Comment on "Chaotic-Integrable Transition in the Sachdev-Ye-Kitaev Model"

In their Letter [1], the authors studied a variant of the Sachdev-Ye-Kitaev model with quadratic and quartic interactions (also known as a mass-deformed SYK). They claimed that the quantum Lyapunov exponent λ_L would vanish below some critical temperature. In this Comment, we show that this is not the case. We calculate λ_L exactly in the perturbative regime where the temperature T and the quartic coupling J are much smaller than the quadratic coupling κ. At leading order in T/κ and J/κ, we find

$$\frac{\lambda_L}{\kappa} = \frac{3T^2J^2}{\kappa^4}. \tag{1}$$

Therefore, this model has no $\lambda_L = 0$ phase at small but nonzero T, J.

To derive Eq. (1), we improve upon the standard approach followed by the Letter's authors and analyze the leading eigenvalue problem of the ladder kernel, Eq. (9) of the Letter:

$$K = K_2K_1, \qquad (K_1f)(t) = [\kappa^2 + 3J^2G_{lr}^2(t)]f(t), \tag{2a}$$

$$(K_2f)(\omega) = \left|G_R\left(\omega + \mathbf{i}\frac{\lambda_L}{2}\right)\right|^2 f(\omega), \tag{2b}$$

where G_R and G_{lr} are retarded and Wightman Green functions of a fermion, respectively. Recall also that λ_L is found by imposing that the largest eigenvalue of K is 1. We shall work perturbatively in J/κ and T/κ up to the leading order. Such a perturbative expansion is controlled since the quartic term is irrelevant in the low-T limit where the model is a Fermi liquid. Our analysis thus differs from the Letter, which considered a perturbative expansion with respect to the relevant term.

To start, we compute G_{lr} and G_R to an adequate order. For G_{lr}, the conformal solution of the $J = 0$ limit [2] suffices:

$$G_{lr}(t)^2 = \frac{T^2}{\kappa^2}\mathrm{sech}(t\pi T)^2. \tag{3}$$

For $|G_R|^2$, the $J = 0$ conformal solution $G_R(\omega) = -\mathbf{i}/\kappa$ [2] is not enough, and we shall incorporate the leading correction. That comes from quasiparticle decay encoded in the imaginary part of the self-energy $\Sigma_4(\tau) := J^2G(\tau)^3$ in

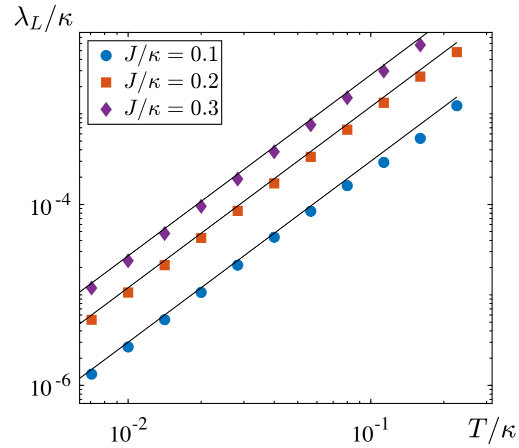

FIG. 1. Temperature dependence of the Lyapunov exponent λ_L for various interaction strengths (markers) compared to the prediction, Eq. (1) (solid lines).

Euclidean time. Continuing to real time, we find that the retarded propagator Σ_4^R satisfies

$$-\mathrm{Im}\Sigma_4^R(\omega) = \frac{J^2T^2}{2\kappa^3} + \frac{J^2\omega^2}{2\pi^2\kappa^3} \tag{4}$$

at leading order in J/κ. Together with the Schwinger-Dyson equation $G_R(\omega)^{-1} = \omega - \kappa^2G_R(\omega) - \Sigma_{4,R}(\omega)$, Eq. (4) implies that

$$\kappa^2\left|G_R\left(\omega + \mathbf{i}\frac{\lambda_L}{2}\right)\right|^2 = 1 - \frac{\lambda_L}{2\kappa} - \frac{J^2T^2}{2\kappa^4} - \frac{J^2}{2\pi^2\kappa^4}\omega^2, \tag{5}$$

where we also omitted higher orders of λ_L as we anticipated it to be small.

With Eqs. (3) and (5), the kernel, Eq. (2), reduces to

$$K = 1 - \frac{\lambda_L}{2\kappa} - \frac{J^2T^2}{2\kappa^4} - \frac{J^2T^2}{\kappa^4}\left[-\frac{1}{2\pi^2T^2}\partial_t^2 - 3\mathrm{sech}(t\pi T)^2\right]. \tag{6}$$

Upon rescaling $s = t\pi T$ in the parenthesis, we recognize the 1D Schrödinger Hamiltonian with a Pöschl-Teller potential: $-\frac{1}{2}\partial_s^2 - 3\mathrm{sech}(s)^2$. Its ground state energy is $E_0 = -2$ [3]. Thus, the largest eigenvalue of K equals $1 - \lambda_L/(2\kappa) + 3J^2T^2/(2\kappa^4)$. Imposing it to be 1 leads to Eq. (1).

We verified our prediction numerically in the large-N limit by directly computing the leading eigenvalue of the kernel, Eq. (2), where the Green functions are (nonperturbative) numerical solutions of the real-time Schwinger-Dyson equations [2]. We find a nice agreement in the perturbative regime (see Fig. 1). Observe, however, that λ_L/κ can become very small and could be mistaken for 0 due to numerical artifacts. Finally, we note that Refs. [4–6] observed $\lambda_L \sim T^2 > 0$ in other large-N models of disordered Fermi liquid.

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Jaewon Kim[1] and Xiangyu Cao[2]

[1]Department of Physics, University of California
Berkeley, California 94720, USA

[2]Laboratoire de Physique de l'Ecole Normale Supérieure
ENS, Université PSL, CNRS
Sorbonne Université
Université de Paris, 75005 Paris, France

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