text{CMSSM/mSUGRA}), \quad (12)$$

where m_0 is the GUT scale unified scalar mass where $m_{H_u} = m_{H_d} = m_0$, $m_{1/2}$ is the unified gaugino mass, A_0 is the unified trilinear soft breaking term and $\text{sign}(\mu) = \pm 1$. The CMSSM has served for many years as a sort of SUSY paradigm model for SUSY collider and dark matter signal predictions. In CMSSM, the weak-scale soft terms are derived from RG running of the unified soft terms from $Q = m_{\text{GUT}}$ to $Q = m_{\text{weak}}$,

FIG. 3. The pocket universe value of m_Z^{PU} vs the SUSY μ parameter for various values of EW finetuning parameter Δ_{EW} . The anthropic band is shown in blue.

where then mixings and mass eigenstates may be evaluated. In the CMSSM model, since $m_{H_u}^2$ is input at the m_{GUT} scale, then its weak-scale value is derived. The μ term is (fine)tuned via Eq. (6) to gain a value in accord with the measured Z-boson mass.⁵

In years past, it was possible to find regions of CMSSM parameter space with small μ as required for naturalness in the hyperbolic branch [51] or focus point region [52] (HB/FP). The HB/FP region can appear for $m_0 \lesssim 10$ TeV for small values of $A_0 \sim 0$. However, the measured value of $m_h \simeq 125$ GeV requires large A_t parameter which then moves the HB/FP region out to huge m_0 values where the $\Sigma_u''(\tilde{t}_{1,2})$ are large, rendering the model unnatural. Detailed scans of the CMSSM model parameter space in accord with requiring $m_h = 125 \pm 2$ GeV find that the minimal value of Δ_{EW} is around 100 but where typically Δ_{EW} can range up to 10^4 [16].

To gain some insight on how frequent models with large values of Δ_{EW} might occur in the landscape, we take the limit of Eq. (6), wherein the radiative corrections are small so that $m_Z^2 \simeq -2m_{H_u}^2 - 2\mu^2$ and then consider SUSY models where $m_{H_u}^2$ is driven to large negative values at the weak scale. We can replace $-m_{H_u}^2(\text{weak})$ by $\Delta_{\text{EW}} \cdot m_Z^2(\text{measured})/2$ to use Eq. (6) to determine the pocket-universe (PU) value of m_Z^{PU} which is expected in SUSY models within the landscape in terms of Δ_{EW} and μ .

In Fig. 3, we plot the value of m_Z^{PU} versus μ for a variety of choices of Δ_{EW} ranging from natural values ~ 5 –20 up to as large as 640. We also show the shaded band in Fig. 3 which corresponds to values of m_Z^{PU} in accord with the Agrawal *et al.* allowed values which should give rise to a habitable pocket universe. Assuming a uniform distribution of μ values in the landscape, then we see from the figure that for low, natural values of Δ_{EW} there are significant ranges of μ which lead to values of m_Z^{PU} within the anthropic zone. However, as Δ_{EW} increases, the expected value of m_Z^{PU} increases well beyond the anthropic allowed zone unless one tunes μ to lie within a tightening range of (finetuned) values. Thus we would expect

⁵We compute SUSY spectra in all models using ISAJET 7.88 [50].

that SUSY models such as CMSSM/mSUGRA—where Δ_{EW} cannot attain natural values for $m_h \sim 125$ GeV—to be rather infrequent occurrences of our fertile patch of the landscape which contains the MSSM as the low-energy effective theory.

V. RADIATIVE NATURAL SUSY FROM STRINGY NATURALNESS

From Fig. 3, we see that models with a weak-scale value of the μ parameter hold a higher likelihood of landing within the narrow band of allowed (pocket universe) weak-scale values from Agrawal *et al.* [38]. Models with nonuniversal Higgs masses where $m_0 \neq m_{H_u}$, such as the two-extra-parameter nonuniversal Higgs model [53] (NUHM2) or its extension for nonuniversal generations NUHM3 (where $m_0(1,2) \neq m_0(3)$) as suggested by minilandscape models [54]) are applicable in this situation since the high-scale Higgs masses $m_{H_u}^2$ and $m_{H_d}^2$ can be traded for weak-scale inputs μ and m_A . Thus the NUHM2 model has input parameters m_0 , $m_{1/2}$, A_0 , $\tan\beta$, μ , and m_A . The added Higgs mass nonuniversality (which is expected since the Higgs multiplets live in different GUT multiplets from matter scalars) allows for small $\mu \sim 100$ –300 GeV for any points in model parameter space.

A. Living dangerously

In Fig. 4, we show the running of the up-Higgs soft mass (actually $\text{sign}(m_{H_u}^2) \cdot \sqrt{|m_{H_u}^2|}$) in the NUHM2 model for parameters $m_0 = 5$ TeV, $m_{1/2} = 1.2$ TeV, $A_0 = -8$ TeV and $\tan\beta = 10$ with $m_{H_d} = 2$ TeV but for $m_{H_u}(\Lambda) = 4, 5, 6.5$, and 8 TeV. We see that for the lower values of $m_{H_u}(\Lambda)$, then $m_{H_u}^2$ runs deeply negative leading to a large negative weak-scale value of $m_{H_u}^2(\text{weak})$. As $m_{H_u}^2(\Lambda)$ increases, then it is driven to smaller and smaller (more natural) weak-scale values. For too large a value of $m_{H_u}^2(\Lambda)$, then its running value is not driven negative at the weak scale and radiative EWSB does not occur. Such unphysical pocket universe solutions must be vetoed. Thus we see that the landscape pull on the soft SUSY

FIG. 4. Running of $m_{H_u}^2$ vs Q for several choices of $m_{H_u}^2(\Lambda)$ in the NUHM2 model for $m_0 = 5$ TeV, $m_{1/2} = 1.2$ TeV, $A_0 = -8$ TeV and $\tan\beta = 10$ for values of $m_{H_u,d}$ shown in the figure. The radiatively driven natural SUSY (RNS) case is green. We also show several unnatural SUSY model parameters.

FIG. 5. Contributions $\text{sign}(\Sigma_u'') \sqrt{|\Sigma_u''(\tilde{t}_{1,2})|}$ to the weak scale vs $A_t(\text{weak})$ in the NUHM2 model with $m_0 = 5$ TeV, $m_{1/2} = 1.2$ TeV, $m_A = 2$ TeV, $\mu = 200$ GeV, and $\tan\beta = 10$.

breaking term $m_{H_u}^2(\Lambda)$ to large values is just what is needed to gain a natural value of $m_{H_u}^2$ at the weak scale. This sort of situation has been dubbed as *living dangerously* in Refs. [55,56] since the soft term is selected to be as large as possible such that EW symmetry is barely broken.⁶ In Refs. [22,23], this situation is called radiatively driven naturalness, or *radiative natural SUSY*, since the largest viable $m_{H_u}^2(\Lambda)$ soft terms lead to natural values of $m_{H_u}^2$ at the weak scale.

A second example of living dangerously within the string landscape is shown in Fig. 5, where we show the contributions $\Sigma_u''(\tilde{t}_{1,2})$ to the weak scale versus A_t for the same NUHM2 parameter choices as in Fig. 4. Here, we see the contribution $\Sigma_u''(\tilde{t}_{1,2})$ are rather large negative for $A_0 \sim 0$ with $\text{sign}(\Sigma_u''(\tilde{t}_{1,2})) \cdot \sqrt{|\Sigma_u''(\tilde{t}_{1,2})|} \sim -400$ GeV (in which case we might expect a pocket universe weak scale of $m_Z^{\text{PU}} \sim 400$ GeV). As A_0 moves to large negative values, then we find a point around $A_0 \sim -5$ TeV where both terms become small, yielding contributions to the weak scale which are indeed comparable to our universe's measured value. For much larger (negative) values of A_0 , then we rapidly move into the zone of charge-and-color-breaking (CCB) minima since top squark squared-mass soft terms are driven negative. Thus, in this case, the A_0 trilinear soft term is statistically preferred to be large (negative) values—but stopping short of CCB minima. This again leads to natural contributions to the weak scale.

A beautiful consequence of the statistical draw to large (negative) A_0 values is that this induces large mixing in the top-squark sector, which also ends up maximizing the value of m_h [20,21]. The case here is shown in Fig. 6 where we plot the light Higgs mass m_h versus A_0 for the same parameters as in Fig. 5. For an unnatural value of $A_0 \sim 0$, then we expect $m_h \sim 119$ GeV. However, as A_0 increases to large (negative) values, then $m_h \rightarrow 124$ –126 GeV, in accord with its measured value in our universe. A third example occurs if we allow the

⁶Arkani-Hamed and Dimopoulos [55] state: “Anthropic reasoning leads to the conclusion that we live dangerously close to violating an important but fragile feature of the low-energy world...,” in this case, appropriate electroweak symmetry breaking.

FIG. 6. Value of m_h vs A_t in the NUHM2 model with $m_0 = 5$ TeV, $m_{1/2} = 1.2$ TeV, $m_A = 2$ TeV, $\mu = 200$ GeV, and $\tan \beta = 10$.

landscape to statistically select large values of $m_{1/2}$. This is shown in Fig. 7(a) where we plot $\sqrt{|\Sigma_u^u(\tilde{t}_{1,2})|}$ versus $m_{1/2}$ for fixed $m_0 = 5$ TeV and other parameters fixed as in the Caption. In this case, then as the gaugino masses increase, especially the gluino mass, then large $SU(3)_C$ contributions M_3 to the top-squark soft masses tend to drive them to large values, thus increasing the $\Sigma_u^u(\tilde{t}_{1,2})$ contributions to values well beyond our measured value of the weak scale. In such cases, we would expect too large a value of the weak scale, typically in violation of Agrawal *et al.* bounds. These would lead to violations of the so-called “atomic principle,” and atoms as we know them would not form with too large a value of m_{weak} . In Fig. 7(b), we show the values of $\Sigma_u^u(\tilde{t}_{1,2})$ versus m_0 for fixed $m_{1/2} = 1.2$ TeV. In this case, if the scalar soft terms become too large, then again the values of $\sqrt{|\Sigma_u^u(\tilde{t}_{1,2})|}$ become too large and we would gain a pocket universe weak-scale value in excess of bounds from nuclear/atomic anthropic requirements.

B. Naturalness versus stringy naturalness

Next, we would like to explore how the notion of stringy naturalness compares to conventional naturalness measures.

FIG. 7. Plot of $\sqrt{|\Sigma_u^u(\tilde{t}_{1,2})|}$ vs (a) $m_{1/2}$ for $m_0 = 5$ TeV and (b) m_0 for $m_{1/2} = 1.2$ TeV in the NUHM2 model. We also take $A_0 = -8$ TeV, $\mu = 200$ GeV, $m_A = 2$ TeV, and $\tan \beta = 10$. The horizontal lines show where contributions to m_{weak} exceed a factor four times its measured value in our pocket universe.

To begin, we plot naturalness contours in Fig. 8(a) in the m_0 versus $m_{1/2}$ plane of the CMSSM/mSUGRA model for $A_0 = 0$, $\tan \beta = 10$ and $\mu > 0$. The $\Delta_{\text{HS}} < 100$ contour shows the region of lighter top squarks for small A_0 parameter in the low m_0 , low $m_{1/2}$ region. The orange contour denotes where $\Delta_{\text{BG}} < 30$ while the green contour denotes where $\Delta_{\text{EW}} < 30$. These latter two measure reflect focus point behavior in that TeV-scale stops are still natural (since the contours are relatively flat with variation in m_0). The LHC limit on $m_{\tilde{g}} > 2.25$ TeV is shown as the magenta contour. This plane might lead to skepticism regarding weak-scale SUSY since the LHC allowed region is so far beyond the naturalness upper bounds. Also, in this plane, the light Higgs mass m_h is always below 123 GeV. The important lesson for now is that the *more* natural regions occur at the lowest m_0 and $m_{1/2}$ values, where the various measures are smallest, and the sparticle masses are closest to the measured weak scale.

For comparison, we show the same m_0 versus $m_{1/2}$ plane in Fig. 8(b), but this time for the NUHM2 model where $\mu = 200$ GeV, $m_A = 2$ TeV, $A_0 = -1.6m_0$ and as before $\tan \beta = 10$. In this case, the lower left region of the plot actually leads to CCB minima (blue contour) so that Δ_{HS} is not computable. Δ_{BG} is still computable and has shrunk down into the lower-left due to large A_0 term contributions. However, we now see that the Δ_{EW} values have expanded out to much larger m_0 and $m_{1/2}$ values since it is largely determined by the $\Sigma_u^u(\tilde{t}_{1,2})$ values, since μ is fixed to be near the measured EW scale. Here, a substantial amount of natural SUSY parameter space lies beyond LHC gluino mass limits. Nonetheless, the lower portions of m_0 , $m_{1/2}$ space are more natural since they yield smaller values of Δ_{EW} . Also, we show contours of $m_h = 123$ and 127 GeV. The region with $m_h = 125 \pm 2$ GeV overlaps nicely with the natural SUSY region, with plenty of parameter space beyond the LHC gluino mass limit.

Now we would like to compare the previous natural SUSY regions against the regions preferred by Douglas’ stringy naturalness. To accomplish this, next we show again in Figs. 9(a)–9(d) the m_0 versus $m_{1/2}$ plane for the NUHM2 model with the same parameters as in Fig. 8(b). We generate SUSY soft parameters in accord with Eq. (7) for various values of

FIG. 8. The m_0 vs $m_{1/2}$ plane of (a) the mSUGRA/CMSSM model with $A_0 = 0$ and (b) the NUHM2 model with $A_0 = -1.6m_0$, $\mu = 200$ GeV, and $m_A = 2$ TeV. For both cases, we take $\tan \beta = 10$. We show contours of various finetuning measures along with Higgs mass contours and LEP2 and LHC Run 2 search limits.

$n = 2n_F + n_D - 1 = 1, 2, 3$, and 4 .⁷ The more stringy natural regions of parameter space are denoted by the higher density of sampled points.

⁷The high n values allow for a consistent sampling of NUHM2 parameter space since here we fix the A_0 parameter in terms of m_0 so it never gets too large (which would lead to CCB minima and nonanthropic vacua) as compared with Ref. [40].

In Fig. 9(a), we show the stringy natural regions for the case of $n = 1$. Of course, no dots lie below the CCB boundary since such minima must be vetoed as they likely lead to an unlivable pocket universe. Beyond the CCB contour, the solutions are in accord with livable vacua. However, now the density of points *increases* with increasing m_0 and $m_{1/2}$ (linearly, for $n = 1$), showing that the more stringy natural regions lie at the *highest* m_0 and $m_{1/2}$ values which are consistent with generating a weak scale within the Agrawal bounds. Beyond these bounds, the density of points of course drops to

FIG. 9. The m_0 vs $m_{1/2}$ plane of the NUHM2 model with $A_0 = -1.6m_0$, $\mu = 200$ GeV, and $m_A = 2$ TeV and (a) an $n = 1$ draw on soft terms, (b) an $n = 2$ draw, (c) an $n = 3$ draw, and (d) an $n = 4$ draw. The higher density of points denotes greater stringy naturalness. The LHC Run 2 limit on $m_{\tilde{g}} > 2.25$ TeV is shown by the magenta curve. The lower yellow band is excluded by LEP2 chargino pair search limits.

zero since contributions to the weak scale exceed its measured value by a factor 4. There is some fluidity of this latter bound as indicated in Fig. 1, so values of $\Delta_{EW} \sim 20\text{--}40$ might also be entertained. The result that stringy naturalness for $n \geq 1$ favors the largest soft terms (subject to m_Z^{PU} not ranging too far from our measured value) stands in stark contrast to conventional naturalness which favors instead the lower values of soft terms. Needless to say, the stringy natural favored region of parameter space is in close accord with LHC results in that LHC find $m_h = 125$ GeV with no sign yet of sparticles.

In Figs. 9(b)–9(d), we show the same m_0 versus $m_{1/2}$ planes but for $n = 2, 3$, and 4. As n steadily increases, the stringy natural region is pushed more strongly to large values of m_0 and $m_{1/2}$ so that relatively few vacua lie below the LHC gluino mass limit. Indeed, we would say that the stringy naturalness prediction is that LHC should see a Higgs mass around 125 GeV with no sign yet of sparticles.

VI. WHY HIGH-SCALE SUSY IS LIKELY A RARE OCCURRENCE IN THE LANDSCAPE

While it is often argued that the landscape opens new territory in model building and motivation for finetuned SUSY models, here we will argue that such models should be, while possible, relatively rare occurrences on the string theory landscape. The rationale is usually that since the cosmological constant is finetuned, then perhaps a similar mechanism allows for a finetuned weak scale. Here we would counter with two observations. First, in Weinberg's approach, the cosmological constant is about as natural as possible subject to allowing for pocket universes that allow for galaxy condensation. Second, there is at present no plausible alternative for a tiny cosmological constant other than the landscape selection. As alternatives to various high-scale SUSY models described below, SUSY with radiatively driven naturalness should be a relatively common occurrence on the landscape, as argued above, whereas finetuned models, while possible, should be relatively rare.

(1) *Split SUSY*. Split SUSY was proposed in the aftermath of the emergence of the landscape as a model which retained the desirable SUSY model feature of gauge coupling unification and a WIMP dark matter candidate whilst eschewing the requirement of weak-scale naturalness. In this approach, then, SUSY scalar masses other than the SM Higgs can range from $\sim 10^3$ TeV up to possibly 10^8 TeV [32,57,58]. Meanwhile, gauginos and higgsinos are highly split from the scalars and can occupy masses in the 0.1–10 TeV range. This type of split spectrum maintains gauge coupling unification and a mixed gaugino-higgsino type WIMP. The ultraheavy scalars suppress flavor- and CP-violating processes and thus explain the lack of such effects in experiments. Recent work comparing split SUSY models with the measured value of $m_h \simeq 125$ GeV favors a lower range of scalar masses $\tilde{m} \sim 10\text{--}10^4$ TeV [59]. Upon integrating out the supermassive scalar particles, then the low-energy effective theory [60] contains SM particles plus the gauginos and higgsinos. Thus quadratic divergences should give rise to models as in Fig. 2 with a scale $\Lambda \sim \tilde{m}$, i.e., the required finetuning leads to a rare occurrence on the landscape for uniformly distributed values of μ_{SM}^{eff} .

(2) *High-scale SUSY*. In high-scale (HS) SUSY, one again discards naturalness in the hope that the landscape will solve the Big hierarchy problem. However, for HS SUSY, one allows *all* the SUSY partners to obtain large masses. The energy scale for HS SUSY may be labeled as Λ_{HS} and values considered in the literature range from $10^2\text{--}10^9$ TeV [33]. Thus the matching conditions between the MSSM and the SM effective field theories are implemented around the scale Λ_{HS} . The resultant Higgs mass dependence on μ_{SM} will be as in Fig. 2 so again we would expect HS SUSY to be a rare occurrence on the landscape as compared to RNS.

(3) *PeV SUSY*. In PeV SUSY [34], SUSY breaking occurs via a charged (i.e., nonsinglet) hidden sector F term leading to scalar masses at the PeV scale ($1 \text{ PeV} = 10^3 \text{ TeV}$), while gaugino soft breaking terms are suppressed and hence dominated by the anomaly-mediation form so that the wino turns out to be LSP. A 2–3 TeV wino is suggested to gain accord with the measured dark matter density which in turn suggests $m_{3/2} \sim \tilde{m} \sim \text{PeV}$. Some motivation for the PeV scale also comes from the neutrino sector by assuming neutrinos charged under an additional $U(1)'$ so that Dirac neutrino masses arise from non-renormalizable operators. The light Higgs mass is expected at $m_h \sim 125\text{--}145$ GeV. The PeV SUSY models would correspond to curves in Fig. 2 with $\Lambda \sim 10^3$ TeV. Thus we would expect PeV SUSY to be relatively rare on the landscape.

(3) *Spread SUSY*. In Spread SUSY [35], three scales of sparticles occur. The first possibility is that scalar masses occur at $\tilde{m} \sim 10^6$ TeV with gauginos at 10^2 TeV and higgsinos around 1 TeV in order to gain accord with the measured dark matter abundance. A second possibility considered has scalars around $\tilde{m} \sim 10^3$ TeV with higgsinos an order of magnitude lower and gauginos at the TeV scale (where winos with mass ~ 3 TeV would saturate the measured dark matter abundance). Thus Spread SUSY models would correspond to curves from Fig. 2 with $\Lambda \sim 10^3\text{--}10^6$ TeV. We would thus expect Spread SUSY to be rare on the string landscape.

(4) *Minisplit*. Minisplit SUSY [36] emerged after the LHC Higgs discovery and attempted to reconcile Split SUSY with a Higgs boson mass of $m_h \simeq 125$ GeV. To accommodate the Higgs mass, the mass scale of the heavy scalars was decreased to (1) $\tilde{m} \sim 10^2$ TeV scale for a heavy $\mu \sim 10^2$ TeV model with TeV-scale gauginos (light AMSB) or (2) $\tilde{m} \sim 10^4$ TeV with 10^2 TeV gauginos but with small $\mu < \text{TeV}$ (heavy AMSB). Models with $U(1)'$ mediation were also considered. The minisplit models would thus correspond to curves in Fig. 2 with $\Lambda \sim 10^2\text{--}10^4$ TeV. These models should be rare on the landscape, but less rare than original Split SUSY models.

VII. IMPLICATIONS OF STRINGY NATURALNESS FOR COLLIDER AND DARK MATTER SEARCHES

In light of the implications of stringy naturalness, how then ought SUSY to be revealed at collider and dark matter search experiments? Since the superpotential μ parameter must not be too far removed from our measured value of the weak scale, then higgsinos must also be light. A compelling signature emerges from $\tilde{\chi}_1^0 \tilde{\chi}_2^0$ production at LHC [61,62] where $\tilde{\chi}_2^0 \rightarrow \tilde{\chi}_1^0 \ell^+ \ell^-$ in recoil against a hard radiated initial state jet. The

soft dilepton plus jet +MET signal should emerge slowly as more and more data accrues at LHC.⁸ Meanwhile, more conventional SUSY signatures such as those from gluino or top squark pair production might be visible at HL-LHC [64], but also may require an upgrade to HE-LHC since gluinos can range up to $m_{\tilde{g}} \lesssim 6$ TeV while stops are required at $m_{\tilde{t}_1} \lesssim 3$ TeV [65,66]. Same-sign diboson signatures from wino pair production are an additional possibility [67,68].

The required light higgsinos within the natural mass range $m(\text{higgsinos}) \sim 100\text{--}300$ GeV provide a lucrative target for an ILC-type e^+e^- collider with $\sqrt{s} > 2m(\text{higgsino})$. Such a machine would act as a higgsino factory where the various higgsino masses could be precisely determined. The higgsino mass splittings are sensitive to the (heavier) gaugino masses; consequent fits to gaugino masses could then allow for tests of hypotheses regarding gaugino mass unification [69].

With regard to dark matter searches, we remark that naturalness in the QCD sector requires the PQ solution to the strong CP problem and the concomitant axion a [70]. To ensure $\bar{\theta} \lesssim 10^{-10}$, then the SUSY axion model must be safe from gravity corrections. This can occur when both PQ symmetry and R -parity conservation emerge from underlying discrete R symmetries, such as the recent example of $\mathbb{Z}_{24}^R$ [46]. In this case, one gains a solution to the SUSY μ problem [45] via a Kim-Nilles operator leading to a SUSY DFSZ axion. While the SUSY DFSZ axion has suppressed couplings to photons (and may not be observable with present technology [71]) the thermally underproduced higgsinolike WIMPs, which make up typically only about 10% of dark matter, should ultimately be detectable via multiton noble liquid detectors [72].

VIII. CONCLUSIONS

In this paper, we have explored Douglas' notion of stringy naturalness—that the value of an observable $\mathcal{O}_2$ is more natural than $\mathcal{O}_1$ if the number of phenomenologically viable vacua giving rise to $\mathcal{O}_2$ is great than the number of vacua giving rise to $\mathcal{O}_1$ —and its relation to conventional naturalness, with regard to the big and little hierarchy problems and why the value of the weak scale m_{weak} in our pocket universe is only $m_{\text{weak}} \sim 100$ GeV. We interpret *phenomenologically viable* to mean a fertile patch of string theory vacua leading to the SM as the low-energy effective theory with a value for the weak scale not too far (a factor four) beyond our measured value, as required by the nuclear physics calculations of Agrawal *et al.* [38].

It is often claimed that the string theory landscape picture allows us to eschew the common notion of naturalness in that certain parameters, such as the cosmological constant or the magnitude of the weak scale, are environmentally determined. With regard to the SM, since there appears to be no theoretical preference for any value of Higgs potential parameter μ_{SM} , we would expect it to be roughly uniformly distributed across the decades of possibilities. For the SM to be valid up to scales $\Lambda_{\text{SM}} \gg m_{\text{weak}}$, only tiny permissible ranges of μ_{SM} could give rise to weak-scale values leading to livable pocket universes (Fig. 2). In contrast, for effective

theories like the MSSM, then since quadratic divergences all cancel, there appears to exist large swaths of superpotential μ values leading to weak scales at ~ 100 GeV. Thus we conclude that stringy naturalness would favor the MSSM over the SM as the appropriate low-energy effective field theory, in agreement with, and not opposed to, conventional notions of naturalness.

We also explored implications of stringy naturalness for various SUSY models. The CMSSM (mMSUGRA) model with $m_h \sim 125$ GeV—where Δ_{EW} is found to be $\sim 10^2\text{--}10^4$ —should be rather infrequent on the landscape since only tiny ranges of μ values lead to $m_{\text{weak}} \sim 100$ GeV. Likewise, the panoply of SUSY models with scalar masses in the $10^2\text{--}10^{11}$ TeV range (Split SUSY, HS SUSY, PeV SUSY, Spread SUSY, and Minisplit SUSY) all appear infrequent on the landscape due to the unlikelihood of vacua which have unrelated parameters compensating for overly large contributions to the weak scale. This is in spite of the rather general expectation that soft terms should be statistically selected for large values, as expected in stringy models with multiple hidden sectors.

The statistical draw to large soft terms is just what is needed for SUSY with radiatively-driven naturalness, where one is *living dangerously* in that if the soft terms are much larger, then one is placed into CCB or no EWSB vacua (which must be anthropically vetoed). This situation, that soft terms are expected as large as possible such that EW symmetry is barely broken and that all independent contributions to the weak scale are within a factor four of our measured value, is just what is needed to radiatively drive the soft terms to natural values.

In these radiative natural SUSY (RNS) models, we further compared the locus of stringy natural regions of parameter space in the m_0 versus $m_{1/2}$ plane to the conventionally natural regions. Here, there is a major difference: conventional naturalness favors soft terms as close to the 100 GeV scale as possible while stringy naturalness favors soft terms as large as possible such that the weak scale is not too far removed from our measured value of ~ 100 GeV. We can read off from Fig. 9 that stringy naturalness predicts a Higgs mass $m_h \sim 125$ GeV whilst sparticles remain (at present) beyond LHC reach. Needless to say, this *postdiction* has been verified by Run 2 data from LHC13.

The stringy natural RNS SUSY model gives rise to specific tests at collider and dark matter search experiment. We expect the soft dilepton plus jet plus MET signature to slowly emerge at HL-LHC as more and more integrated luminosity accrues, while gluino, top squark and wino pair production signals might require a HE-LHC for discovery. Stringy naturalness cries out for construction of an ILC e^+e^- collider with $\sqrt{s} > 2m(\text{higgsino})$ which would act as a higgsino factory. We still expect WIMPs at multi-ton noble liquid dark matter detection experiments but SUSY DFSZ axions will likely be difficult to detect.

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⁸Excess events of this type seem to be emerging already in a recent Atlas analysis with 139 fb^{-1} [63].

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