Erratum: Quantum Thermodynamics of Correlated-Catalytic State Conversion at Small Scale [Phys. Rev. Lett. **126**, 150502 (2021)]

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Theorem 3 of this Letter states that

Consider four quantum states ρ , ρ' , η , and η' of the system S . Then, $S(\rho||\eta) \geq S(\rho'||\eta')$ holds if and only if there exists a catalyst C and its two states c , d , and a CPTP map which converts $\eta \otimes d$ to $\eta' \otimes d$ and $\rho \otimes c$ to τ satisfying (i) $\text{Tr}_S[\tau] = c$, (ii) $\text{Tr}_C[\tau]$ is arbitrarily close to ρ' , (iii) the correlation between S and C in τ is arbitrarily small.

In this theorem, the following additional condition C on catalysts was missing: (C) The support of d contains that of c ; $\text{supp}(c) \subset \text{supp}(d)$. Without this condition C, $S_1(c||d)$ diverges, and the transformation in (S.1) in the Supplemental Material of this Letter no longer holds [1]. Moreover, without condition C, any state transformation becomes possible, as pointed out by Rubboli and Tomamichel (footnote 3 of Ref. [2]). We note, however, that the entire argument of this Letter remains correct only by adding condition C to the statement of Theorem 3. We also note that this modification only touches the definition of catalytic state conversions, and therefore Theorem 3 still applies to any four quantum states ρ , ρ' , η , and η' of the system S . The correct full statement of Theorem 3 is the following:

Consider four quantum states ρ , ρ' , η , and η' of the system S . Then, $S(\rho||\eta) \geq S(\rho'||\eta')$ holds if and only if there exists a catalyst C and its two states c , d satisfying $\text{supp}(c) \subset \text{supp}(d)$, and a CPTP map which converts $\eta \otimes d$ to $\eta' \otimes d$ and $\rho \otimes c$ to τ satisfying (i) $\text{Tr}_S[\tau] = c$, (ii) $\text{Tr}_C[\tau]$ is arbitrarily close to ρ' , (iii) the correlation between S and C in τ is arbitrarily small.

A similar correction is needed for the description of Theorem 3 in the Supplemental Material [1].

- [1] N. Shiraishi and T. Sagawa, Quantum thermodynamics of correlated-catalytic state conversion at small-scale, *Phys. Rev. Lett.* **126**, 150502 (2021).
- [2] R. Rubboli and M. Tomamichel, Fundamental limits on correlated catalytic state transformations, [arXiv:2111.13356](https://arxiv.org/abs/2111.13356).