García-García *et al.* Reply: In a recent Comment [1], it was claimed that the Lyapunov exponent λ_L of the N Majoranas Sachdev-Ye-Kitaev model of strength J with a quadratic perturbation of strength κ is

$$\lambda_L/\kappa = 3T^2J^2/\kappa^4 \quad (1)$$

for a fixed and large N , $T/\kappa \ll 1$, $J/\kappa \ll 1$. Therefore, according to Ref. [1], the dynamics is quantum chaotic for a sufficiently large κ assuming the rest of parameters (T , N , J) are fixed. This is in tension with the results of Ref. [2] where we show that, for T , N , J fixed, this model is only quantum chaotic for sufficiently small κ . Here we show why the semiclassical formalism employed to obtain Eq. (1) is not applicable in this range of parameters and show explicitly that the system is no longer quantum chaotic in the large κ limit. We note that the perturbative $1/N$ expansion leading to Eq. (1) in [1] is valid for $t_E \leq t \ll t_H$ where $t_H \sim 1/\Delta$ is the Heisenberg time, Δ is the mean level spacing, t_E is the Ehrenfest or scrambling time, and $\hbar = 1$. Physically, this is the timescale for which nonperturbative in N quantum effects, related to the discreteness of the spectrum, become dominant. The analytical treatment of Ref. [1] is semiclassical, namely, it captures only the leading $1/N$ corrections. Therefore, it is not in principle applicable when t_E becomes of the order or larger than t_H . In our model, we can estimate $t_H \sim t_0 2^{N/2}$, where t_0 is N independent, so the formalism of Ref. [1] is not applicable for

$$t_E \geq t_H \Leftrightarrow \lambda_L(\kappa, T, J) \leq 2^{1-N/2} \ln N. \quad (2)$$

It is straightforward to estimate the maximum $\kappa = \kappa_*$ for which Eq. (1) applies:

$$\kappa_*(T, N, J) \sim \left(\frac{TJ}{\epsilon_0^{1/2}} \right)^{2/3} \frac{2^{N/6}}{(\ln N)^{1/3}}, \quad (3)$$

where $\epsilon_0 = 1/t_0$ and we omit numerical factors $\mathcal{O}(1)$.

This is in contradiction with the claim of Ref. [1] that, for any fixed value of $N \gg 1$, T , J , Eq. (1) applies for any sufficiently large κ . The fact that this is only true for $\kappa < \kappa_*$ casts doubts about the quantum chaotic nature of the motion for sufficiently large κ which is the main claim of Ref. [1]. We show next that this claim is indeed incorrect. The fact that the system is not quantum chaotic for a fixed J , N and sufficiently large κ is rather evident from the level statistics analysis of Ref. [2]; see the adjacent gap ratio in Fig. 11 of the Supplemental Material of Ref. [2] or the lower panels of Fig. 1 and Fig. 2 in the Letter. As a further

FIG. 1. Left: distribution function $P(r)$ of the adjacent gap ratio for $\kappa = 50$ (see definition of r around Eq. 35 of Ref. [3]), for different temperatures (T is defined in Ref. [2]). Agreement with Poisson statistics contradicts the claim of Ref. [1] that the system is quantum chaotic in this region. Right: Lyapunov exponent as a function of $\beta = 1/T$ for various κ 's, circles are the results of Ref. [2], and the squares are the recalculated results.

confirmation of this point, in the left panel of Fig. 1, we depict results of the distribution function of the adjacent spacing ratio $P(r)$ for $\kappa = 50$ and different temperatures. In contradiction with the claims of Ref. [1] about the persistence of quantum chaos for any $\kappa/T \gg 1$, we find that, in this region, $P(r)$ is already very close to Poisson statistics that characterizes an integrable or many-body localized phase.

Moreover, it has recently been shown analytically [3] that for $\kappa > \kappa_c \sim N^2 \ln N$, the system is no longer quantum chaotic, *all* states are many-body localized, and spectral correlations are well described by Poisson statistics. In the language used in Ref. [1], this means that the Lyapunov exponent is zero for any temperature. The results of Ref. [2] point to a much weaker N dependence, if any, of κ_c in the tail of the spectrum corresponding to the low temperature limit. However, the analytical result of Ref. [3] refutes already the main claim of Ref. [1] that λ_L is always finite.

Finally, in the right panel of Fig. 1, we compare Fig. 3 of [2] with the recalculated λ_L after we noticed that for a few κ 's in the low temperature limit, our code did not pick up the largest λ_L . The corrections are only appreciable for a few points. A fitting of $\lambda_L(T)$ still suggests the possible existence of a critical temperature $T_c(\kappa)$ below which $\lambda_L = 0$. However, the values of T_c resulting from the fitting are not conclusive about the vanishing of T_c at finite temperature. In any case, the conclusions of this Reply, or of Ref. [2], are not modified because, as argued earlier, the theoretical formalism employed in Ref. [1] breaks down for a sufficiently large $\kappa/T \gg 1$ and, more importantly, in stark contrast with the claim of Ref. [1], level statistics results together with the analytical findings of Ref. [3], confirm that, for a fixed N , the model is not quantum chaotic but many-body localized/integrable for sufficiently large κ .

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