Erratum: Semirelativistic Schrödinger Equation for Relativistic Laser-Matter Interactions [Phys. Rev. Lett. **121**, 253202 (2018)]

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In this Letter, a Foldy-Wouthuysen-like transformation is applied in order to derive a semirelativistic equation of Schrödinger form from the time-dependent Dirac equation [1]. However, one of the expressions in this regard is erroneous.

The transformed Hamiltonian reads

$$H' = THT^\dagger - i\hbar T \frac{d}{dt} T^\dagger, \quad (1)$$

where

$$T = e^S, \quad (2a)$$

$$S = \frac{1}{2d} \tan^{-1} \left(\frac{d}{mc} \right) \beta \boldsymbol{\alpha} \cdot \mathbf{d} = \frac{1}{2mc} \beta \boldsymbol{\alpha} \cdot \mathbf{d} + \mathcal{O}(c^{-3}), \quad (2b)$$

$$\mathbf{d} = \mathbf{p} + e\mathbf{A} + \frac{e^2}{2mc} A^2 \cdot \hat{\mathbf{k}}. \quad (2c)$$

$\mathbf{A}$ is the vector potential of the laser field, which, in this context, is taken to be dependent on time only, and $\hat{\mathbf{k}}$ is a unit vector pointing in the propagation direction. In Eq. (15) of the Letter it is claimed that Eq. (1) may be written

$$H' = e^S H e^{-S} + i\hbar \frac{d}{dt} S = \beta \sqrt{m^2 c^4 + d^2 c^2} + e^S V e^{-S} - \frac{e^2}{2m} A^2 + i \frac{\hbar}{2} \beta \boldsymbol{\alpha} \cdot \boldsymbol{\lambda}, \quad (3)$$

where

$$\boldsymbol{\lambda} = \frac{d}{dt} \frac{\mathbf{d}}{d} \tan^{-1} \left(\frac{d}{mc} \right) = -\frac{e}{mc} \mathbf{E}_{\text{eff}} + \mathcal{O}(c^{-3}), \quad (4)$$

and

$$\mathbf{E}_{\text{eff}} = -\frac{d}{dt} \left(\mathbf{A} + \frac{e}{2mc} A^2 \hat{\mathbf{k}} \right) = \mathbf{E} + \frac{e}{mc} \mathbf{A} \cdot \mathbf{E} \hat{\mathbf{k}}. \quad (5)$$

However, in Eq. (3), a term originating from the last term in Eq. (1) is missing. The transformed Hamiltonian should read

$$\begin{aligned} H' &= THT^\dagger - i\hbar T \frac{d}{dt} T^\dagger = e^S H e^{-S} + \frac{\hbar}{2} \beta \boldsymbol{\alpha} \cdot \boldsymbol{\lambda} + \frac{\hbar e}{4m^2 c^2} \boldsymbol{\sigma} \cdot (\mathbf{E}_{\text{eff}} \times \mathbf{d}) + \mathcal{O}(c^{-3}) \\ &= \beta \sqrt{m^2 c^4 + d^2 c^2} + e^S V e^{-S} - \frac{e^2}{2m} A^2 - i \frac{\hbar e}{2mc} \beta \boldsymbol{\alpha} \cdot \mathbf{E}_{\text{eff}} + \frac{\hbar e}{4m^2 c^2} \boldsymbol{\sigma} \cdot (\mathbf{E}_{\text{eff}} \times \mathbf{d}) + \mathcal{O}(c^{-3}). \end{aligned} \quad (6)$$

As the spin degree of freedom is not considered in the numerical examples presented in the Letter, the results are not affected by this error.

[1] L. L. Foldy and S. A. Wouthuysen, *Phys. Rev.* **78**, 29 (1950).