Erratum: Does Ignorance of the Whole Imply Ignorance of the Parts? Large Violations of Noncontextuality in Quantum Theory [Phys. Rev. Lett. 107, 030402 (2011)]

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We first describe an error in our Letter. To derive Equation (22) in the Supplemental Material of the Letter, as well as the expression at the top of page 5, which are key in proving Theorem II.2, the sum $\sum_{\lambda}$ over all ontic states is interchanged with the maximum over the dits $y = y_0y_1$ or y_0 , respectively. This requires a restriction to deterministic ontological models, and can be justified by coarse-graining ontic states that assign the same outcome. As we will show, minor modifications to the proof of Theorem II.2 extend its validity to general, nondeterministic ontological models.

Without interchanging $\sum_{\lambda}$ and $\max_y$, the assumption in Theorem II.2 that $H_{\infty}(Y_0Y_1|E) \geq \alpha$ can be written as

$$2^{-\alpha} \geq \sum_k p(k|\mathcal{M}) \max_{y_0y_1} p(\mathcal{P}_{y_0y_1}|k, \mathcal{M}), \quad (1)$$

where the inequality holds for all guessing strategies $\mathcal{M}$ (d^2 -outcome measurements) for y_0y_1 . Here, $\mathcal{P}_{y_0y_1}$ is the preparation that encodes the dits y_0y_1 . Assuming a general ontological model description with ontic states $\lambda \in \Lambda$, (1) can be rewritten as

$$2^{-\alpha} \geq \sum_k \max_{y_0y_1} \sum_{\lambda} p(\lambda) p(\mathcal{P}_{y_0y_1}|\lambda) p(k|\lambda, \mathcal{M}). \quad (2)$$

To arrive at the statement of Theorem II.2 it remains to lower bound the min-entropy $H_{\infty}(Y_C C|E)$ as $H_{\infty}(Y_C C|E) \geq \alpha/2$. Recall that random counter variable C points to the dit whose value should be recovered. Analogous to (1), this corresponds to the upper bound

$$2^{-\alpha/2} \geq \max_{\mathcal{M}'} \sum_{k'} p(k'|\mathcal{M}') \max_{y_c c} p(y_c, c|k', \mathcal{M}'), \quad (3)$$

which holds for any $2d$ -outcome guessing strategy $\mathcal{M}'$ for $y_c c$. If we define the preparations $\mathcal{P}_{y_0y_10}$, $\mathcal{P}_{y_0y_11}$ that extend $\mathcal{P}_{y_0y_1}$, with the only difference that we interchange the disjoint subsets of Λ on which these are supported, we find that

$$\sum_{k'} p(k'|\mathcal{M}') \max_{y_0} p(y_0, 0_C|k', \mathcal{M}') \leq 2^{\alpha/2} \sum_{k'} \max_{y_0y_1} \sum_{\lambda} p(\lambda) p(\mathcal{P}_{y_0y_1}|\lambda) p(k'|\lambda, \mathcal{M}'), \quad (4)$$

and

$$\sum_{k'} p(k'|\mathcal{M}') \max_{y_1} p(y_1, 1_C|k', \mathcal{M}') \leq 2^{-\alpha/2},$$

for all $\mathcal{M}'$. In deriving these inequalities, we made use of the fact that, by construction,

$$2^{\alpha/2} \sum_{y_0} p[\mathcal{P}_{y_0 y_1 0} | (\lambda, 0_C)] \geq 1.$$

The term on the right-hand side of (4) is identical to the term on the right-hand side of (2). Extending the $2d$ -outcome measurement to $d^2 \geq 2d$ effects $\Lambda \rightarrow [0, 1]$, we can use (2), which holds for an arbitrary collection of d^2 effects that sum to the unit effect. As such, $H_\infty(Y_C C | E) \geq \alpha/2$, which proves Theorem II.2.

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