Investigating and improving student understanding of quantum mechanics in the context of single photon interference

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Single photon experiments involving a Mach-Zehnder interferometer can illustrate the fundamental principles of quantum mechanics, e.g., the wave-particle duality of a single photon, single photon interference, and the probabilistic nature of quantum measurement involving single photons. These experiments explicitly make the connection between the abstract quantum theory and concrete laboratory settings and have the potential to help students develop a solid grasp of the foundational issues in quantum mechanics. Here we describe students' conceptual difficulties with these topics in the context of Mach-Zehnder interferometer experiments with single photons and how the difficulties found in written surveys and individual interviews were used as a guide in the development of a Quantum Interactive Learning Tutorial (QuILT). The QuILT uses an inquiry-based approach to learning and takes into account the conceptual difficulties found via research to help upper-level undergraduate and graduate students learn about foundational quantum mechanics concepts using the concrete quantum optics context. It strives to help students learn the basics of quantum mechanics in the context of single photon experiment, develop the ability to apply fundamental quantum principles to experimental situations in quantum optics, and explore the differences between classical and quantum ideas in a concrete context. We discuss the findings from in-class evaluations suggesting that the QuILT was effective in helping students learn these abstract concepts.

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I. INTRODUCTION

Learning quantum mechanics is challenging partly due to the abstract and nonintuitive nature of the subject matter. It is difficult to visualize and reason about quantum concepts partly because one does not generally observe quantum phenomena in everyday experience and the formalism of quantum mechanics is unintuitive. Indeed, several prior studies have found that many upper-level undergraduate and graduate students struggle with the foundational topics in quantum mechanics (see, e.g., Refs. [1–6]), and focused on diverse pedagogical approaches for helping students learn quantum mechanics better (see, e.g., Refs. [7–14]).

It is possible that research-based learning tools focusing on concrete experimental situations in which the fundamentals of quantum mechanics manifest themselves can aid students in learning the basics. However, there are very few concrete experimental contexts that can reveal these foundational issues and help students learn quantum mechanics better. Fortunately, in the past few decades, quantum optics has emerged as a vibrant research area and single photon experiments have played an important role in elucidating the foundational issues in quantum mechanics. For example, quantum optics experiments involving the Mach-Zehnder interferometer (MZI) with single photons (see, e.g., Refs. [15–20]) elegantly illustrate the fundamental concepts of quantum mechanics such as the wave-particle duality of a single photon, single photon interference, and the probabilistic nature of quantum measurement. Therefore, using research-based tools to learn about how the abstract quantum concepts can be applied in the concrete context of the MZI experiment with single photons can assist students in developing a coherent understanding of the foundational issues in quantum mechanics.

Since few prior studies have focused on student understanding of the foundational topics in quantum mechanics in the context of quantum optics experiments [21] involving a MZI with single photons, it is important to investigate students' conceptual difficulties in order to use them as resources in the development of the learning tools to help students. Therefore, we conducted research on students' conceptual difficulties with the foundational topics in quantum mechanics such as wave-particle duality of a single photon, single photon interference, and the probabilistic nature of quantum measurement in the context of a MZI with single photons. We used the research on students' conceptual difficulties as a guide to develop a Quantum Interactive Learning Tutorial (QuILT) to help students learn about these concepts using the context of MZI experiments. The QuILT uses gedanken (thought) experiments and simulations involving a MZI to help students learn about these concepts using single photon interference. It also strives to help students learn about the probabilistic nature of the quantum measurement and how adding photodetectors and optical elements such as beam splitters in the path of the MZI affect the outcomes of quantum measurement. The QuILT uses an inquiry-based approach to learning and was developed using an iterative approach to development and assessment.

We start with a brief background on the MZI with single photons. Then, we discuss the theoretical frameworks informing the investigation of student difficulties and the development of the QuILT. We then describe the methodology for the investigation of students' conceptual difficulties and categorize the difficulties found. We then discuss the development and assessment of the QuILT including data from upper-level undergraduate and graduate students suggesting that the QuILT was effective in improving students' understanding of the foundational issues in quantum mechanics in the context of MZI experiments involving single photons. Physics education researchers and instructors of upper-division quantum mechanics courses can both benefit from these findings.

II. BACKGROUND

Before we discuss research on student conceptual difficulties and how the research was used as a guide to develop and evaluate the QuILT, we describe the basic MZI setup in Fig. 1 and summarize how single photon experiments using a MZI can illustrate many of the foundational topics in quantum mechanics. For simplicity, the following assumptions are made: (i) all optical elements are ideal; (ii) the nonpolarizing beam splitters (BS1 and BS2) are infinitesimally thin such that there is no phase shift when a single photon propagates through them; (iii) the monochromatic single photons travel the same distance in vacuum in the upper path (U) and lower path (L) of the MZI; and (iv) the MZI without the phase shifter is set up such that there is completely constructive interference at photodetector 1 (D1) and completely destructive interference at photodetector 2 (D2).

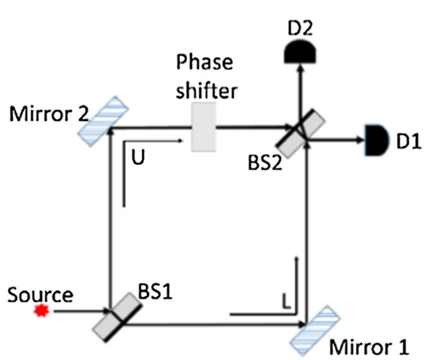

FIG. 1. MZI setup with a phase shifter in the U path.

If single photons are emitted from the source in Fig. 1, BS1 causes each photon to be in a superposition state of the path states U and L. The photon path states reflect off the mirrors. Then, beam splitter BS2 mixes the photon path states such that each component of the photon state along the U and L paths can be projected into the photodetectors D1 and D2 in Fig. 1. Here, we define the term "projection" of a photon path state into a detector to mean that the component of the photon path state arrives at (reaches) the detector. In Fig. 1, the projection of both components of the photon path state leads to interference at the photodetectors (called detectors D1 and D2 from now on). Depending upon the thickness of the phase shifter, the interference observed at detectors D1 and D2 can be constructive, destructive, or intermediate. Observing interference at the detectors D1 and D2 can be interpreted in terms of not having "which-path" information (WPI) about the single photon [13–18]. WPI is a common term associated with these types of experiments and was popularized by Wheeler [22]. WPI is unknown (as in the setup shown in Fig. 1) if both components of the photon state can be projected into D1 and D2 and the projection of both components at each detector leads to interference. When a large number of single photons are sent through the setup in Fig. 1 and the thickness of the phase shifter is varied, the probability of photons arriving at D1 and D2 will change due to the interference of the components of the single photon state from the U and L paths. A detector clicks when a photon is detected by it and absorbed. However, there is no way to know *a priori* which detector will click when a photon is emitted until the photon state collapses at either detector D1 or D2.

On the other hand, if the components of the photon path state are not recombined at the detectors, there is no possibility for interference of the photon path states to occur at the detectors. In this case, WPI is known about a photon that arrives at a detector D1 or D2. In other words, WPI is "known" about a photon if only one component of the photon path state can be projected into each detector. For example, if BS2 is removed from the setup (see Fig. 2), WPI is known for all single photons arriving at the detectors and each detector (D1 and D2) has an equal probability of clicking. Changing the thickness of a phase shifter in one of the paths will not affect the probability of each detector clicking when photons are registered.

The different setups of the MZI experiment with single photons, e.g., the ones shown in Figs. 1 and 2, illustrate many of the fundamental principles of quantum mechanics. For example, the wave-particle duality of a single photon is observed in the MZI experiment with single photons. When a single photon propagates through the MZI, it exhibits properties of a wave. However, when a photon is registered at one of the detectors, it exhibits properties of a particle because only one detector will click when it detects a photon and the state of the single photon collapses.

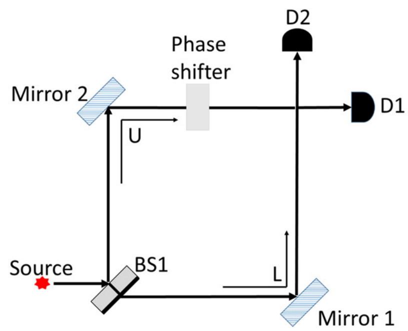

FIG. 2. MZI setup with beam splitter 2 (BS2) removed.

The MZI with single photons also demonstrates single photon interference. In Fig. 1, interference is observed because both of the components of the photon state can be projected into D1 and D2 and the projection of both components at each detector leads to interference. In contrast, in Fig. 2, interference is not observed at the detectors because only one component of the photon path state can be projected in each detector. Furthermore, the MZI with single photons illustrates the probabilistic nature of quantum measurements. As the thickness of the phase shifter in Fig. 1 is varied, the probability of photons arriving at D1 and D2 will change and there is no way to know *a priori* which detector will click (i.e., whether the photon will arrive at D1 or D2). Similarly, in Fig. 2, the photon state collapses either at detector D1 or at detector D2 with equal likelihood and there is no way to know *a priori* which detector will click when a particular photon arrives.

III. THEORETICAL FRAMEWORKS INFORMING THE INVESTIGATION OF STUDENT DIFFICULTIES AND DEVELOPMENT OF THE QuILT

The MZI experiment with single photons elucidates foundational issues in quantum mechanics. Therefore, research-based learning tools that focus on this experiment can help students make connections between abstract quantum mechanics concepts and concrete experiments and learn the concepts better. The development of the QuILT was inspired by several cognitive theories. All of these cognitive theories suggest that learning tools should take into account students' difficulties and build on them to help students develop a functional understanding of the underlying concepts.

For example, Hammer's "resource" model suggests that students' prior knowledge, including their learning difficulties, should be used as resources to help students learn better [23]. Similarly, the Piagetian model of learning emphasizes an "optimal mismatch" between what a student knows and is able to do and the instructional design [24,25]. In particular, this model focuses on the importance of knowing students' difficulties and using this knowledge to design instruction to help them assimilate and accommodate new ideas and build a good knowledge structure. Similarly, Bransford and Schwartz's framework, "preparation for future learning" (PFL), suggests that instructional design should include elements of both innovation and efficiency to help students transfer their learning from one context to another [26]. While there are multiple interpretations of the PFL model, efficiency and innovation can be considered to be two orthogonal dimensions in instructional design. If instruction only focuses on efficiently transferring information, cognitive engagement will be diminished and learning will not be effective. On the other hand, if the instruction is solely focused on innovation, students will struggle to connect what they are learning with their prior knowledge and learning and transfer will be inhibited. Incorporating the efficiency and innovation elements into an instructional design based upon this framework and being in the "optimal adaptability corridor" demands that the instruction build on students' existing knowledge and skills and takes into account their difficulties [26]. Vygotsky's notion of the "zone of proximal development" (ZPD) is another synergistic model of student learning. The ZPD refers to the zone defined by the difference between what a student can do on their own and what a student can do with the help of an instructor who is familiar with their prior knowledge and skills [27]. Scaffolding is at the heart of the ZPD model and can be used to stretch students' learning beyond their current knowledge using carefully crafted learning tools that provide scaffolding support. These frameworks are synergistic in that one can provide an optimal mismatch by ensuring that instruction is in the zone of proximal development and by designing instructional tasks that are in the optimal adaptability corridor.

All of these frameworks point to the fact that one must determine the initial knowledge states of students in order to design effective instructional tools commensurate with students' current knowledge and skills. The research on student conceptual difficulties involving the MZI with single photons and the development of the QuILT were based upon these synergistic models of student learning, which involve building on students' prior knowledge (including their conceptual difficulties) to help them develop a coherent knowledge structure of relevant concepts.

IV. METHODOLOGY FOR THE INVESTIGATION OF STUDENTS' CONCEPTUAL DIFFICULTIES

Students' conceptual difficulties involving the MZI with single photons were investigated by administering written open-ended questions to upper-level undergraduate and graduate students in various quantum mechanics classes after instruction in relevant concepts. The instruction included an overview of the MZI setup and students learned about the propagation of light through the beam splitters, phase difference introduced by the two paths of the MZI, and the meaning of what happens when a detector "clicks". Based upon student responses to questions in one quantum mechanics course, some of the written open-ended questions were revised to further probe student understanding. The open-ended questions were graded using rubrics which were developed by the two investigators together. A subset of the questions was graded separately by the investigators. After comparing the initial grading (in which the agreement was better than 85%), we discussed any disagreements in grading and resolved them. The final interrater reliability in the grading of open-ended questions is better than 90%.

We also investigated student conceptual difficulties via individual interviews with upper-level undergraduate and graduate students. Fifteen individual interviews were conducted using a semi-structured, think-aloud protocol [28] to better understand the rationale for student responses before, during, and after the development of different versions of the QuILT and the corresponding pretest and post-test. During the semi-structured interviews, upper-level undergraduate and graduate students were asked to verbalize their thought processes while they answered the questions. Students read the questions related to the MZI setup and answered them to the best of their ability without being disturbed. We prompted them to think aloud if they were quiet for a long time. After students had finished answering a particular question to the best of their ability, we asked them to further clarify and elaborate issues that they had not clearly addressed earlier.

Students' conceptual difficulties were analyzed using open coding to generate initial categories of difficulties [29]. After the initial categories emerged from the data, a subset of the open-ended questions and interviews were coded by two of the researchers separately using the initial categories. After comparing codes, any disagreements were discussed until better than 90% agreement was reached. Below, we discuss categories of students' conceptual difficulties.

V. STUDENTS' CONCEPTUAL DIFFICULTIES

Both before and during the preliminary development of the QuILT, we investigated the conceptual difficulties students have with the relevant quantum mechanics concepts in order to effectively address them. Below, we describe some of the common difficulties found in written responses and interviews.

Ignoring the wave properties of classical light by treating it as a large number of individual photons (i.e., particles).—Interviews suggest that many students did not take into account the interference phenomenon of a classical beam of light. In written questions and interviews, students were asked to explain why they agreed or disagreed with the following statement about sending a beam of monochromatic light through the MZI setup shown in Fig. 1: "If the source produces light with intensity I, the intensity of light at the point detectors D1 and D2 will be $I/2$ each." In response to this question, one student stated "There will be billions of photon[s] in one beam so… approximately half go through U and half go through L. When going through BS2 they also have equal chance to reach D1 [and] D2. So the [intensity] on each [detector] will be $I/2$." Similar to this student, other students in the interviews also invoked the concept of photons when reasoning about a classical beam of light. Further probing indicates that students with these types of responses had some idea that a beam of light can be treated as a stream of photons but they often failed to invoke the wave nature of light which would lead, e.g., to constructive interference at D1 and destructive interference at D2 for the setup given in Fig. 1 without a phase shifter.

Ignoring the interference of a single photon at the detectors after passing through the MZI.—Students often struggled with the fact that the U and L components of the photon state can interfere at the detectors D1 and D2. In written questions and interviews, students were asked to explain whether they agreed or disagreed with the following statement for the setup shown in Fig. 1 without the phase shifter and why: "If the source emits N photons one at a time, the number of photons reaching detectors D1 and D2 will be $N/2$ each." Many students incorrectly agreed with this statement because they had difficulty with the fact that the U and L path components of the photon state can interfere at the detectors D1 and D2. Interviews and written responses indicate that even students who were aware that a photon was in a superposition of the path states U and L after passing through BS1 often claimed that $N/2$ photons would arrive at each detector D1 and D2. They often justified their reasoning by stating that each detector would have an equal probability of clicking since the photon is in an equal superposition of the upper and lower path states. These students often ignored the interference of the single photon path states and did not take into account the phase shifts of each photon path component. Students' conceptual difficulties with interference have been found in other contexts as well [30].

Ignoring the wave nature of a photon.—Students sometimes ignored the wavelike nature of a single photon and treated a single photon as a point particle. Written responses and interviews suggest that students' difficulties involving a single photon as a point particle involved the following three categories: (i) claiming that the photon takes either the U or L path, (ii) claiming that the photon splits into two photons that take the U and L paths, and (iii) claiming that the photon is in a superposition after beam splitter BS1 but interpreting superposition to mean that the photon splits into two photons and the two photons interfere at the detectors.

Some students with the photon as a point particle model incorrectly claimed that the photon can only take either the U or L path in the MZI. In written questions and interviews, students were asked to explain whether they agreed or disagreed with the following statement for the setup shown in Fig. 1 without the phase shifter and why: "If the source emits photons one at a time, the number of photons reaching detectors D1 and D2 will be $N/2$ each." One student stated, "I agree because the photon has equal probability of reflecting or transmitting when it hits the beam splitter [BS1]." Further probing suggests that this student thought that if the photon gets reflected by BS1, it will be in the U path of the MZI. On the other hand, if the photon gets transmitted by BS1, it will be in the L path of the MZI. Students with this type of response had difficulty reasoning about how the photon can have wave properties and the beam splitter BS1 causes a photon to be in a superposition of the U and L path states. They also did not take into account the phase shifts of each photon path component and how the phase difference between the U and L paths causes constructive and destructive interference of single photons at the detectors. Students' conceptual difficulties involving superposition states in quantum mechanics has been found in other contexts as well [7].

Other students claimed that the beam splitter BS1 splits the photon into two photons. In written questions and interviews, students were asked to explain why they agreed or disagreed with the following statement for the setup shown in Fig. 1: "The beam splitter BS1 causes the photon to split into two parts and the energy of the incoming photon is also split in half." Students who agreed with this statement often claimed that the photon has an equal probability of being in the U and L path and incorrectly claimed that the detectors D1 and D2 would click with equal probability. These students struggled with the fact that a single photon can behave as a wave passing through the MZI and be in a superposition of the U and L path states. They did not take into account the fact that the two components of the photon path state can interfere at the detectors D1 and D2.

Some students had a hybrid model in which they claimed that a photon is a point particle but applied the principle of superposition to particles to reason about interference. For example, some students claimed that a single photon can be split into two photons and it is these two photons that interfere at the detectors (instead of the fact that interference is due to the wave nature of single photons). One student with this type of view stated, "it seems like (each photon with half of the energy of the incoming photon traveling along the U and L paths of the MZI is) the only way for a photon to interfere with itself and have some probability of going through either path until getting measured." Interviews and written responses indicate that some students knew that the photon would be in a superposition of the two path states of the MZI but incorrectly interpreted this to mean that the photon is split into two photons. They incorrectly claimed that these two photons interfere at the detectors.

Not recognizing how beam splitter BS2 affects interference.—Several students incorrectly claimed that either removing or inserting beam splitter BS2 will not change the probability of the single photons arriving at each detector. In written questions and interviews, students were asked the following question: "Suppose we remove BS2 from the MZI setup as shown in Fig. 2. How does the probability that detector D1 or D2 will register a photon in this case differ from the case when BS2 is present as in Fig. 1?" One student stated that removing beam splitter BS2 would not change the probability of the single photons arriving at each detector and supplemented his claim as follows: "I don't see how BS2 affects/causes any asymmetry to make probabilities D1 $\neq$ D2 or how BS2 causes a loss of photons." Another student who made similar claims about what happens at the detectors with and without BS2 stated, "I say still 50% each since it's symmetric." Students who treated a single photon as a point particle and ignored its wave nature did not take into account the phase shifts affecting the components of the photon state along the U and L paths due to BS1 and BS2 (e.g., in Fig. 1), which influence the interference of single photons at the detectors D1 and D2.

Not recognizing how a detector placed in the U or L path affects the single photon state.—Students often asserted that inserting an additional detector in the U or L path of the MZI would not affect the interference at the detectors D1 and D2 at the end (see Fig. 3). For example, students were shown a MZI with an additional detector placed in the L path between BS1 and BS2 (see Fig. 3) and were asked the following question: "How does what you observe at detectors D1 and D2 in Fig. 3 compare to the case in Fig. 1 in which the photodetector is not present?" Some students had difficulty with the fact that an additional detector, e.g., in the L path of the MZI in Fig. 3, would collapse the state of the photon to the U or L path state so that the detectors D1 or D2 after BS2 would click with equal probability and the interference would be destroyed.

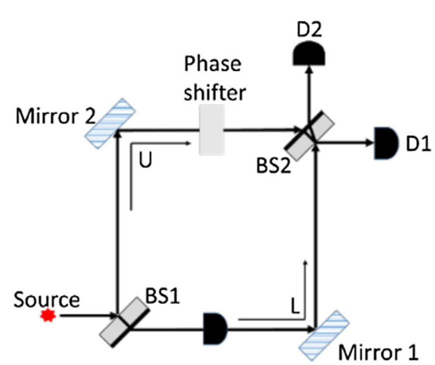

FIG. 3. MZI setup with an additional detector placed in the L path.

Instead, many students claimed that the photon state would remain delocalized in a superposition of the U and L path states (as in Fig. 1) and interference would be observed at D1 and D2 even in the situation in Fig. 3. Some students correctly stated that a detector placed in the L path would absorb some photons but incorrectly inferred that there would still be interference displayed by the photons that are not absorbed earlier and reach detector D1 and/or D2. For example, one student stated, "Now path L is blocked [by a detector in the L path], so only ½ as many photons should hit the [detector D1 or D2 at the end]. I don't see how there can be any but constructive interference since path lengths are the same." Further probing of students with these types of responses suggests that they struggled with how placing a detector in the U or L path amounts to a measurement and destroys the delocalized single photon state which was in a superposition of the U and L path states before the measurement.

Unfamiliarity with the concept of which-path information (WPI).—The concept of WPI can be a useful tool for reasoning about whether interference will be observed at detectors D1 and D2. In particular, if WPI is known for the single photons arriving at D1 and D2, interference will not be observed at the detectors. If WPI is not known, interference will be observed. Students rarely mentioned the concept of WPI in interviews and written responses to questions about the MZI experiment. Most students had difficulty reasoning that interference would be observed at the detectors if WPI is unknown about a photon arriving there (i.e., when both components of the photon path state are projected in each detector D1 and D2). In addition, the majority of students had difficulty reasoning that interference would not be observed at the detectors D1 and D2 if WPI is known about a photon (i.e., when only one component of the photon path state is projected into each detector).

In summary, students had difficulty reasoning about single photon interference in a MZI. They often treated a single photon as a point particle and ignored its wavelike nature when it was in a superposition of the two path states of the MZI. Because students treated the photon as a point particle, they sometimes did not take into account the phase shifts of a single photon propagating through the MZI and did not recognize that a single photon could display interference. Furthermore, some students claimed that adding or removing beam splitter BS2 would not change the probability of a photon arriving at the detectors D1 or D2 because they ignored the wave nature of a single photon and did not take into account the phase shifts affecting the components of the photon state. In addition, students struggled with the notion that adding an additional detector into one of the paths of the MZI would amount to a measurement and interference would not be observed at the detectors D1 and D2 in Fig. 3. Also, most students had difficulty reasoning about single photon interference using the concept of WPI.

VI. QuILT DEVELOPMENT

A. Development and validation of the QuILT

The conceptual difficulties described in Sec. V indicate that many students struggle with foundational issues in quantum mechanics in the context of the MZI experiment after instruction in relevant concepts. Therefore, we developed a QuILT on the MZI with single photons that strives to help students develop a coherent understanding of these concepts using the concrete contexts in the MZI experiment. The QuILT is inspired by the theoretical frameworks discussed earlier (i.e., the synergistic models of learning such as Hammer's resources model, the Piagetian model of optimal mismatch, the PFL framework of Schwartz and Bransford, and Vygotsky's ZPD) and was developed based upon the research on students' conceptual difficulties. Furthermore, a cognitive task analysis of the underlying concepts from an expert perspective [31] was also used as a guide to develop the research-based QuILT. The cognitive task analysis from an expert perspective involves a careful analysis of the underlying concepts in the order in which those concepts should be invoked and applied in each situation to accomplish a task. The research-based QuILT actively engages students in the learning process using an inquiry-based approach in which various concepts build on each other. The QuILT can be used in upper-division quantum mechanics courses after students have had instruction in the relevant topics (here we will describe its effectiveness for upper-level undergraduate and graduate students).

The development of the QuILT went through a cyclic, iterative process which included the following stages before the in-class implementation:

(i) Development of the preliminary version based on a cognitive task analysis of the underlying knowledge and research on student difficulties with relevant concepts.
(ii) Implementation and evaluation of the QuILT by administering it individually to students and obtaining feedback from faculty members who are experts in these topics.
(iii) Determining its impact on student learning and assessing what difficulties were not adequately addressed by the QuILT.
(iv) Refinements and modifications based on the feedback from the implementation and evaluation.

As noted, in addition to written free-response questions administered to students in various classes, individual interviews with 15 students were carried out using a think-aloud protocol to better understand the rationale for their responses throughout the development of various versions of the QuILT and the development of the corresponding pretest and post-test, given to students before and after they engaged in learning via the QuILT. The QuILT asks students to predict what should happen in a particular situation. After their prediction phase is complete, students use a simulation to check their predictions and then reconcile the differences between their predictions and what the simulation shows. After each individual interview with a particular version of the QuILT (along with the administration of the pretest and post-test), modifications were made based upon the feedback obtained from the interviewed students. For example, if students got stuck at a particular point and could not make progress from one question to the next with the scaffolding already provided, suitable modifications were made to the QuILT. Thus, the administration of the QuILT to the graduate students and upper-level undergraduate students individually was useful to ensure that the guided approach was effective and the questions were unambiguously interpreted. The QuILT was also iterated with three faculty members and two additional graduate students who conduct physics education research several times to ensure that the content and wording of the questions were appropriate. Modifications were made based upon their feedback. When we found that the QuILT was working well in individual administration and the post-test performance was significantly improved compared to the pretest performance, it was administered in classes.

B. Structure of the QuILT

The QuILT on the MZI with single photons includes a pretest to be administered right after traditional instruction on the basics of MZI with single photons but before students engage with the QuILT and a post-test to be administered after students work on the QuILT. The questions on the pretest and post-test are open ended. The open-ended format requires that students generate answers based upon a robust understanding of the concepts as opposed to memorization. Before the development of the QuILT, some students were given the pretest or post-test questions before traditional instruction and after traditional instruction in relevant concepts and their responses did not improve significantly despite the fact that the same questions were used before and after instruction. Therefore, we decided to administer the same questions on the pretest and post-test.

The QuILT begins with a warm up that builds on students' prior knowledge about the interference of light. Then, students transition to the main section of the QuILT that focuses on the fundamentals of quantum mechanics in the context of MZI with single photons. The QuILT is best used in class to give students an opportunity to work together in small groups and discuss their thoughts with peers, which provides peer learning support. However, students can be asked to work on the parts they could not finish in class at home as homework.

The QuILT strives to provide optimal mismatch by explicitly bringing out common conceptual difficulties found via research and then providing appropriate scaffolding to help students develop a coherent understanding. Throughout the QuILT, students make predictions about a particular MZI setup, check their prediction via a computer simulation, and then reconcile the differences between their prediction and observation. If the students' predictions and observations are inconsistent, further scaffolding is provided throughout the QuILT to ensure that students remain in the optimal adaptability corridor or ZPD. We give some typical examples of how some of the common difficulties found via research are incorporated as resources and how student learning is scaffolded via the QuILT.

C. Addressing student difficulties with interference of light in a classical situation via the QuILT warm up

The QuILT warm up helps students review the basics of interference at the detectors due to the superposition of light from two paths of the MZI in the classical situation. It uses students' conceptual difficulties with the interference of light in a classical situation [30] as a resource to help students learn better. Previous work has shown that students sometimes fail to recognize how path length difference and phase shift affect interference [30]. The warm up helps students to determine the phase shifts of a light wave propagating through the MZI and the type of interference (e.g., constructive or destructive) observed at each detector in Fig. 1. To help students reason about the phase shifts of a light wave, e.g., when it propagates through the MZI, they are guided to make an analogy with a wave pulse on a rope since a wave pulse on a rope is easier to visualize and make sense of than light waves. Students use this concrete example that they have learned about in introductory physics to visualize the phase shift associated with reflection or transmission of a wave pulse based upon the physical properties of the rope. They then use the rope analogy to build intuition about the more abstract case of light waves and the phase shifts associated with the reflection, transmission, and propagation of light through different optical media.

For example, one of the questions in the guided inquiry sequence in the warm up helps students recognize that a wave pulse propagating through a rope that has a fixed end will be reflected, a concept that they have learned earlier in introductory physics. Students can visualize that a wave pulse propagating along the rope reflects at the fixed end and is inverted (it undergoes a phase shift of π at the fixed end). Other questions help students learn about the phase shifts associated with a wave propagating through different media. For example, students are given a situation in which a wave pulse propagates through a rope consisting of two ropes tied together—one of the ropes has a low mass density and the other rope has a high mass density. Students reason about how a wave pulse traveling along the low mass density rope is partially transmitted and partially reflected when it reaches the high mass density rope. At the

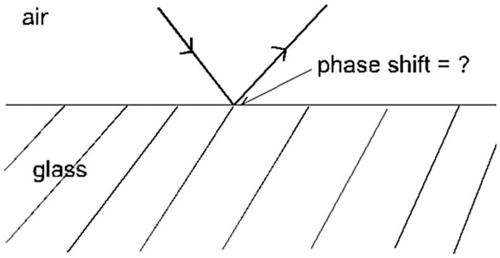

FIG. 4. Reflection of light at an air-glass interface.

interface of the two ropes, the reflected pulse undergoes a phase shift of π (it is inverted) and the transmitted pulse does not undergo a phase shift (it is not inverted).

Students are then given the opportunity to build intuition about a light wave propagating through different media based upon their prior knowledge of a wave pulse propagating on a rope. For example, the following part of the QuILT warm up helps students learn about the phase shift when a light wave is incident on a medium of higher refractive index:

We can see a parallel between the mass density of a rope and the refractive index of a medium. Lower mass density is analogous to lower refractive index and higher mass density is analogous to higher refractive index. We can use this analogy to calculate the phase shift (change in phase) of light introduced by reflection or transmission at the interface between two media. We will also discuss the propagation of light through a refractive medium.

Light (plane harmonic electromagnetic wave) is incident from air onto a glass surface. The light gets partially reflected back into the air after striking the air-glass interface (see Fig. 4). Which one of the following phase shifts is introduced in the reflected light due to the reflection at the interface? Always assume that the angle of incidence is smaller than Brewster's angle.

(a) Zero (b) $\pi/2$ *(c)* π *(d) None of the above.*

Throughout the QuILT, students are given checkpoints (i.e., summaries of the concepts so that they can infer the correct answers to the preceding questions) to help them reflect upon their responses in the guided approach to learning and ensure that they are answering the questions correctly. If students' answers are not consistent with the checkpoints, students are asked to reconcile the differences and additional scaffolding is provided.

D. Addressing student conceptual difficulties about single photons via the QuILT

The QuILT takes into account students' common conceptual difficulties with single photon interference and helps them learn about fundamentals of quantum mechanics in the context of a MZI. Below, we provide a few examples from the QuILT to illustrate how some common student conceptual difficulties found via research were used as resources in the development of the QuILT and how the QuILT strives to provide students appropriate scaffolding (based upon the synergistic theoretical frameworks discussed earlier) to help them develop a coherent understanding of relevant concepts.

Wave nature and interference of a single photon.— Students had conceptual difficulties with the wave nature of a photon and the fact that the U and L components of the photon state can interfere at the detectors D1 and D2. The following question in the QuILT uses students' common difficulties found via interviews and written responses about this topic as a resource to help them learn the concepts better:

Consider the following conversation between Student A and Student B about Fig. 1 without the phase shifter:

- *Student A: If we send one photon at a time, there is no way to observe interference at detectors D1 and D2. Interference is due to the superposition of waves from the U* and *L paths. A single photon must choose either the U or the L path.*
- *Student B: I disagree. We should observe interference because a single photon can go through both the U and L paths simultaneously and can interfere with itself. We can observe constructive, destructive or intermediate interference at the detectors depending on the phase difference between the U and L paths of the photon state arriving at a detector from the U and L paths. However, only one of the detectors will click for each photon. This is because the measurement collapses the photon state when the photon is registered in the detector and the state becomes localized.*

With whom do you agree? Discuss your preceding answer with a partner and explain your reasoning.

Students are encouraged to discuss and articulate their thoughts about whom they agree with to a peer, which can aid learning. After this question in the guided inquiry-based sequence, further scaffolding is provided which strives to ensure that students are in the ZPD (or equivalently, they are in the optimal adaptability corridor or are provided optimal mismatch). The scaffolding helps them reflect upon whether their responses are consistent with help in checkpoints, reconcile possible differences between their initial responses and the correct concepts, and build a coherent understanding of the underlying concepts.

For example, to check if interference occurs at the detectors for the MZI setup shown in Fig. 1 without the phase shifter, students are also asked to use a computer simulation and reconcile the difference between their prediction and observation. In the computer simulation, a screen is used in place of point detector D1 and the photon has a transverse Gaussian width as opposed to being a collimated beam having an infinitesimally small transverse width. Students are guided to think about how the transverse Gaussian profile of the photon may yield constructive or destructive interference at different points on the screen, creating an interference pattern on the screen (in situations in which interference should be observed). Students learn that the advantage of the screen (as opposed to point detectors D1 and D2) is that an interference pattern is observed without placing a phase shifter in one of the paths and changing the path length difference between the two paths. For the case with point detectors D1 and D2, the thickness of the phase shifter must be changed in order to observe interference (if interference is displayed in a particular case). Students can use the computer simulation to verify that a single photon can exhibit wave properties while propagating through the MZI setup and interference fringes are observed on the screen (see Fig. 5). They are then given an opportunity to reconcile possible differences between their prediction and observation and learn that a single photon in the MZI experiment can behave as a wave and the U and L components of the photon state can interfere at the detectors D1 and D2.

Since students struggled with the concept of the wave-particle duality of a single photon and the collapse of the state of the photon upon measurement, the following question in the QuILT uses common student difficulties as a resource to help students learn the concepts better:

Consider the following conversation between three students:

- *Student A: How can a single photon be in both the U and L paths of the MZI simultaneously if only one detector D1 or D2 clicks and registers a photon? It must go through only one path if only one detector clicks.*
- *Student B: Registering of a photon at the detector corresponds to a measurement of the photon's position via its interaction with the atoms in the detector. The photon is absorbed by the detector during the detection process.*
- *Student C: I agree with Student B's statement. A single photon can be delocalized or localized depending on the situation. For example, the single photon state is delocalized while going through the U and L paths but becomes localized upon detection because measurement collapses the state. Then, the photon gets absorbed by the material in the detector.*

With whom do you agree? Discuss your answer with a partner and explain your reasoning.

Following this question and peer discussion, the inquiry-based approach employed in the QuILT uses student difficulties as resources and strives to scaffold their learning and help them develop a coherent knowledge structure about the wave-particle duality of a photon and the collapse of a photon state upon measurement. Computer simulations are used as appropriate to help students check their predictions and students are then provided support to reconcile the difference between their predictions and observations.

Role of beam splitter 2.—As noted earlier, students also struggled with how beam splitter 2 (BS2) affects interference of a single photon and claimed that either removing or inserting beam splitter BS2 will not change the probability of the single photons arriving at each detector D1 or D2. The following question in an inquiry-based sequence was designed to have students think about the role of BS2 based upon their own understanding before they were provided guidance and support:

Choose all of the following statements that are true about the case in which the second beam splitter BS2 is removed (see Fig. 6):

(I) *The point detectors D1 and D2 can only project the superposition state of the photon along the U path state or L path state, respectively.*

(II) *No interference is observed at either detector and each detector has a 50% probability of registering a photon, regardless of the phase difference between the U and L paths.*

(III) *It is useless to calculate the phase difference between the photon state due to the U and L paths for information about interference because we have WPI about each photon that arrives at detectors D1 or D2 (because detector D1 can only project the component along the U path and detector D2 can only project the component along the L path).*

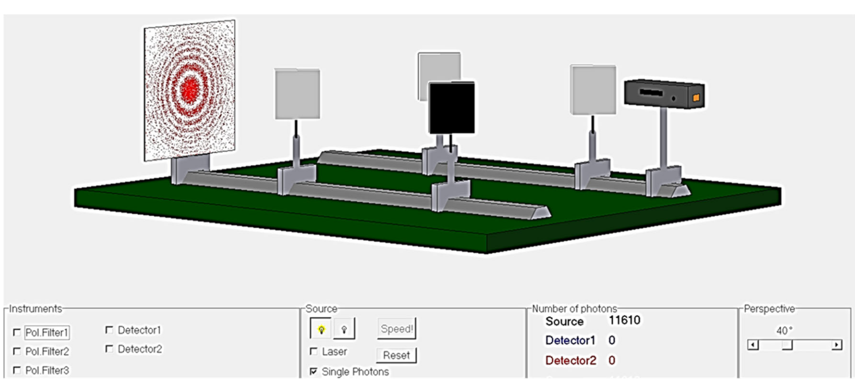

FIG. 5. Screen shot of the computer simulation of a large number of single photons propagating through the MZI. Simulation developed by Albert Huber.

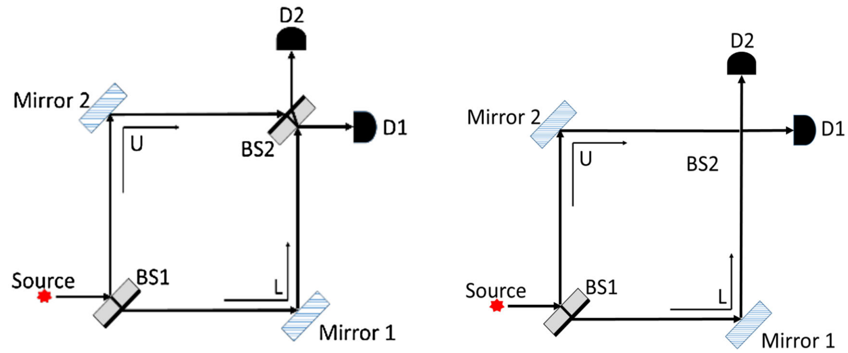

FIG. 6. MZI setup with beam splitter 2 (BS2) inserted (left) and MZI setup with beam splitter 2 (BS2) removed (right).

All options (I)–(III) are correct. Following this question and peer discussions, an inquiry-based approach is used in the QuILT that provides scaffolding support and strives to help them learn about the role of BS2 and whether interference is observed for single photons without BS2.

Role of additional detectors inserted into one of the paths of the MZI.—In interviews and written responses, students often claimed that inserting an additional detector in the U or L path of the MZI would not affect the interference at the detectors D1 and D2 (see Fig. 3). They struggled to reason about why an additional detector would collapse the state of the photon to the U or L path state (instead of the single photon state being a superposition of the U and L path states). They also had difficulty with how the collapse of the photon state to the U or L path state causes the detectors D1 or D2 after BS2 to click with equal probability and destroys the interference at the detectors. The following questions in the guided inquiry sequence were designed to have students think about the effect of an additional detector placed in one of the paths of the MZI on the interference at the detectors based upon their understanding up to that point in the QuILT before further scaffolding is provided.

A. Choose all of the following statements that are correct if you insert an additional detector into the lower path (see Fig. 3) and the source emits a large number (N) of single photons.

(I) *The interference is unchanged (without the phase shifter, N photons reach D1 and no photons reach D2).*

(II) *The interference vanishes.*

(III) *Changing the thickness of the phase shifter will not affect the number of photons reaching detectors D1 and D2.*

Explain your reasoning for the preceding question.

B. In Fig. 3, why will changing the thickness of the phase shifter not affect the number of photons arriving at the detectors? Explain your reasoning below.

Options (II) and (III) are correct in part A. Following these questions, students are also given the opportunity to use a computer simulation to check their responses to questions about whether placing additional detectors into one or more of the paths of the MZI will affect the interference at detectors D1 and D2. Figure 7 shows a screen shot of the simulation in which an additional detector was placed in one of the paths of the MZI. Students can use this computer simulation to observe that there are no interference fringes on the screen when an additional detector is placed in one of the paths of the MZI (see Fig. 7). The QuILT strives to achieve optimal

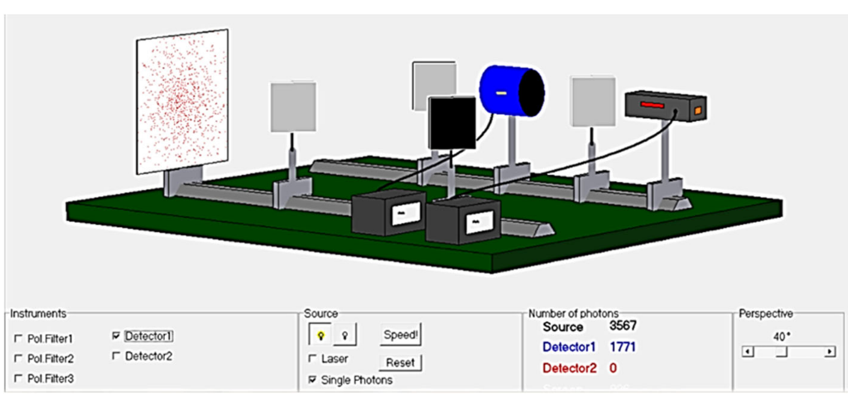

FIG. 7. Screen shot of the computer simulation in which an additional detector (blue device) is placed in one of the paths of the MZI. Simulation developed by Albert Huber.

mismatch and students are given the opportunity to reconcile the differences between their prediction and what they observe (using the computer simulation). Additional scaffolding is provided to help them assimilate and accommodate the concepts via the guided inquiry-based approach.

Unfamiliarity with the concept of which-path information (WPI).—Students were often unfamiliar with the concept of WPI and the relationship between interference of a single photon and whether WPI is known or unknown. Therefore, the QuILT gives students an opportunity to reason about how adding or removing optical elements can yield WPI about the photon arriving at the detector and destroy the interference at detectors D1 and D2. For example, students were asked to predict whether WPI is known and interference would be observed in the case in which an additional detector is placed in one of the paths of the MZI (see Fig. 3). Then, they were asked to check their prediction using the computer simulation and reconcile differences between their prediction and what they actually observed (see Fig. 7). Students observe that no interference occurs and WPI is known about a single photon when an additional detector is placed in one of the paths of the MZI (i.e., only one component of the photon path state is projected into each detector after beam splitter BS2). Then, the guided inquiry-based approach in the QuILT strives to scaffold student learning such that students are in the ZPD (or they are in the optimal adaptability corridor or they have an optimal mismatch) and help students learn the relationship between WPI and whether interference is observed.

After working through the QuILT, students are expected to be able to qualitatively reason about how a single photon can exhibit the properties of both a wave and a particle. They are also expected to be able to describe how a photon can be delocalized or localized depending on the situation and that the measurement of a photon's position at the detector collapses the photon path state. Students are also expected to be able to explain the roles of BS1, BS2, and additional detectors placed in the MZI and how these affect the interference at the detectors. Students should also be able to reason about whether a particular MZI setup gives WPI about a single photon and destroys the interference observed at the detectors and whether inserting a phase shifter will change the number of photons arriving at detectors D1 and D2.

VII. EVALUATION OF THE QuILT

Once we determined that the QuILT was effective in individual administration, it was administered to upper-level undergraduate and graduate students. Students ($N = 44$) in two upper-level undergraduate quantum mechanics courses first had instruction in relevant topics. The instruction included an overview of the MZI setup and students learned about the propagation of light through the beam-splitters, phase difference introduced by the two paths of the MZI, and the meaning of what happens when the detectors click. Then, students were given a pretest on the topics in class. All students had sufficient time to work through the pretest. The QuILT warm up was given to undergraduate students to work on at home. Students worked through part of the main QuILT in class and were given one week to work through the rest of the QuILT as homework. The pretest and QuILT counted as a small portion of their homework grade for the course. However, the undergraduate students were not given credit for working through the QuILT warm up. The undergraduate students were then given a post-test in class (all students had sufficient time to take the post-test). The post-tests were graded for correctness as a quiz for the quantum mechanics course. In addition, the upper-level undergraduate students were aware that topics discussed in the tutorial could also appear in future exams since the tutorial was part of the course material for the quantum mechanics course.

The QuILT was also administered to graduate students ($N = 45$) who were simultaneously enrolled in the first semester of a graduate level core quantum mechanics course and a course for training teaching assistants in two consecutive years. In the teaching assistant training class, the graduate students learned about instructional strategies for teaching introductory physics courses (e.g., tutorial-based approaches to learning). They first worked on the pretest (all students had sufficient time to take the pretest). The QuILT warm up was given to the graduate students to work on at home. The graduate students worked through the QuILT in the teaching assistant training class to learn about the effectiveness of the tutorial approach to teaching and learning. They were given one week to work through the rest of the QuILT as homework. Then, a post-test was administered to the graduate students in class (all students had sufficient time to take the post-test). The graduate students were given credit for completing the pretest, QuILT, and post-test, but they were not given credit for correctness. The graduate students' scores on the post-test did not contribute to the final grade for the teaching assistant training class (which was a pass or fail course).

A. Students' overall performance on the pretest and post-test

Table I shows the common conceptual difficulties and percentages of students displaying them on the pretest or post-test questions and Table II displays the average percentage scores on pretest and post-test questions. Any student who did not work through the QuILT completely was omitted from the post-test data. Average normalized gain [32] is commonly used to determine how much the students learned and takes into account their initial scores on the pretest. It is defined as, $\langle g \rangle = (\%\langle S_f \rangle - \%\langle S_i \rangle)/(100 - \%\langle S_i \rangle)$, in which $\langle S_f \rangle$ and $\langle S_i \rangle$ are the final (post) and initial (pre) class averages, respectively [32]. Table II

TABLE I. Common difficulties and percentages of undergraduate students (UG) and graduate students (G) displaying them on the MZI pretest or post-test questions involving single photons. The number of students who took the pretest does not match the post-test because some students did not work through the QuILT completely and their answers on the post-test were disregarded.

Common difficulty	Pretest UG ($N = 44$)	Post-test UG ($N = 38$)	Pretest G ($N = 45$)	Post-test G ($N = 45$)
Q1. Ignoring interference phenomena	66%	21%	56%	36%
Q2. BS1 causes the photon to split into two parts and halves the photon energy	32%	11%	24%	20%
Q2. Photon must take either U or L path	43%	11%	36%	16%
Q3 and Q4. Removing or inserting BS2 does not affect the probability of the detectors D1 and D2 registering photons	41%	16%	47%	9%
Q5. A photodetector placed in the U or L path may absorb photons but does not affect whether interference is observed if photons arrive at detectors D1 and D2	41%	0%	40%	7%

TABLE II. Average percentage scores and normalized gain $\langle g \rangle$ on the MZI pretest or post-test for undergraduate students (UG) and graduate students (G). The number of students who took the pretest does not match the post-test because some students did not work through the QuILT completely and their answers on the post-test were disregarded.

	Pretest UG ($N = 44$)	Post-test UG ($N = 38$)	$\langle g \rangle$ UG	Pretest G ($N = 45$)	Post-test G ($N = 45$)	$\langle g \rangle$ G
Q1	8%	72%	70%	21%	66%	57%
Q2	31%	86%	80%	41%	76%	59%
Q3	18%	87%	84%	22%	86%	82%
Q4	11%	70%	66%	13%	72%	68%
Q5	61%	97%	92%	50%	87%	74%
All questions	28%	82%	75%	29%	77%	68%

shows that the average normalized gain on questions related to difficulties involving interference of light, the wave-particle duality of a single photon, the probabilistic nature of quantum measurement, the role of BS2, and WPI was 0.75 for undergraduate students and 0.68 for graduate students. We also calculated the effect size denoted by d in the form of Cohen's d [$d = (\mu_1 - \mu_2)/\sigma_{\text{pooled}}$, where μ_1 and μ_2 are the averages of the two groups being compared and $\sigma_{\text{pooled}} = \sqrt{({\sigma_1}^2 + {\sigma_2}^2)/2}$, where σ_1 and σ_2 are the standard deviations of the two groups] [33]. The effect size on the five pretest questions is 2.4 for undergraduate students and 1.4 for graduate students, which is considered large [33].

B. Students' performance on individual questions on the pretest and post-test

Below, we describe student performance on the individual questions on the pretest and post-test that focus on the conceptual difficulties discussed earlier.

Question 1: Interference phenomena.—The following question on the pretest and post-test assessed student understanding of the interference of light in the MZI. In the first year of administration, 36 students were asked the following question about a beam of light propagating through the MZI:

Consider the following statement about sending a beam of monochromatic light through the MZI setup shown in Fig. 1: "If the source produces light with intensity I, the intensity of light at the point detectors D1 and D2 will be I/2 each." Explain why you agree or disagree with this statement.

In the second year of administration, this question was modified to involve single photons and 53 students were asked the following question:

Consider the following statement about single photons emitted from the source in Fig. 1: "If the source emits N photons one at a time, the number of photons reaching detectors D1 and D2 will be N/2 each." Explain why you agree or disagree with this statement.

Both statements are incorrect because the MZI setup is such that there is completely constructive interference at D1 and completely destructive interference at D2. Therefore, the light (or single photons) from the U and L paths arrives completely in phase at detector D1 with intensity I (N photons arrive there) and arrives out of phase at D2 and no light (or no photon) arrives there.

Table I shows that 66% of the undergraduate students and 56% of the graduate students incorrectly agreed with this statement in the pretest, indicating that they did not take into account the interference phenomenon taking place at the detectors. In interviews and written responses, students often did not take into account the phase shifts of a beam of light propagating through the MZI and how the phase difference between the U and L paths causes constructive and destructive interference at the detectors. In addition, students struggled with the fact that the U and L components of the photon state can interfere at the detectors D1 and D2. Even when students were aware that a single photon would be in a superposition of the U and L path states after passing through BS1, they often had difficulty with how the phase shifts of the U and L components of the photon path state would cause constructive or destructive interference at the detectors. After working on the QuILT, this difficulty was reduced and many students took into account interference phenomena of a beam of light or a single photon. Students were given full credit for this question if they stated that they disagreed with the statement and explained that there would be constructive interference at detector D1 and destructive interference at D2.

Question 2: Wave nature of a photon.—The pretest and post-test also evaluated students' understanding of the wave nature of a photon. Students were asked the following question:

Consider the following conversation between Student 1 and Student 2:

- *Student 1: The beam splitter BS1 causes the photon to split into two parts and the energy of the incoming photon is also split in half. Each photon with half of the energy of the incoming photon travels along the U and L paths of the MZI and produces interference at detectors D1 and D2*.
- *Student 2: If we send one photon at a time through the MZI, there is no way to observe interference in the detectors D1 and D2. Interference is due to the superposition of waves from the U and L paths. A single photon must choose either the U or the L path.*

Do you agree with Student 1, Student 2, both, or neither? Explain your reasoning.

Neither student is correct because a photon does not split into two parts with half the energy of the incoming photon but a single photon can be in a superposition of the U and L path states.

On the pretest, students often treated a single photon as a point particle, ignoring its wavelike nature. 43% of the undergraduate students and 36% of the graduate students incorrectly agreed with Student 2 in Question 2 on the pretest claiming that a photon must take either the U or L path. In interviews and written responses, these students were aware that neither a photon nor its energy would be split in half after BS1, but they claimed each photon is localized in either the U or L path. 32% of the undergraduate students and 24% of the graduate students incorrectly agreed with student 1 in the pretest. These students sometimes claimed that either (i) the photon splits into two photons that take the U and L paths or (ii) the photon is in a superposition after beam splitter BS1 but incorrectly interpreted superposition to mean that the photon splits into two photons and the two photons interfere at the detectors (instead of the fact that interference is due to the wave nature of single photons). After working on the QuILT, Table I shows that the difficulties involving interference of a single photon were reduced. The majority of students were able to explain that a single photon is in a superposition of the U and L path states and the U and L components of the photon path state can interfere. Students who stated that they disagreed with both students and provided correct reasoning were given full credit. On the post-test, some students who agreed with student 1 (i.e., that the photon is split with half the energy) wrote statements that were partially correct, e.g., "I agree with student 1 because the photon goes into a superposition state and interferes with itself." Students who wrote these types of statements received half credit since the statement that the photon goes into a superposition of path states after BS1 is correct. Students who agreed with student 2 (i.e., that the photon must choose either the U or L path) were given a score of zero.

Questions 3 and 4: Role of BS2.—These questions on the pretests or post-tests evaluated student understanding of the role of BS2. Students were asked the following two questions:

Question 3. Suppose we remove BS2 from the MZI setup as shown in Fig. 2. How does the probability that detector D1 or D2 will register a photon in this case differ from the case when BS2 is present as in Fig. 1? Explain your reasoning.

Question 4. Suppose we have an MZI setup initially without BS2 (see Fig. 2). If we suddenly insert BS2 after the photon enters BS1 but before it reaches the point where BS2 is inserted (see Fig. 1), with what probabilities do detectors D1 and D2 register the photon? Explain your reasoning. Assume that the situation after BS2 is inserted is identical to Fig. 1.

If BS2 is present, it evolves the state of the photon such that both the U and L path components of the photon state can be projected into each detector and the U and L path components of the photon state interfere at the detectors D1 and D2. In the setup students were given, without the phase shifter in Fig. 1 (when BS2 is present), constructive interference occurs at D1 (the single photons always arrive at D1) and destructive interference occurs at D2 (no photon reaches D2). If BS2 is not present, the photon is still in a superposition of U and L path states after BS1, but only the U path component can be projected in detector D1 and only the L path component can be projected in detector D2. Thus, the photons do not display interference and each detector registers the photons with 50% probability.

In the pretest, 41% of the undergraduate students and 47% of the graduate students incorrectly claimed that removing or inserting BS2 will not change the probabilities of the photon arriving at D1 and D2. This high percentage is consistent with the fact that these students did not acknowledge the wave nature and interference effects of single photons in response to other questions as well. Students often explicitly claimed that the photon behaves as a point particle and it would not matter whether BS2 was present or not—each detector would register the photon with 50% probability. Table I shows that in the post-test, students performed better. After working through the QuILT, many students were able to explain that if BS2 was removed, each detector would register a photon with 50% probability. Students were given zero credit if they stated that the probabilities do not change whether BS2 is present or missing. Students were given full credit on these questions if they stated that (i) when BS2 is present, D1 registers all photons and D2 registers zero photons, and (ii) when BS2 is removed, D1 registers 50% of the photons and D2 registers 50% of the photons. Students were given half credit if they stated that the probabilities would change depending on whether BS2 was present or missing, but wrote the wrong probabilities. In addition to writing the correct probabilities when BS2 was present or missing on the post-test, many students also mentioned how adding or removing beam splitter BS2 would affect whether WPI is known. Most students noted that removing beam splitter BS2 would give WPI about a photon arriving at a detector D1 or D2.

Question 5: Role of additional detectors.—On the pretests post-tests, students were shown a MZI with an additional detector placed in the L path between BS1 and BS2 (see Fig. 3). They were then asked the following question:

How does what you observe at detectors D1 and D2 in Fig. 3 compare to the case in Fig. 1 in which the photodetector is not present?

In the situation in which an additional detector is placed in the L path between BS1 and BS2, if the detector does not absorb the photon, the photon path state must collapse to the U path. WPI is known and interference is not displayed. The detector absorbs half of the emitted photons, and one-quarter of the emitted photons arrive at each detector D1 and D2. Table I shows that in the pretest, 41% of the undergraduate students and 40% of the graduate students incorrectly claimed that adding a detector in the L path would not change anything or would cause fewer photons to arrive at detectors D1 and D2 because some photons are absorbed. These students struggled with the fact that the detector in the L path acts as a measurement device and will collapse the photon state of the photons not absorbed by it to the U path state. After working on the QuILT, the difficulty with the effect of an additional detector placed in the L path of the MZI was reduced (see Table I). On the post-test, the majority of the students noted that the detector collapses the photon state to either the U or L path and that WPI would be known if an additional detector was placed in one of the paths of the MZI. Students were given full credit if they stated either that there would be no interference or that one-quarter of the emitted photons reach each of the detectors (as opposed to all of the photons reaching detector D1 and 0 photons reaching D2) when an additional detector is placed in one of the paths of the MZI.

As shown in Table II, many students still had difficulty with questions 1 and 4 on the post-test. These questions relate to the interference phenomenon and how the phase difference between the U and L paths causes constructive and destructive interference at the detectors. Students were supposed to have learned about this topic in the QuILT warm up at home (ungraded) before the actual QuILT in class. In the future, the warm up should be administered as a graded homework to ensure that students work on it before working on the QuILT in class. Regarding the difficulty with question 4 focusing on the role of BS2 on measurement outcomes, students who had difficulty on the post-test were often partially correct. In particular, many correctly claimed that inserting BS2 would remove WPI, but incorrectly claimed that the probabilities of detection of the photons at D1 and D2 would not change. For example, one student stated, “the probabilities do not change, but we no longer have ‘which-path’ information about each incident photon.” Some students displayed another difficulty and claimed that D1 would register a photon 50% of the time and D2 would never register a photon because although the photon arrives there, it “gets killed.” We have taken into account these findings from in-class administration in the next version of the QuILT.

Table II shows that the performance of graduate students was approximately equal to (or slightly higher than) undergraduate students on the pretest. In contrast, on the post-test, undergraduate students performed slightly better than the graduate students (although the difference in the performance of the two groups is not statistically significant). The difference between undergraduate and graduate students’ performance in the post-test may be due, in part, to the fact that the graduate students’ performance on the post-test was not part of their final grade for the teaching assistant training class. In particular, if students are aware that they are not going to be graded on their performance on a post-test, they may be less motivated to engage with the material in the tutorial (especially if they are working on the tutorial on their own as a homework assignment). It is possible that some graduate students did not work through the tutorial in an engaged manner, i.e., they may not have contemplated their difficulties and attempted to repair their knowledge structure in the same manner as the undergraduate students. This dichotomy could be at least one reason that resulted in the persistence of conceptual difficulties and a smaller overall increase in the scores of graduate students from the pretest to post-test compared to the undergraduates.

VIII. SUMMARY AND FUTURE PLAN

There are few real world experiments that can provide concrete contexts to help students learn the abstract and nonintuitive concepts of quantum mechanics. The context of quantum optics experiments involving the MZI with single photons provides such a concrete context that can be helpful in elucidating the fundamental concepts of quantum mechanics such as the wave-particle duality of a single photon, single photon interference, and the probabilistic nature of quantum measurement and the measurement of the photon at a detector. We conducted research on conceptual difficulties that students have with these concepts in the context of a MZI and used research as a guide to develop a QuILT. The performance on the post-test compared to the pretest suggests that the research-based QuILT on a MZI with single photons was effective in helping upper-level undergraduate and graduate students learn about foundational topics in quantum mechanics in the context of MZI experiments. On the post-test, many students took into account the wave nature of a single photon, were able to explain the interference phenomena of a single photon, and correctly reasoned about the collapse of a photon state and WPI in the context of a MZI. Many students in the courses for which the pretest and post-test are discussed in the preceding section stated that it was one of their favorite QuILTs. For example, one student stated "The [MZI QuILT] was pretty cool because I had no idea what the concept of which path information was before."

Since the development of the conceptual QuILT involving a MZI with single photons, we have also developed additional QuILTs which strive to help students integrate conceptual aspects of the MZI involving single photon inference with mathematical formalism using a two state system and a four state system involving photon path states and polarization states [21]. These QuILTs help students connect the qualitative understanding of single photon interference in a MZI with mathematical formalism using a product space for the photon path and polarization states. They also help students develop a quantitative understanding of how beam splitters and polarizers affect interference and measurement outcomes. We are currently conducting in-class evaluation of these QuILTs.

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