Student conceptual resources for understanding mechanical wave propagation

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Much of the literature contributing to physics instructors' knowledge of student ideas (KSI) reports common patterns of reasoning that are framed as discontinuous with canonical concepts. Our work contributes new KSI about mechanical wave propagation from a *resources* perspective, framing student thinking in terms of context-sensitive pieces of knowledge that are continuous with canonical physics concepts. The intent of this work is to inform instruction on mechanical waves by identifying and illustrating some of the conceptual resources that instructors might expect their students to use. To support instructor predictions about student thinking, we identify resources that are common across multiple samples and questions. Our data include written responses to three versions of a conceptual question about mechanical pulse propagation. We use an emergent coding scheme to characterize a total of 851 written responses from 6 universities in the United States. Our analysis reveals three common conceptual resources: (i) properties of the medium either impede or facilitate the motion of the pulse, (ii) the speed or duration of transverse motion affects pulse speed, and (iii) the speed of the pulse is affected by its kinetic energy. We show how each of these resources can be viewed as continuous with formal understandings of pulse propagation.

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I. INTRODUCTION

Knowledge of students' ideas is essential for effective teaching, according to conceptualizations of pedagogical content knowledge [1–3]. Most existing research that provides knowledge of student ideas (KSI) emphasizes ways in which student thinking is inconsistent with scientific understandings [4,5]. These research efforts have guided the development of instructional materials designed to target specific difficulties or misconceptions (see, e.g., Refs. [6,7] in the context of waves) and thereby improve students' conceptual understanding. Investigations of common student difficulties or misconceptions and the curricula informed by them have had a widespread impact on physics education: the pragmatic, high-leverage KSI that they provide supports instructors in anticipating content areas where students may struggle and in designing instruction that considers students' existing ideas [8,9].

Physics education research focused on common student difficulties and misconceptions is grounded in constructivist learning theory [9–11], which assumes that “all individuals must construct their own concepts, and the knowledge they already have...significantly affects what they can learn” [9].

Also consistent with constructivism is the *resources* theory of knowledge, which sees student thinking as composed of inherently sensible pieces of knowledge that are activated in the moment to construct concepts, arguments, or explanations [11–15]. Research that embodies a resources perspective tends to focus on student ideas that “serve as significant input to the process of conceptual growth” [13] rather than on ideas that represent obstacles to learning. In light of the significant impact that investigations of common incorrect patterns of student reasoning have had on physics education, authors have called for “complementary research to identify possible conceptual progenitors of expert understanding in students' intuitions” [16].

This “complementary research” is important for both pedagogical and theoretical reasons. From a pedagogical point of view, a resources approach to KSI motivates instruction to refine, extend, and build from existing knowledge [13,14,16], rather than to confront, remedy, or overcome incorrect ideas. From a theoretical point of view, resources theory makes claims that have not yet been empirically demonstrated for large numbers of students (e.g., that resources are context dependent). Resources-oriented KSI research—particularly research into ideas that are common across many students—has the potential to support theoretical development and instructional practice.

Relatively few studies have investigated the common conceptual resources that students use to reason about specific physics topics. Most resources-oriented research has focused on developing theory or building a case for the

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pedagogical significance of resources theory, and most research documenting *common* patterns of student thinking has focused on misconceptions or difficulties. For example, research on student thinking about mechanical waves mainly attends to the ways in which student thinking is inconsistent with canonical physics concepts, and provides instructors with KSI in the form of common patterns of incorrect reasoning (see, e.g., Refs. [17–22]). A notable exception is Ref. [23], which identifies some possible conceptual resources that students might use to understand mechanical wave propagation. Our research extends such work both pedagogically and theoretically by empirically investigating some common conceptual resources that students use to reason about mechanical wave propagation.

Mechanical wave propagation is especially interesting to us because of its prominence as a foundational topic in both K–12 and introductory university physics [24], and thus its potential relevance and impact for physics instructors. The topic of mechanical wave propagation (a) often introduces a scientific understanding of what a wave *is*, (b) serves as a concrete and accessible foundation for understanding more abstract wave phenomena (namely, electromagnetic waves) that are central to many areas of advanced physics study, and (c) is challenging for students, according to the literature [25].

In this paper, we address the following research questions: What are some of the common conceptual resources that introductory university physics students use to reason about mechanical wave propagation? How does student use of these resources vary across questions about mechanical wave propagation? We report three common conceptual resources for mechanical wave propagation: (i) properties of the medium either impede or facilitate the motion of the pulse, (ii) the speed or duration of transverse motion affects pulse speed, and (iii) the speed of the pulse is affected by its kinetic energy. In Sec. IV, we illustrate variation in how each of these resources was used by students in our study and we discuss how each of these resources may be productive for learning, even as they are not always fully correct. We also discuss some possibilities for leveraging each one instructionally. In Sec. V, we show that these resources are elicited at different frequencies for different wave propagation questions: that is, we demonstrate patterns in context dependence of these resources that emerged in our analysis.

Our study contributes new, topic-specific KSI in the form of student resources for understanding mechanical wave propagation, and thereby serves the broader goal of supporting instructors in anticipating and building on student thinking about propagation in their classrooms. Further, our research illustrates what it might look like for an instructor to view their students' ideas as inherently sensible and potentially productive, which may be an entry point for a broader resources framing of instruction. Instructional approaches consistent with a broader resource-oriented framing have empirically demonstrated benefits, including promoting students' conceptual understanding (beyond

more traditional approaches) [26,27] and engagement in disciplinary practices [28–30], and supporting student agency in classrooms [28].

II. RESOURCES THEORETICAL FRAMEWORK

Our study draws on the resources theoretical framework, which models cognition in terms of pieces of knowledge—i.e., resources—that are activated in the moment in a context-sensitive way to construct arguments, theories, and concepts. Resources theory highlights the dynamic, emergent, complex-systems-like nature of student thinking [16] and emphasizes the continuity of student thinking with formal physics concepts. Our work draws on the resources theoretical framework as we identify patterns in student thinking about mechanical wave propagation, particularly by emphasizing the inherent sensibility and continuity of student thinking with formal physics.

Theoretical conceptualizations of the structure, deployment, role, and pedagogical significance of resources [11–15,31] have been developed by a number of researchers, and vary to some degree across different sources. Our work is most informed by the following tenets from resources theory:

- *Resources are sensible to the learner* [12–15,32]. Resources (for physics) are thought to be derived from a person's experiences of the physical world and prior learning and thus inherently sensible to the learner [12–15,32]. That is, learners have good reason for thinking in the ways that they do based on ideas that are useful for understanding many phenomena, even as these ways may be different than formal physics [15,32,33]. For example, diSessa [15] says that phenomenological primitives (“*p* prims,” which we consider to be a kind of resource) such as “closer means stronger” are best understood as “serv[ing] individuals well in dealing effectively with the physical world.” From a resources lens, even canonically incorrect ideas are seen as arising from activation of resources that support the student in making sense of physical phenomena, and that may be correct in other contexts. In the resources perspective, instructors can expect that learners' ideas are fruitful for sense making in some context(s), and that there is a way to “see the science” in learners' thinking [34].
- *Resources are activated in context-sensitive ways* [11,12,15,16,28]. Resources theory is a cognitivist perspective in that resources are understood to exist in the minds of individual students. They are “activated” as relevant in the moment to construct explanations, arguments, and concepts, and are not necessarily activated consistently across contexts. However, resources are generally considered stable in the sense that they can be reused and applied to multiple situations [35]. Resources are activated when they “make sense” or are deemed applicable by the student;

activation of a particular resource is influenced by a constellation of conditions related to the learner, the environment, and the question at hand [11,12,31]. Thus, resources are not expected to be consistently activated across a set of physically similar problems. For example, instructors can expect that questions that seem conceptually similar to them may elicit very different resources from students, and they can expect that the same student may display one idea in one context and a very different idea (or set of ideas) in another [12,32].

- *Learning involves changing the structure or activation of resources, and thus resources are continuous with more sophisticated scientific understandings* [11–13, 15,33,34]. Resources are depicted as the raw material out of which learners build understanding. For example, Smith *et al.* define resources as “any feature of the learner’s present cognitive state that can serve as significant input to the process of conceptual growth [13].” Learning involves refining resources and reorganizing the structure and conditions for their activation. This may include refining or increasing the degree of formality of individual resources, changing the role resources play, and/or changing the structure and connectedness of resources [12,13,15]. According to this view, the resources that novices activate as they seek to sense make about the natural world are also part of the structure of expert physics thinking. This tenet of resources theory affords an instructional expectation that learners’ existing understandings can be used to construct scientific explanations and concepts in the classroom [31,36]. It also embeds an understanding that resources themselves are *continuous* with formal physics: according to resources theory, resources have the potential to be fruitful for students science learning, and even ideas that are incorrect can be seen as “seeds” of sophisticated scientific concepts and practices [14,31,37–40]. That is, students’ own thinking can develop toward canonical physics understandings, support their engagement in rich disciplinary practices, or help to solve problems or create products that are meaningful to learners [31].

Our project takes up these three tenets: We identify resources that we see as sensible and we make an effort to make that sensibility visible to our readers. We also highlight an aspect of the context sensitivity of these ideas by showcasing question-dependent patterns in resource activation that recur across instructors and universities. In our work here, we primarily attend to resources that we see as continuous with canonical understandings of mechanical wave propagation. That is, we identify resources that we see as “seeds of science,” that resemble pieces of canonical models for wave propagation, or that represent scientific ways of thinking about wave phenomena [37]. In Sec. IV, we articulate ways in which the resources we report are continuous with formal physics in such a sense. Finally, we suggest ways in which instructors support their students

in refining and changing the structure and connectedness of these resources, toward a formal understanding of mechanical wave propagation.

The resources we report are phrased in terms of statements about mechanical wave propagation, e.g., “the speed or duration of transverse motion affects pulse speed.” Thus, the resources we report are smaller in grain size than scientific concepts (e.g., the concept of force), and concrete in description. This is consistent with depictions of resources in the literature, which vary in their grain size and in their degree of abstraction from particular physical phenomena. Specifically, resources are depicted as knowledge elements or ideas that are often of smaller grain size than scientific concepts (e.g., Ref. [15]), though they can also be more complex structures [35,41]. Examples of resource-like cognitive structures can be concrete, such as *the (less massive) car reacts twice as much (in a collision)* [14] or *active objects can exert forces* [42], and they can be abstract, such as *bouncing or force as mover* [15].

The resources we report are not the full set of student ideas in our data that are continuous with formal physics ideas. Our analysis focused on a subset of resources, informed by our goals of (i) contributing to KSI and (ii) supporting instructors in noticing and building on students’ potentially productive thinking. To support instructional predictions, we investigate *commonly activated* resources. We further focus on those resources that we judge to be most *instructionally relevant*, i.e., most likely to support shifts in instructional practice. For mechanical waves, we report students’ ideas about physical mechanisms for wave phenomena and about the ontology of waves. We do not report students’ resources for use of equations or discrete steps to solving a problem (although we have noticed such resources in our analysis).

III. THE PHYSICS OF MECHANICAL WAVE PROPAGATION

The motion of a single pulse that propagates without change in shape can be described by a single, constant velocity v . If the leading edge (or any other point) of the disturbance is located at position x_0 at some instant t_0 , the location of the disturbance at any later time can be described in the same way that the location of a Newtonian object could be described (i.e., $x_f = x_0 + vt$) [43]. Thus, we may treat the pulse as an abstract objectlike entity whose speed is defined by the distance it travels in a given time interval [44]. In a string or spring, the speed of the disturbance can be demonstrated—either empirically or mathematically via more detailed modeling—to increase when tension applied to the medium is increased, and to decrease when the mass density of the medium is increased (e.g., Ref. [7]).

To construct a more detailed mathematical depiction of transverse pulse propagation on a (nondispersive) string (or spring), the string can be modeled as a number of small beads of mass dm connected by massless strings of length dx and equal tension T [45,46]. In the continuum limit, one

can analogously consider a small segment of a massive string with uniform tension T , with length dx and mass μdx . According to this model a propagating pulse is described as the sequential, transverse disturbance and return to equilibrium of individual beads or segments of the medium. The wave equation for a disturbance on a string can be derived by considering the acceleration of a single bead or segment of string (we follow the derivations in Refs. [25,47]). According to Newton's second law

$$F_{\text{net}} = \mu dx \frac{\partial^2 y}{\partial t^2}. \quad (1)$$

The nonzero net force acting on the bead is due to the small difference in the directions of the two tension forces acting on it (Fig. 1). Assuming that tension is uniform throughout the system—i.e., the tension in all inextensible string segments is equal—the vertical and horizontal components of the net force are

$$F_{\text{net},x} = T \cos(\theta + d\theta) - T \cos \theta, \quad (2)$$

$$F_{\text{net},y} = T \sin(\theta + d\theta) - T \sin \theta. \quad (3)$$

Further assuming that all transverse displacements are small enough that the angle between any segment of string and the x axis is small (i.e., θ and $d\theta$ are small), the horizontal force is negligible. Using the small angle approximation $\sin \theta \approx \theta$, the vertical force is reduced to $F_{\text{net},y} = T d\theta$, and $d\theta$ can be written as $(\partial\theta/\partial x)dx$. Finally, θ is approximately equal to the slope of the string, $\partial y/\partial x$. Thus, the net force on the particle can be given by

$$F_{\text{net}} \cong T dx \frac{\partial^2 y}{\partial x^2}. \quad (4)$$

Combining Eqs. (1) and (4) yields

$$T \frac{\partial^2 y}{\partial x^2} dx = \mu dx \frac{\partial^2 y}{\partial t^2}. \quad (5)$$

This is the wave equation

$$c^2 \frac{\partial^2 y}{\partial x^2} = \frac{\partial^2 y}{\partial t^2},$$

where the speed of the propagating disturbance is

$$c = \sqrt{\frac{T}{\mu}}. \quad (6)$$

In short, the pulse is modeled in terms of infinitesimal segments of a string that are displaced from equilibrium. The disturbance is shown to propagate in that it obeys the wave equation, and the motion of neighboring parts of the string is coupled by the tension force between them. Propagation is thus determined by properties of the medium that affect the transverse acceleration of small segments of the medium.

In the string-and-bead model, there is kinetic energy associated with each point mass, and the amount depends on its mass and transverse velocity [43]. The kinetic energy density is given by

$$\frac{dK}{dx} = \frac{1}{2} \mu \left(\frac{\partial y}{\partial t} \right)^2. \quad (7)$$

As the wave propagates, energy is transferred from one point mass or segment of the string to the next via the tension force interaction between them. The system also has potential energy associated with the slight stretching of its disturbed segments, which can be modeled by considering the connecting segments as massless springs that obey Hooke's law [43,45]. As point masses are displaced, the springs attached to them are stretched to a length of $dl = \sqrt{dx^2 + dy^2}$. Since the tension in the string is assumed uniform, the potential energy gained by the segment is $dU = T(dl - dx)$, or the work done in stretching the string. Assuming that the transverse displacement is small, the potential energy density can be given by

$$\frac{dU}{dx} = \frac{1}{2} T \left(\frac{\partial y}{\partial x} \right)^2. \quad (8)$$

Equations (7) and (8) show that a disturbance carries with it both kinetic and potential energy: $\partial y/\partial t$ and $\partial y/\partial x$ are only nonzero in regions where the medium is disturbed. Equations (7) and (8) also show that the amount of energy carried by the pulse is affected by the properties of the medium and by the shape of the pulse (which is determined by the motion of the source). It can be shown that for any

FIG. 1. String-and-bead model.

traveling wave, Eqs. (7) and (8) are equal, and therefore a pulse with greater kinetic energy must also have greater potential energy by the same amount. In Sec. VI, we will describe how three conceptual resources that were common in our analysis resemble aspects of this model for mechanical wave propagation.

IV. PREVIOUS INVESTIGATIONS OF STUDENT UNDERSTANDING OF MECHANICAL WAVE PROPAGATION

Previous investigations of student understanding of transverse mechanical wave propagation report two clusters of student ideas: (i) the speed of a pulse or wave depends on or is determined by how it is generated, and (ii) wave speed, wavelength, and frequency can all be manipulated (and held constant) experimentally.

Several authors report that students commonly use the idea that the speed of a pulse or wave depends on how it is generated when answering the original pulse-flick question [22,46,48] (Fig. 2). Many students answer that flicking a string harder will generate a faster-moving pulse [6,17,20,22,23,48–50]. Caleon and Subramaniam [19] report a slightly more nuanced version of this idea: that as frequency increases, wave speed also increases because more energy is being imparted to the particles of the medium. Wittmann [6,22,49] has interpreted this idea as treating a pulse like a Newtonian particle: just as throwing a ball harder makes it move faster, “throwing” a pulse on a string harder makes it move faster.

Multiple authors report that students rely on the equation $v = \lambda f$ to reason about the relationship between propagation speed, wavelength, and frequency. Based on this equation, students may think that wave speed depends

on frequency (or vice versa) [17–19,21]. This idea may be associated with reasoning that a higher frequency means “more energy is imparted to the particles of the medium [19],” so the wave travels faster. Kryjevskaja, Stetzer, and Heron [21] summarize this kind of idea as a difficulty understanding how the variables v , λ , and f can be manipulated experimentally—that is, that wave speed is a property of the medium, frequency can be manipulated at the source, and wavelength is determined by the frequency and propagation speed in a given medium.

With a few exceptions [23,49] these studies of student thinking about mechanical waves have attended to the ways in which student thinking is inconsistent with canonical physics concepts and provide instructors with KSI in the form of common patterns of incorrect reasoning that are the result of the interaction between student thinking and the discipline.

V. DATA COLLECTION & METHODS OF ANALYSIS

We analyzed student responses to three versions of a conceptual question about pulse propagation on a string or spring (the “pulse-flick question”). The original question was drawn from Tongchai *et al.* [48] and is a modified version of a question from the Waves Diagnostic Test [6]. We chose this question as a starting point for our investigation because we expected the pulse flick phenomenon to be accessible to students with a range of prior experiences and because student difficulties in the context of this question have been well documented [6,18,22,23,48], which affords the opportunity to compare students’ resources and difficulties in the context of this question. Since we were interested in students’ reasoning in addition to their answer choices, we modified

<u>Original Pulse-Flick Question</u>	<u>Tension Pulse-Flick Question</u>	<u>Mass Density Pulse-Flick Question</u>
<p>One end of a long taut string is tied to a distant pole and the other end of the string is held by a student (see figure below). The student quickly flicks her hand up and down to create a pulse moving toward the pole.</p> <p>a. She now wants to produce a pulse on the same string that takes less time to reach the pole, keeping her hand motion the same as in the initial experiment. How can she do this? Explain your reasoning for your answer.</p> <p>b. She still wants the pulse to reach the pole in less time by making a change to the string. How can she do this? Explain your reasoning for your answer.</p>	<p>Consider the following two scenarios: In scenario 1, your Teaching Assistant (TA) creates a pulse by flicking the end of a spring, as in the figure above. In scenario 2, your TA pulls the spring so that it is more taut (<i>i.e.</i>, increases the tension in the spring) and then creates a pulse by flicking the end of the spring in the same way. The pulse in scenario 2 travels down the spring faster (<i>i.e.</i>, has a larger speed) than the pulse in scenario 1.</p> <p>Why would it make sense for a pulse to move faster on a higher-tension spring?</p>	<p>Consider the following two scenarios: In scenario 1, your Teaching Assistant (TA) creates a pulse by flicking the end of a spring, as in the figure above. In scenario 2, your TA uses a heavier spring (<i>i.e.</i>, increases the mass density of the spring) and then creates a pulse by flicking the end of the spring in the same way. The pulse in scenario 2 travels down the spring more slowly (<i>i.e.</i>, has a lower speed) than the pulse in scenario 1.</p> <p>Why would it make sense for a pulse to move more slowly on a heavier spring?</p>

FIG. 2. Three versions of the pulse-flick question. The original question is drawn from Tongchai *et al.* and is modified from Wittmann’s Waves Diagnostic Test.

the original question to be free response rather than multiple choice. After receiving an initial set of student responses, we made several modifications to the question to elicit more about *why* students think that changing the tension or mass density—the two most common answers to the original pulse flick question—would change the pulse speed. This effort produced the tension pulse-flick question and the mass density pulse-flick question. All three versions of the pulse-flick question are shown in Fig. 1.

In total, we analyzed 851 student responses across the three versions of the pulse-flick question. These questions were given in introductory physics courses at five U.S. Universities: Baylor University, California Polytechnic State University—San Luis Obispo (CPSLO), Seattle Pacific University (SPU), University of Washington (UW), and Western Washington University (WWU). A typical introductory university-level physics course in the U.S. includes instruction on wave propagation, superposition, and reflection, as well as wave optics and interference. In all samples, our questions were given as a required part of the coursework (as homework or on an exam, not for extra credit) after some instruction on mechanical wave propagation. At least 70% of students enrolled in each course participated in our study. In most cases, the questions were administered to students after some or all instruction. Table I provides additional details about the samples.

To analyze this data, two authors (L. M. G. and A. D. R.) created an emergent coding scheme [51] that characterized the resources that students were using to reason about mechanical wave propagation in the pulse flick question. To develop this coding scheme, L. M. G. and A. D. R. separately examined a subset of student responses from each version of the question and each participating institution, identifying preliminary resources—ideas that we deemed continuous with canonical wave propagation concepts—used by students in the subset. Our attention in this process was toward students’ *conceptual* resources, rather than resources for mathematical procedures, problem-solving steps, or epistemological resources. In many cases, these early-analysis resources we identified were quite specific and

concrete. For example, one early-analysis resource was “increasing tension means the pulse ‘wants’ to reach equilibrium faster.” When we had completed our preliminary analyses for all three versions of the pulse-flick question, we looked for themes across questions—the *common* resources—by grouping early-analysis resources according to our sense of their productive substance. For example, the resources “higher tension means every particle moves faster,” “pulse speed is related to how fast particles return to equilibrium,” and “increasing tension means the pulse ‘wants’ to reach equilibrium faster” were grouped into the code the speed or duration of transverse motion affects pulse speed. The development of our final coding scheme was an iterative process; several versions of the scheme were proposed and tested on larger subsets of data to assess whether they accurately reflected students’ ideas and appropriately captured the continuity of these ideas with formal physics. Ultimately, this process resulted in a coding scheme with five codes, three of which appeared frequently in our dataset; these are the three discussed in Sec. IV.

Our interpretation of the resources we present here is shaped by our own understandings of wave propagation (as we describe in Sec. III) and by the orientation toward student ideas that is embodied by resources theory (i.e., students’ ideas can be fruitful for their learning). We expect that another researcher with different understandings and commitments would interpret these data differently. Our effort here has been to describe the categories of student thinking that we infer, expecting that others may be able to “f[i]nd or recogniz[e]” these categories “once [they] ha[ve] been described to them by the original researcher[s]” (us) [52]. It is not to suggest that “other researchers [would] find the same conceptions or categories if they were doing the study for the first time” [52]. Further, when we say students “use” a particular resource, we mean that their response is consistent with that resource—that is, we acknowledge that the existence of these resources is hypothetical, as is their use in these specific cases.

To be considered “common,” a resource had to be used by at least 10% of students in at least one sample, and it had to be used by students in response to more than one question. Because these resources are used in many student responses across multiple sources of heterogeneity (i.e., instructor, institution, and version of the question) [53], we expect that these are resources that instructors of comparable courses might anticipate in propagation contexts. In this sense, our results contribute KSI that may support instructors in anticipating and leveraging these ideas in their classroom. Because much of our data come from student responses after some instruction, it is appropriate to consider these resources that may be leveraged *during* instruction, after students have had some exposure to mechanical waves concepts.

In the final analysis for this paper, L. M. G. and A. D. R. independently coded each student’s response according to the final scheme. A single response could be given more

TABLE I. Administration contexts and participation rates for three versions of the pulse-flick question. For all samples, participation was least 70% of students enrolled in the course.

Question	Institution and academic term	Number of students in sample
Original	CPSLO, Winter 2017	67
	UW, Autumn 2016	134
Tension	CPSLO, Spring 2017	159
	Baylor, Spring 2018	173
	WWU, Autumn 2017	103
Mass density	CPSLO, Autumn 2017	35
	SPU, Spring 2018	42
	UW, Winter 2018	138

than one code when responses were seen as using more than one resource. We compared the codes L. M. G. and A. D. R. separately assigned to each response to establish a sense of percentage agreement.

Standard statistical measures of interrater agreement, such as Cohen's kappa, require that codes are mutually exclusive and/or exhaustive [54]. Our coding scheme, in which it is possible for a single response to be assigned multiple (or no) resource codes, reflects our theoretical perspective that multiple resources can be activated to construct an idea. As a single idea can be assigned multiple resource codes, and we do not claim our codes are exhaustive, standard statistical measures are not appropriate for our data. As a measure of percentage agreement, we took the normalized difference between the total number of codes possible and the total number of disagreements between the two coders, represented by

$$1 - \frac{N \text{ total disagreements}}{(N \text{ possible codes})(N \text{ coded responses})}$$

For example, for a scheme with two codes, if a student's response was assigned codes 1 and 2 by the first coder and code 1 by the second, there would be 1 disagreement for this dataset of one response. Since there are two possible codes, this would represent a percentage agreement of 50%. The percentage agreement for our analysis of the dataset for this paper was 91%.

In tabulating the fraction of responses that used each resource (reported in Secs. VI and VII), we included only those codes that were in 100% agreement. That is, a response was included in these fractions only if both coders agreed that the response used the resource. However, the *percentage agreement* we report above—91%—was determined from our coding of the full dataset, including all disagreements between coders.

VI. DESCRIPTION OF COMMON CONCEPTUAL RESOURCES

Here, we report three resources for understanding mechanical wave propagation that were common across institutions and versions of the pulse-flick question. Students use the resources (i) properties of the medium either impede or facilitate the motion of the pulse (the properties of the medium resource), (ii) the speed or duration of transverse motion affects pulse speed (the transverse speed resource), and (iii) the speed of the pulse is affected by its kinetic energy (the energy resource). These three do not represent all of the resources that we identified in our data, or even all that may have been common. The resources we report are those that represent student thinking about physical mechanisms for wave phenomena and about the ontology of waves. We have focused on these because we judge them to be pedagogically significant.

In this section, we provide examples to illustrate the various ways in which these resources manifest in student responses. Our goal in doing so is to highlight diversity—that the same resource (e.g., properties of the medium) may show in quite different ways among different students and in response to different questions; It is not to make claims about the relative frequency of any one instantiation of a given resource. We also discuss here how these resources, while not necessarily complete or canonically correct, are continuous with the scientific understanding of mechanical wave propagation that we have described in Sec. III.

A. Properties of the medium either impede or facilitate the motion of the pulse

The properties of the medium resource was the most commonly used resource across the three versions of the pulse-flick question: approximately 60%–80% of students used it in response to the original question, ~10%–15% in response to the tension question, and ~25%–30% in response to the mass density question. This resource models pulses as unified entities whose motion is distinct from the motion of the medium and is described by a single velocity v ; the pulse velocity is related to the properties of the medium that the pulse moves through (e.g., tension or mass density). This resource is continuous with the model for a pulse propagating on a string that we described in Sec. III.

In response to the pulse-flick question, some students explain that making changes to the medium means pulses will move faster or slower through it, or that changing the medium makes it easier or harder for the pulse to move through it. Some responses describe a qualitative relationship between properties of the medium and pulse speed, such as

“If the string¹ is more taut, the wave wants to pass through faster.” (tension question)

“Decreas[ing] the tension in the string [increases pulse speed because] if the string is less tense, or not as stiff, the wave can travel faster.” (original question)

These responses and others like them state directly that the speed of the pulse depends on a property of the medium (in this case, the tension). Other responses use an equation to relate pulse speed to properties of the medium:

¹The original version of the pulse-flick question refers to a pulse on a *string*, while the mass-density and tension versions refer to a pulse on a *spring*. We do not notice any clear relationship between (a) the types of responses students offered and (b) whether the question refers to a spring or a string. Further, we notice that students do not consistently take up the wording in the question they were asked: some students refer to a *string* when the question refers to a *spring*, and vice versa. In the examples in this section, we have chosen to preserve the original language used by each student, and we view the words “string” and “spring” as interchangeable.

“My intuition actually is from the equation $v = \sqrt{T/\mu}$. So when $\mu \uparrow$, and T doesn’t change, the $v \downarrow$.” (mass density question)

“As we have witnessed by the equation $v_{\text{string}} = \sqrt{T/\mu}$ ($\mu = m/L$), if she still wants the pulse to reach the pole in less time (wave speed) then she needs to find a string with a smaller m/L ratio or smaller μ .” (original question)

“ $v = \sqrt{T/\mu}$ [so] the speed of propagation of the pulse is directly related to the square root of the tension.” (original question)

These examples, and others like them, describe the motion of the pulse by a single speed v , and they use a (correct) quantitative expression to show that pulse speed depends on the properties of the medium. In the first and second examples, students use the equation $v = \sqrt{T/\mu}$ to support their arguments that higher pulse speed is associated with lower mass density. The third example interprets the same equation as demonstrating that tension directly affects pulse speed. Responses like these were especially common for the original pulse-flick question, where up to 90% of students cited an equation in their answers. However, simply stating the formula for pulse speed was not sufficient, on its own, to receive the properties of the medium code; such responses accompanied their use of the equation $v = \sqrt{T/\mu}$ with qualitative statements that relate pulse speed to properties of the string. For example, in all three responses above students use the equation $v = \sqrt{T/\mu}$ to support conceptual arguments that changing the mass density or tension in the string would change the speed of a pulse moving through it.

Some examples of this resource describe nascent mechanisms by which changing the tension or mass density will change the speed of the pulse:

“The higher the tension of the spring, the more sensitive it is to movement so the pulse can travel faster.” (tension question)

“Because tension is directly related to speed in waves, higher tension means that the pulse and speed can travel faster because higher tension makes a more linear path to travel.” (tension question)

“It makes sense for a pulse to move more slowly on a heavier spring because the velocity of the pulse depends on the string’s medium. A heavier string moves more slowly because it takes longer for the pulse to move more mass.” (mass density question)

These three examples treat the pulse as a distinct macroscopic entity moving on or through the medium and say that changing tension or mass density affects how the medium responds to the pulse. The first example explains that the speed of a pulse depends on the tension in the medium because increased tension means the medium can react more

quickly to the pulse. The second example connects increased tension with increased pulse speed, implying that the pulse can move more efficiently through a tauter string. The mechanism implied in this response is incorrect: pulses are not like objects that move along curved paths that can be straightened by increasing tension. However, the essence of the properties of the medium resource is still evident in this example; it appropriately associates tension with pulse speed. The third example argues that increasing mass density will decrease the pulse speed because it takes longer to displace something heavier, suggesting that the pulse is thought of as an entity that moves the medium out of the way. These examples only hint at mechanisms by which the medium affects pulse speed; details about the implied causal relationships are unstated. Nonetheless, nascent causal associations like these can be the beginnings of a more sophisticated mechanistic explanation of the relationship between pulse speed and properties of the medium, even if they are incorrect or incomplete [36].

Other examples of the properties of the medium resource make more formal mechanistic connections between properties of the medium and pulse speed, drawing on mechanics concepts such as energy, momentum, or Newton’s laws:

“It takes more energy to move something of greater mass. The pulse also must travel through more particles per unit of distance therefore it makes sense intuitively that it would take longer.” (mass density question)

“Springs have inertia and heavier springs will have more inertia because inertia is proportional to mass. Since inertia is the tendency to resist change in motion, a heavier spring will oppose the motion of a pulse more.” (mass density question)

Responses such as these include a sense of the physical mechanism by which a change to the medium affects pulse speed. The first example offers both macroscopic and microscopic arguments that mass density affects pulse speed because mass resists motion. The second example makes a clear distinction between the pulse and medium—the medium opposes the motion of the pulse—and explains that this is because mass or inertia resists motion. These, and other similar responses, not only state a relationship between the speed of a pulse and the properties of the medium it moves through, but also theorize a mechanism by which the medium affects the pulse.

The examples we provide in this section show that manifestations of this resource were quite varied, particularly with respect to the level of formality, quantification, or causality they embodied. As these examples illustrate, the properties of the medium resource treats the medium and the pulse as distinct—though not independent—macroscopic entities. This resource bears some resemblance to the previously reported difficulty treating a pulse as a Newtonian object because both treat pulses as distinct macroscopic entities. However, the resource focuses on

propagation as an interaction between the pulse and medium—i.e., it focuses on *propagation*—rather than on what the pulse *is*. This resource is continuous with formal physics: it qualitatively—and in some cases, quantitatively—connects pulse speed to properties of the medium. In responses that use this resource, students correctly identify one or more of the factors that influence pulse speed, consistent with the commonly taught model for the speed of a pulse on a spring or string ($v = \sqrt{T/\mu}$).

B. The speed or duration of transverse motion affects pulse speed

Students also commonly used the transverse speed resource. Students used this resource in all three versions of the pulse-flick question, though it was more common in the tension and mass density versions: between ~15 and ~25% used this resource in the tension question, and between ~10% and ~20% used it in the mass density question, but only 0%–5% used it in the original question. This resource acknowledges that the pulse is a propagating disturbance of parts of the string and that propagation speed is linked to the transverse motion of the medium. The transverse speed resource represents a nascent mechanistic model for propagation: particles or segments of the medium are sequentially displaced and return to equilibrium at a rate that depends on parameters of the pulse and/or medium. According to a canonical understanding of pulse propagation, the transverse motion of a part of the medium is determined solely by the motion of the source. Therefore, assuming the same source motion, the velocity and acceleration of part of the medium are not affected when properties of the medium change. Even so, the transverse speed resource is continuous with mathematical models for propagation that arise from considering the dynamics of the system (e.g., Ref. [47]): the string-and-bead model described in Sec. III begins by examining the transverse motion of a small segment of the medium, just as this resource does.

In some examples of this resource, faster or slower displacement is seen as being caused by the forces acting on, or acceleration of, a piece or particle of the medium. For example,

“In increasing the tension, the TA puts more force on the spring towards making it follow a straight line. This makes its acceleration towards the center greater, and thus, each individual point does ‘the wave’ more quickly. For the pulse to complete one length of the string, each point must complete the ‘wave’ motion. Therefore, since the points complete the wave more quickly, the pulse moves more quickly.” (tension question)

“When the tension is higher, the particles (or small part [s]) of the springs are pulling each other with more forces. If one particle goes upward, then the particle next to it will be dragged upward. With more forces

(=higher tension), it gets dragged even faster (tightly bound). This speed of getting dragged is the speed of the wave, so the higher the tension [the] higher the speed. ($c = \sqrt{T/\mu}$, with same μ .)” (tension question)

“It would make sense for the pulse to move more slowly on a heavier spring because you could represent each point of mass on the string with a free body diagram and the F_{net} on each point of mass is equal to mass times acceleration. ($F_{\text{net}} = ma$) Therefore, if the F_{net} on each spring is the same as scenario 1, when the mass increases in scenario 2, the acceleration must decrease to compensate. This means that each point on the spring will accelerate more slowly when moving away from the ‘center line’ of the spring, effectively resulting in the pulse moving more slowly.” (mass density question)

“If he used the same motion on the heavier string it would move slower because each part of the string is slower at moving up or down due to greater inertia which results in a slower wave propagation.” (mass density question)

All of these examples focus on a “point,” a “particle,” or a “part” of the string or spring, and in doing so they imply a discretized model of the medium such as the string-and-bead model. The first and second responses above use the transverse speed resource as they argue that increasing tension makes each particle accelerate more quickly, increasing the wave speed. The second response supports such an argument with a diagram illustrating a spring as a series of point objects connected by strings. The third and fourth responses also use this resource to connect the dynamics of a particle in the medium to propagation speed, but in these examples an increased mass supposedly makes each part of the medium more resistant to movement.

Other examples of this resource explicitly describe the dynamics of the system in terms of the tendency of the medium to return to equilibrium or the magnitude of the restoring force, as in these responses:

“The greater the tension in a wave, the higher the restoring force that returns it to equilibrium.” (tension question)

“If the tension is higher, the particles will move faster in the pulse because they want to return to equilibrium to have the lowest energy state. Whereas [with] loose rope the difference in energy of particles [at] rest and in a pulse isn’t as big so they’ll take longer to return to equilibrium.” (tension question)

Both responses imply that increased restoring force, or tension in the medium, causes a higher wave speed because the medium responds more quickly to the disturbance and tends toward equilibrium more strongly. These and other responses like them center on the tendency of the medium toward equilibrium, which can be a way to investigate the tendency of the medium to “pass displacement along” [45].

Another illustration of this resource is the answer that decreasing the amplitude of the pulse means that the vertical motion of part of the medium takes less time, therefore the pulse moves faster. For example,

“The higher tension will decrease the amplitude if the same amount of force is used, which means each displacement is shorter, [and] tak[es] less time to do [therefore] moving faster.” (original question)

This example instantiates the transverse speed resource by explaining that when each displacement takes less time, the pulse moves faster. In canonical models, amplitude and duration of the displacement are both determined by the source rather than by properties of the medium; thus, taken as a whole, this statement is incorrect. However, we argue that this instantiation of the transverse speed resource is still continuous with formal physics because it expresses a (correct) connection between the transverse motion of the medium and the propagation of the pulse.

Taken together, these examples illustrate that the transverse speed resource is a mechanistic connection between transverse motion of the medium and longitudinal motion of the pulse. The transverse speed resource resembles previously-reported difficulties or misconceptions related to the idea that flicking a string up and down faster generates a faster-moving pulse in that a faster flick represents a faster displacement and return to equilibrium.² In our study, we characterize this idea as a resource: students are deploying this idea in a mechanistic way that is continuous with the way physicists model mechanical wave propagation. In particular, it models a mechanical pulse as sequential, local disturbances of a medium communicated via interactions between parts of the medium. In using the transverse speed resource, students attend to the dynamics of the transverse motion of particles or disturbed parts of the string, which is necessary for constructing a mathematical model of a wave on a string [25,47]. Further, this resource is appropriate for predicting the relationship between particle velocity and pulse speed when other parameters of the pulse (e.g., amplitude, width) have been held constant.

C. The speed of the pulse is affected by its energy

Finally, students used the energy resource in answering the three versions of the pulse-flick question. Students deploying this resource use energy accounting (i.e., considering how energy is increased, decreased, transferred, or transformed) to explain or predict changes in pulse speed. In order to receive this code, responses had to explicitly associate the amount of energy in the system with the speed of the pulse or a part of the medium. Some responses use

the term “energy” to speak about the “ease” with which a pulse moves through a medium (e.g., *“It takes more energy to move something of greater mass...therefore it makes sense intuitively that it would take longer”*). This response and others like it focus on how the properties of the medium affect pulse speed, and thus are better represented by the properties of the medium resource. In contrast, responses coded as using the energy resource focus on how changing the amount of energy in the system affects the speed of the pulse propagating through it. This resource applies a powerful physical model to wave propagation and is continuous with a scientific conception of waves as carriers of energy because it associates potential and/or kinetic energy with the pulse.

The energy resource was most commonly used in the mass density version of the question—~10%–20% of students used it there—but was also present in the other two versions: ~0%–5% used this resource in the original question, and ~5%–10% in the tension question. Though all of the responses that received this code connected changes in energy to changes in pulse speed, there were a variety of ways students used this resource in their responses.

Some responses imply that the energy added to the system is equal in the two experiments proposed in the pulse-flick question. These responses use the energy resource to reason about how changes to the medium affect the speed of the pulse:

“There is kinetic energy being transferred into the spring when the professor creates the pulse. Since the mass of the second string is increased, it would make sense for the second spring to move slower because it has less velocity (if the amount of energy transferred is kept the same).” (mass density question)

“In a heavier spring, there is more mass in a given unit of length. It would take more energy to displace this mass than in a lighter spring. Since more energy is spent on actually displacing the spring in a wave there will be less energy to drive the pulse’s velocity, given that the same amount of energy was applied each time.” (mass density question)

“Having a tighter string means less energy gets represented as vertical displacement so that more energy goes into the pulse’s forward movement, i.e., speed.” (tension question)

The first example correctly identifies that energy is added to the system by the source and relates the velocity of the medium to the kinetic energy of the system and its mass, consistent with the equation for kinetic energy density $dK/dx = 1/2\mu(\partial y/\partial t)^2$. The second example similarly identifies that the kinetic energy of the system depends on the mass density and velocity, though in this response it is unclear whether the velocity mentioned is the (longitudinal) pulse speed or the (transverse) speed of the medium. In the third example, increased tension means that energy added

²However, in our dataset, students rarely focused on the speed of the flick.

to the system manifests differently. Though it is incorrect, the third response associates energy with the shape (“vertical displacement”), and associates the speed of the pulse with the tension in the medium.

Other responses using this resource describe how changes to the medium or pulse generation increase or decrease the amount of energy in the system and relate these changes to pulse speed:

“If the tension of the string is greater, the system would have more potential energy, then once the system is given a pulse, that potential energy is converted to kinetic energy which is a function of velocity. Therefore it would make sense for a pulse to move faster on a high-tension spring by the law of conservation of energy.” (tension question)

“When you think about the energy in the spring, the higher the tension, the higher potential energy it would have. [Therefore] you would expect it to have a higher kinetic and in turn velocity.” (tension question)

“[You can generate a faster-moving pulse] by adding an enormous amount of motion that gives the motion a small “T” period per wave cycle {because} the more energy you add, the faster the wave will travel.” (original question)

In the first two examples here, potential energy is appropriately associated with tension in the medium. Both examples relate the kinetic energy of the pulse to its potential energy and use this to explain how tension affects propagation speed. The third response argues that more energy in the system means that the wave will travel faster. While it is not true that pulses generated with more energy move faster (the energy will instead affect the shape of the pulse), this response still associates energy with the propagating pulse and reasons about speed using energy principles.

The examples we provide here do not illustrate all of the ways in which the energy resource was used by students, but they do represent some of the more common ways that we noticed students using this resource to answer versions of the pulse-flick question. Common to all of the responses we assigned this resource code is a sense that energy is associated with or carried by the pulse itself and that energy transfers and transformations play a role in the speed or motion of the pulse.

Students using this resource to answer the pulse-flick question apply a fundamental physical concept—energy—to explain wave propagation. Energy is a particularly powerful physical model for understanding waves because waves carry energy, but not matter. This idea has been emphasized as an important learning goal both because it expresses fundamental trait of waves and because of its important consequences for technological applications of wave phenomena [24]. Students’ uses of this resource are consistent with energy models typically taught in mechanics (i.e., kinetic energy depends on mass and

velocity, potential energy depends on how much a spring is stretched) and continuous with wave mechanics in some situations. Mathematical derivations of energy densities begin from considerations of mass, tension, and speed, which students enact when they use this resource to answer the pulse-flick question. This suggests to us that the energy resource is worth instructors’ attention. Yet, the continuity between instantiations of this resource and scientific notions of mechanical wave propagation can be subtle—more so than for the first two resources we have discussed. Listening to and coming to understand the meaning students make when they use this resource is likely a prerequisite to building on it in instruction.

VII. FREQUENCY OF COMMON CONCEPTUAL RESOURCES

Figures 3–5 show the fraction of student responses that used each resource for the three versions of the pulse-flick question, given at a total of six universities. (These fractions represent the percentages of student responses that both coders agreed instantiate each code.)

Resources theory hypothesizes that resource activation is context sensitive [11,12,14,16,31]. Our study supports this hypothesis: the frequency of resource codes appears sensitive to the version of the question students are asked, as shown in Figs. 3–5. However, the frequencies of resource codes do not clearly depend on the university or course instructor—the bar charts are generally the same shape for each question. That is, for a given question, the frequency with which students use these resources is relatively

FIG. 3. Fraction of responses using three common resources for the original pulse-flick question.

FIG. 4. Fraction of responses using three common resources for the tension question.

FIG. 5. Fraction of responses using three common resources for the mass density question.

consistent across universities. For example, in the original pulse flick question, students at both UW and CPSLO used the properties of the medium resource at high frequency and transverse speed at low frequency (or not at all). However, the frequency with which students use these resources is qualitatively different across questions—the bar charts for the original question have very different shape than the ones for the tension question, and so on. For example, the original pulse-flick question appears to much more commonly elicit the properties of the medium resource than any other question, and at much higher frequency than in other question contexts.

Instructors who want to plan instruction to elicit and build on particular resources may find it especially useful to observe the pairings of resources and questions indicated by this data. For example, instructors who wish to elicit and build on the properties of the medium resource may wish to choose the original or mass density questions rather than

the tension question. Instructors who wish to elicit a multiplicity of resources may wish to choose the tension or mass density questions. More research on the reproducibility of resource-question pairings may help to more firmly establish the relationship between context and resource activation.

The bar charts in Figs. 3–5 prompt questions as to why, for example, the properties of the medium resource is more frequently elicited in the original question, or why a diversity of resources is more likely for the other two. Our hypothesis is that this is due at least in part to the question type. The original question asks students to *predict* what change to the experiment would produce a certain outcome, whereas the other two questions ask students to *explain* an outcome that is described to them. The interplay between the type of question asked and the diversity and frequency of resources is a phenomenon we hope to explore in future work.

VIII. IMPLICATIONS FOR RESOURCES THEORY

We have described three common conceptual resources that students used to reason about mechanical wave propagation (the properties of the medium, transverse speed, and energy resources), contributing to instructors' repertoire of KSI. We have used classic tools of physics education research, looking at large numbers of student responses to written questions and coding student thinking according to common patterns. Yet, our contribution is new because we discuss common patterns of reasoning that can be viewed as continuous with canonical models for wave propagation. In doing so, this work takes up the tenets of resources theory described in Sec. II: the resources we report are sensible and continuous with formal physics, context sensitive, and may constitute the raw material for learning.

Our analysis suggests that in certain contexts, student ideas that have formerly been reported as difficulties can be viewed as continuous with formal physics. Wittmann reports that treating a pulse as a Newtonian particle can lead to difficulties in the context of pulse *generation* [6,22]—particularly the incorrect idea that flicking the string faster makes the pulse move faster, which has been reported by several authors [6,17,20,48]. This kind of thinking could be seen in terms of the transverse speed resource, which (as we showed in Sec. VI) can be viewed as continuous with a canonical understanding of pulse propagation.

Our study contributes to resources theory by demonstrating patterns of context sensitivity for a large number of students. Previous resources-oriented physics education research has used case studies to illustrate the context sensitivity of student thinking. These previous studies demonstrate a single student or small number of students using different ideas in different settings or when asked different questions (e.g., [12,55]). These studies do not demonstrate whether there are patterns in this context sensitivity or identify patterns in resource-context pairings; i.e., they do not answer questions about which resources are reliably elicited in which contexts. Our study illustrates how context dependence plays out on a large scale and supports what has been previously shown in case studies.

An important limitation in the generalizability of this work is that we do not know the demographic data of the students in our sample. We have collected data from a number of different universities and instructors in an effort to sample across multiple sources of heterogeneity. At the same time, we know that our dataset oversamples from selective, majority-white, four-year universities as compared to populations of U.S. undergraduate students, as is the case with most PER studies [56]. Thus, it is most appropriate to say that the resources we report represent ideas that instructors of courses similar to those in our study might expect. In other words, these resources cannot be taken as useful KSI without careful attention to the ideas that students are bringing to bear in a particular classroom.

IX. IMPLICATIONS FOR INSTRUCTION

The primary intent of this work is to inform instruction on mechanical waves by identifying and illustrating some of the conceptual resources that instructors might expect their students to use. Our study calls attention to the ways in which student thinking is continuous with physics concepts and suggests possibilities for eliciting and leveraging such ideas. Each of the resources we report can be seen as the beginnings of a scientific model for mechanical wave propagation. Students use the properties of the medium resource to associate the speed of a pulse on a string with its tension and/or mass density, the transverse speed resource models a pulse as a propagating local disturbance of the medium, and the energy resource applies scientific energy concepts—kinetic energy, potential energy, and conservation—to propagation. None of these resources is entirely correct or complete. As our examples demonstrate, each is sometimes activated in ways that are limited or incorrect. For example, the transverse speed resource is correct only when comparing pulses of the same shape on different media. Even so, these resources are sensible and continuous with formal physics: all three are central to students' sense making about changes to the speed of a pulse, embody physics ideas such as force and energy, and connect to a scientific understanding of pulse propagation.

The context-dependence of these resources shows in the fact that they appear with different frequencies in response to different questions. The fact that the frequency pattern is fairly consistent across universities helps attribute the variation to the question asked, rather than to some other factor. We notice that different versions of the pulse-flick question elicit different resources most frequently, and some questions tend to elicit a diversity of resources whereas others elicit a single resource at high frequency. For instructors, this offers knowledge not only of the resources that may be likely to be used by their students, but also of which questions are more likely to elicit particular resources. Effective implementation of these question-resource pairings in instruction is an important direction for future work.

Resources theory describes learning in terms of reorganizing, refining, properly activating, increasing the degree of formality of, or changing the role of resources. Instructors might facilitate student learning about mechanical wave propagation, first, by listening to and sensemaking with students, given that the resources we report manifest in diverse ways and the connections to canonical physics may not be immediately recognizable. We notice that in many instantiations, these resources are causal or mechanistic in nature. Our attention to these conceptual resources also draws attention to rich mechanistic reasoning and model-building that students are doing in their answers to these questions. In many instantiations, these resources incorporate features of causal explanations and model-based reasoning. For instructors who have a goal of highlighting

and building from their students' own thinking, our results may suggest an instructional focus on mechanisms for propagation—which are not common in introductory physics courses—and attention to more detailed explanatory models (not just an equation) for pulse speed.

For example, instructors might intentionally elicit the properties of the medium resource to support predictions about the behavior of waves in a variety of scenarios (standing waves on strings, for example). Likewise, the properties of the medium resource might be generalized and extended through instruction to be useful for waves in media other than strings (such as water or air) and could even be generalized to situations when the “medium” is the vacuum with permittivity ϵ_0 and permeability μ_0 . Both the properties of the medium resource and the transverse speed resource may serve as starting points for students' construction of more detailed models of propagation (such as the string-and-bead model) [36]. For example, in using the transverse speed resource, students often model a pulse in terms of the motion of small parts of the medium and relate propagation speed to the dynamics of the transverse motion of the medium. The resemblance of this resource and the string-and-bead model suggests that eliciting the transverse speed resource may be a particularly fruitful starting point for instruction on wave mechanics. By eliciting students' own ideas about how propagation works, and then comparing and generalizing from them, instructors might leverage this resource to facilitate students' development of explanatory models for pulse propagation. Instructors might

leverage the energy resource by eliciting it in the context of a more detailed exploration of the energy transfers and transformations that occur in the pulse-flick scenario. Finally, instruction might build on or refine the resources we identified by inviting students to consider parameters that affect the energy or speed of a pulse. For example, in many of the examples given in this paper, variables such as width, amplitude, or the source motion have been implicitly held constant or unspecified. We are currently designing and testing instructional materials that elicit and build on student resources for understanding pulse propagation.

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