Exponentially Small Cosmological Constant in String Theory

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We construct a supersymmetric vacuum of string theory in which the magnitude of the vacuum energy is $< 10^{-123}$ in Planck units. The mechanism that we introduce generates a well-controlled exponential hierarchy from an appropriate choice of the quantized parameters in the superpotential. Our approach provides a new angle on the cosmological constant problem.

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Introduction.—Explaining the smallness of the observed dark energy density [1,2],

$$\rho_{\text{obs}} \approx 10^{-123} M_{\text{pl}}^4, \quad (1)$$

is a profound challenge [3–5]. The idea that anthropic selection in a landscape could account for the smallness of ρ_{obs} [6] has been influential and, in broad strokes, appears to align with known and suspected properties of string theory [7]. Toy landscapes of flux configurations in compactifications of string theory [8–10] admit a vast number $\mathcal{N}_{\text{vac}}$ of solutions, and the values that the classical vacuum energy ρ_{class} takes in such ensembles can be finely spaced, with differences $\Delta\rho_{\text{class}} \sim M_{\text{pl}}^4/\mathcal{N}_{\text{vac}} \ll \rho_{\text{obs}}$.

One natural objection is that the true explanation might be a richer one, involving dynamics or a different sort of mechanism, and we might fail to seek it by too-readily accepting an anthropic approach. We will do nothing to overcome this concern, remarking only that, in the absence of any framework for a dynamical explanation, perhaps the best one can do is to sharpen the landscape argument.

A more practical complaint is that exploring a landscape of vacua of string theory and finding solutions therein, in which the actual value ρ_{vac} of the vacuum energy obeys $\rho_{\text{vac}} \approx \rho_{\text{obs}}$, appears out of reach. The mechanism proposed by Bousso and Polchinski [7] relies on the high dimensionality of the space of fluxes, and so one faces a search for special, exponentially rare solutions in a high-dimensional energy landscape. At least in its full generality, this problem’s computational complexity makes it inaccessible by a direct assault [11–13]. Even worse, given a candidate de Sitter vacuum in string theory, one can at best imagine computing ρ_{vac} order by order and

nonperturbatively: in the string loop expansion, in the α' expansion, and perhaps in other approximations at the same time. Achieving in this way a precision of order ρ_{obs} does not seem possible to us. The difficulty of exhibiting de Sitter vacua of string theory with $\rho_{\text{vac}} \approx \rho_{\text{obs}}$ thus presents an obstruction to bringing the landscape argument for the cosmological constant problem into sharp focus in quantum gravity.

An essential part of the problem is achieving *scale separation* in cosmological solutions of quantum gravity: can one find isolated solutions of string theory in which the Kaluza-Klein radius R_{KK} of the internal space is not extremely large in units of the Planck length ℓ_p , and yet the noncompact spacetime has a cosmologically large radius of curvature $R_{\text{cosm}} \sim 10^{60} \ell_p$? In a theory with unbounded continuous parameters, this question might not arise. However, the known classes of realistic solutions of string theory come in large but finite families (see, however, [14]), and their low-energy parameters are ultimately determined by inherently *quantized* parameters of the string vacuum, such as the topological data of a Calabi-Yau flux compactification. Whether or not $R_{\text{cosm}} \gtrsim 10^{60} \ell_p$ can occur in a particular setting evidently depends on the ranges of values that these quantized parameters can take and on how ρ_{vac} depends on these parameters.

In this Letter, we exhibit a solution of type IIB string theory with $|\rho_{\text{vac}}| \approx 10^{-144} M_{\text{pl}}^4$, in which the internal space—an orientifold of a Calabi-Yau threefold—has radius $R_{\text{KK}} \approx 10^4 \ell_p$. This vacuum is an example of extreme scale separation, but in four-dimensional anti-de Sitter space (AdS_4), not dS_4 : the solution preserves $\mathcal{N} = 1$ supersymmetry, and the cosmological constant is negative.

Although this compactification cannot describe our Universe, it provides an intriguing angle on the cosmological constant problem. To stumble upon a solution with vacuum energy ϵM_{pl}^4 in a complex landscape of vacua in which the characteristic scale is M_{pl} , one would naively expect to have to explore $\mathcal{O}(1/\epsilon)$ distinct vacua. The actual distribution of cosmological constants could play a role, of

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course, but the above expectation should be a reasonable guide unless the set of solutions manifests an exponentially strong concentration around $\rho = 0$ —which is to say, unless the landscape furnishes a bona fide statistical solution to the cosmological constant problem, a possibility that we shall discount in this Letter. In particular, to find vacuum energy ϵM_{pl}^4 in a high-dimensional landscape of flux vacua, one generally has to search through $\mathcal{O}(1/\epsilon)$ choices of quantized flux.

We have arrived at $|\rho_{\text{vac}}| \ll 10^{-123} M_{\text{pl}}^4$ *without* performing such a costly search. Instead, we have exploited structures in the quantized parameters occurring in string compactifications: specifically, 3-form flux quanta in an orientifold of a Calabi-Yau threefold X and the Gopakumar-Vafa invariants [15,16] of curves in the mirror threefold $\tilde{X}$. We have found choices of flux for which the part of the flux superpotential [17] that descends from perturbative contributions to the prepotential in compactification of type IIA string theory on $\tilde{X}$ vanishes *exactly*, while the nonperturbative terms, which arise from worldsheet instantons on $\tilde{X}$, fall into the form of a racetrack. The competition of two such instanton terms then generates an exponentially small number [18]. When the fluxes and Gopakumar-Vafa invariants are modest integers like 1 or 2, the resulting hierarchy is similarly modest. But there exist, and we have found, Calabi-Yau threefolds for which the integers in question are, for example, 2 and 252, and the hierarchy in the vacuum energy is proportional to

$$\left(\frac{2}{252}\right)^{58} \approx 10^{-122}. \quad (2)$$

In this Letter, we will present one such example.

A Vacuum.—To define a compactification [19] of type IIB string theory on an orientifold of a Calabi-Yau threefold hypersurface X in a toric variety V , we consider the reflexive polytope Δ [21] with vertices given by the columns of

$$\begin{pmatrix} 1 & -3 & -3 & 0 & 0 & 0 & -5 & -2 \\ 0 & -2 & -1 & 0 & 0 & 1 & -3 & -1 \\ 0 & 0 & -1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & -1 & -1 \end{pmatrix}. \quad (3)$$

We define toric varieties $\tilde{V}$ and V , respectively, in terms of fine regular star triangulations of Δ and its polar dual Δ° . The anticanonical hypersurfaces in $\tilde{V}$ and V are a mirror pair of Calabi-Yau threefolds $\tilde{X}$ and X , with $h^{1,1}(X) = h^{2,1}(\tilde{X}) = 113$, and $h^{2,1}(X) = h^{1,1}(\tilde{X}) = 5$. We find an O3/O7-orientifold involution of X with 26 O7-planes and 48 O3-planes, and obeying $h_{-}^{1,1} = h_{+}^{2,1} = 0$. The D3-brane tadpole is 60.

In this compactification, the 3-form fluxes

$$\begin{aligned} \vec{f} &= (10 \ 12 \ 8 \ 0 \ 0 \ 4 \ 0 \ 0 \ 2 \ 4 \ 11 \ -8), \\ \vec{h} &= (0 \ 8 \ -15 \ 11 \ -2 \ 13 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0), \end{aligned} \quad (4)$$

which carry D3-brane charge 56, lead to an exponentially small flux superpotential. The dominant instantons along the perturbatively flat valley, as defined in [18], have Gopakumar-Vafa invariants -2 and 252 , respectively. The flux superpotential takes the form

$$W_{\text{flux}}(\tau) = \sqrt{\frac{2^3}{\pi^5}} (-2e^{2\pi i \tau \frac{7}{29}} + 252e^{2\pi i \tau \frac{7}{28}}) M_{\text{pl}}^3, \quad (5)$$

up to corrections of order $\mathcal{O}(e^{2\pi i \tau (43/116)})$. The string coupling is $g_s \approx 0.011$, and we find

$$W_0 := \langle |W_{\text{flux}}| \rangle \approx 0.526 \times \left(\frac{2}{252}\right)^{29} M_{\text{pl}}^3 \approx 6.46 \times 10^{-62} M_{\text{pl}}^3. \quad (6)$$

Turning now to the Kähler moduli, we find 114 prime toric divisors D_I that are rigid and whose uplifts to divisors in F-theory have trivial intermediate Jacobian. These divisors therefore generate nonvanishing superpotential terms [22], and their Pfaffian prefactors $\mathcal{A}_{D_I}$ are pure constants [23]. Thus the total superpotential takes the form

$$W = W_0 + \sum_I \mathcal{A}_{D_I} \exp\left(-\frac{2\pi}{c_I} T_{D_I}\right) + \dots, \quad (7)$$

where $c_I = 6$ if there is an $\text{so}(8)$ stack of seven-branes on D_I , and $c_I = 1$ otherwise.

With this superpotential, we find a $\mathcal{N} = 1$ supersymmetric AdS_4 vacuum at which the volume of the internal space in string units is $\mathcal{V} \approx 945$. At this point in Kähler moduli space there are perturbative and nonperturbative corrections to the Kähler potential $\mathcal{K}$, but these lead to a controllably small *multiplicative* correction to the vacuum energy [20]. The overall scale of the vacuum energy is dictated by the superpotential, which does not suffer renormalization at any perturbative order [24,25]. Convergence of the series of worldsheet instanton corrections to $\mathcal{K}$ is shown in Fig. 1.

We have therefore found a supersymmetric AdS_4 vacuum with vacuum energy

$$V_0 = -3M_{\text{pl}}^{-2} e^{\mathcal{K}} |W|^2 \approx -1.68 \times 10^{-144} M_{\text{pl}}^4. \quad (8)$$

This vacuum is very well controlled as a result of the residual supersymmetry and the smallness of the string

FIG. 1. Left: the logarithm of the size ξ_n of the n th term in the series of worldsheet instantons resulting from a sample of 1728 rays in $H_2(X)$. Right: histogram of the slopes of the lines in the left panel. The sum evidently converges.

coupling. The Kaluza-Klein radius is $R_{\text{KK}} \approx 10^4 \ell_p$, so the hierarchy of scales is $R_{\text{cosm}}/R_{\text{KK}} \approx 10^{68}$.

We have constructed a number of comparable vacua in compactifications on orientifolds of Calabi-Yau hypersurfaces with $4 \leq h^{2,1} \leq 7$ and $51 \leq h^{1,1} \leq 214$ [20].

Discussion.—The cosmological constant problem demands that we explain how exponentially large universes arise in theories with small fundamental length scales. In this Letter, we have taken a step forward by showing how an exponentially large *supersymmetric* universe can arise in string theory. Specifically, we exhibited an AdS_4 solution of string theory in which the AdS length is approximately $10^{72} \ell_p$, which is much larger than the diameter of the observable Universe in which we live. Our construction thus solves a supersymmetric version of the cosmological constant problem and may shine a light toward a solution of the much harder general problem.

A key mechanism underlying our class of constructions is the exact cancellation of all perturbative terms in the superpotential. Specifically, we have arranged the quantized 3-form fluxes so that all perturbative [26] contributions to the flux superpotential cancel perfectly: this is possible because the superpotential is ultimately determined by integer data. All remaining terms in the scalar potential are proportional to nonperturbative effects. We have made further discrete choices to arrange that these effects balance in a controlled minimum, finding an orientifold of a Calabi-Yau threefold X that supports an array of nonperturbative superpotential terms from Euclidean D3-branes and gaugino condensation, and whose mirror $\tilde{X}$ enjoys a pattern of Gopakumar-Vafa invariants that give rise to a racetrack of worldsheet instantons [18]. These choices lead to stabilization of all the moduli in a supersymmetric vacuum, as foreseen in [27], with a cosmological constant that is exponentially small in Planck units.

The smallness of the vacuum energy in our construction is *natural*, in the sense that it is determined by a competition among exponentials, once we have ensured the absence of all perturbative terms in the superpotential by

making discrete choices of topology and fluxes. Of course, it is very well known that small scales can arise dynamically in supersymmetric theories [28], and in four-dimensional $\mathcal{N} = 1$ effective supergravity one can easily write a racetrack superpotential whose minimization yields a supersymmetric AdS_4 vacuum with small vacuum energy. But presented with such an effective description, one could ask how finely balanced a racetrack is allowed by ultra-violet completion in quantum gravity. Thus, something is gained by realizing such a construction in string theory, where all the underlying parameters are quantized, and their allowable values can be determined.

The principal result of this Letter is our demonstration, through the explicit example of Sec. II, that exponentially small vacuum energy, on a scale comparable to the observed dark energy density in our Universe, is possible in a compactification of string theory. Our solution is not realistic: the cosmological constant is negative, and we have *not* addressed the fundamental problem of arranging for small positive vacuum energy after supersymmetry breaking, electroweak symmetry breaking, and the QCD phase transition. However, as we explained in the Introduction, our result is already very surprising from the perspective of prior studies of the statistics of string vacua [8–10] and of the computational complexity of finding small vacuum energy therein [11–13] by choosing appropriate flux quanta [7]. In this sense, we have uncovered a new perspective on the cosmological constant problem: one can speculate that the smallness of the observed vacuum energy in our Universe could be governed by nonperturbative effects in a configuration in which perturbative contributions exactly vanish, and the exactness of the cancellation is ensured by the quantization of fundamental parameters.

Our construction is, at the same time, the first explicit example of a vacuum with a hierarchically small cosmological constant in the celebrated framework proposed by Kachru *et al.* (KKLT) [27] (for important early works in this direction, see [29–31]). The methods laid out in this Letter allow for construction of ensembles of explicit and well-controlled KKLT vacua [20] and so pave the way for direct exploration of a corner of the landscape of string theory that has been the subject of intense investigation without a corresponding wealth of examples.

Exploring solutions of the sort we have presented here may lead to an improved understanding of the cosmological constant problem in quantum gravity.

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