Double Copy for Celestial Amplitudes

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Celestial amplitudes which use conformal primary wave functions rather than plane waves as external states offer a novel opportunity to study properties of amplitudes with manifest conformal covariance and give insight into a potential holographic celestial conformal field theory at the null boundary of asymptotically flat space. Since translation invariance is obscured in the conformal basis, features of amplitudes that heavily rely on it appear to be lost. Among these are the remarkable relations between gauge theory and gravity amplitudes known as the double copy. Nevertheless, properties of amplitudes reflecting fundamental aspects of the perturbative regime of quantum field theory are expected to survive a change of basis. Here we show that there exists a well-defined procedure for a *celestial double copy*. This requires a generalization of the usual squaring of numerators which entails first promoting them to generalized differential operators acting on external wave functions and then squaring them. We demonstrate this procedure for three- and four-point celestial amplitudes and give an argument for its validity to all multiplicities.

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Introduction.—Scattering amplitudes are usually calculated using asymptotic states in a plane wave basis. Applying a Mellin transform to the energy of each external state in an amplitude amounts to a change in basis of the asymptotic states from plane waves to so-called *conformal primary wave functions* [1]. In this basis, the four-dimensional Lorentz group $SL(2, \mathbb{C})$ acts manifestly as the group of conformal transformations of the *celestial* two-sphere at null infinity. Amplitudes in which asymptotic states are in this conformal basis make conformal covariance manifest [2–8] and, thus, offer a novel opportunity to study properties of amplitudes that may be obscured in a plane wave basis. Moreover, it is hoped that studying these *celestial amplitudes* will shed light on aspects of a possible flat space holographic principle, where a conformal field theory (CFT) living on the celestial sphere would be dual to a bulk theory living on asymptotically flat spacetime.

Celestial amplitudes have a rather complicated analytic structure already at tree level which is further complicated by the fact that the Mellin transform mixes the UV and the IR. It is therefore curious yet reassuring that universal aspects of amplitudes such as soft factorization theorems survive in the conformal basis [9–19]. In general, features

of amplitudes that are expected to reflect fundamental properties of the perturbative regime of quantum field theory should survive a change of basis.

One such remarkable feature are the double copy relations [20], which state that gravitational amplitudes can be obtained by a well-defined “squaring” of gauge theory amplitudes. These relations are known to hold to all multiplicities at tree level [21–24] and can be seen as implied by string theory relations [21,22,25–34] in their low-energy limit. They also hold at loop level in a plethora of cases and for many pairs of field theories (see [35] for a comprehensive review). Moreover, there are versions of the double copy relating full nonlinear solutions of gauge theory and gravity [36–50]. Thus, it is reasonable to expect the double copy to also hold in some form after a change of basis for the asymptotic particles.

However, because translation invariance is obscured in the conformal basis, a generalization of the double copy that does not rely on momentum conservation is required. This is not dissimilar to the situation for amplitudes in curved backgrounds for which a notion of the double copy was shown to persist [51–56]. This provides another incentive to study the double copy for celestial amplitudes, as it presents aspects of curved space amplitudes without the complications of nontrivial backgrounds.

Here we propose a well-defined procedure for a *celestial double copy*. This requires a generalization of the usual squaring of numerators to first promoting them to differential operators acting on external scalar wave functions and then squaring them. Moreover, our proposal offers an

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interesting presentation for gauge and gravity amplitudes as operators acting on scalar amplitudes.

Momentum basis vs conformal basis.—We consider massless scattering processes in four-dimensional Minkowski space $\mathbb{R}^{1,3}$, with spacetime coordinates X^μ for $\mu = 0, 1, 2, 3$, where states are labeled by null momentum four-vectors k_μ , such that $k^2 = 0$.

Scattering problems of gauge bosons are usually studied in the plane wave basis which consists of spin-one wave functions $\epsilon_{\mu;\ell}(k)e^{ik \cdot X}$ in Lorenz gauge and spin-two wave functions $\epsilon_{\mu\nu;\ell}(k)e^{ik \cdot X}$ in harmonic (de Donder) gauge. Here $\epsilon_{\mu;\ell}(k)$ are the polarization vectors for helicity ℓ one-particle states which satisfy $\epsilon_\ell(k) \cdot k = 0$, $\epsilon_\pm(k)^* = \epsilon_\mp(k)$, and $\epsilon_\ell(k) \cdot \epsilon_{\ell'}(k)^* = \delta_{\ell\ell'}$. The polarization tensor is taken to be $\epsilon_{\mu\nu;\ell}(k) = \epsilon_{\mu;\ell}(k)\epsilon_{\nu;\ell}(k)$. Plane wave solutions are thus labeled by the spatial momentum $\vec{k}$, the four-dimensional helicity ℓ , and a sign distinguishing incoming from outgoing states.

To make conformal symmetry manifest, an alternative basis of massless wave functions was constructed in Ref. [1], called *conformal primary wave functions*. These are labeled by a point $(z, \bar{z})$ on the celestial sphere and the quantum numbers under the conformal group, namely, the conformal dimension Δ and the two-dimensional spin J , as well as a sign s distinguishing incoming (−) from outgoing (+) states. Writing the momentum four-vector

$$k_\mu = s\omega q_\mu(z, \bar{z}), \quad (1)$$

massless spin-zero conformal primaries satisfying the scalar wave equation are related to the plane wave functions by a Mellin transform [57]

$$\phi^{\Delta,\pm}(X^\mu; z, \bar{z}) = \int_0^\infty d\omega \omega^{\Delta-1} e^{\pm i\omega q \cdot X} = \frac{(\mp i)^\Delta \Gamma(\Delta)}{(-q \cdot X)^\Delta}. \quad (2)$$

We construct massless spin-one wave functions as

$$V_{\mu;J}^{\Delta,\pm}(X^\mu; z, \bar{z}) = \epsilon_{\mu;J}(z, \bar{z}) \phi^{\Delta,\pm}(X^\mu; z, \bar{z}) \quad (3)$$

and spin-two wave functions as

$$V_{\mu\nu;J}^{\Delta,\pm}(X^\mu; z, \bar{z}) = \epsilon_{\mu\nu;J}(z, \bar{z}) \phi^{\Delta,\pm}(X^\mu; z, \bar{z}), \quad (4)$$

where we identified the two-dimensional spin J with the four-dimensional helicity ℓ and the polarization vectors of, respectively, helicity +1 and −1 one-particle states propagating in the q^μ direction are $\partial_z q^\mu = \sqrt{2}\epsilon_+^\mu(q)$ and $\partial_{\bar{z}} q^\mu = \sqrt{2}\epsilon_-^\mu(q)$. The polarization tensors of helicity +2 and −2 states are obtained by the product of same helicity polarization vectors. The wave functions (3) and (4) satisfy, respectively, the Maxwell and linearized Einstein equations and, up to pure gauge terms, transform covariantly under $SL(2, C)$ Lorenz transformations and as two-dimensional conformal primaries with, respectively, spin $J = \pm 1$ and

$J = \pm 2$ and conformal dimension Δ [1,9]. In Ref. [1], they were shown to form a complete δ -function normalizable basis for $\Delta \in 1 + i\mathbb{R}$ on the principal continuous series of the $SL(2, \mathbb{C})$ Lorentz group [58].

Celestial amplitudes.—Give an amplitude in the plane wave basis

$$\mathcal{A}_n(\{\omega_j, \ell_j; z_j, \bar{z}_j\}) = A_n \delta^{(4)}\left(\sum_{j=1}^n k_j^\mu\right), \quad (5)$$

the celestial amplitude is obtained by a Mellin transform on each of the external particles as

$$\tilde{\mathcal{A}}_n(\{\Delta_j, J_j; z_j, \bar{z}_j\}) = \prod_{j=1}^n \left(\int_0^\infty d\omega_j \omega_j^{\Delta_j-1} \right) \mathcal{A}_n. \quad (6)$$

Here $k_j^\mu = s_j \omega_j q_j^\mu(z_j, \bar{z}_j)$ are the momenta of the massless particles with $\omega_j > 0$ and null vectors q_j^μ directed at points $(z_j, \bar{z}_j)$ on the celestial sphere at null infinity where the particles cross, with $s_j = \pm$ labeling outgoing and incoming states. In Refs. [1,3,5], celestial amplitudes were shown to share conformal properties with correlation functions on the celestial sphere. At the same time, translation invariance becomes obscured in the conformal basis. Hence, properties of amplitudes that make heavy use of momentum conservation appear to be lost in celestial amplitudes. An example of this is the perturbative double copy relating Einstein gravity amplitudes to the “square” of Yang-Mills amplitudes [35].

Double copy.—Let Γ_n be the set of trivalent graphs with n external legs and no internal closed loops, corresponding to tree-level amplitudes [59]. Yang-Mills amplitudes can be presented as a sum over these trivalent graphs as

$$\mathcal{A}_n^{\text{YM}} = \delta^{(4)}\left(\sum_{j=1}^n k_j^\mu\right) \sum_{\gamma \in \Gamma_n} \frac{c_\gamma n_\gamma}{\Pi_\gamma}, \quad (7)$$

where, for a particular trivalent graph γ , the numerators c_γ are its color factors, that is, traces of the Lie algebra matrices of the external particles. The numerators n_γ are the kinematical factors given by polynomials in the momenta k_j and polarizations ϵ_j of the external particles. The denominator Π_γ is a product of the scalar propagators associated with the graph γ . The color factors c_γ are not all independent due to the Lie algebra Jacobi identity. For a triple of graphs related by a Bern-Carrasco-Johansson (BCJ) move [60], the color factors obey $c_s - c_t + c_u = 0$. At tree level we can always find kinematical numerators n_γ that obey the same identities as the color factors $n_s - n_t + n_u = 0$. These are called color-kinematics dual numerators or BCJ-satisfying numerators [60]. Substituting the color factors in Eq. (7) by these numerators, we obtain

$$\mathcal{A}_n^G = \delta^{(4)} \left(\sum_{j=1}^n k_j^\mu \right) \sum_{\gamma \in \Gamma_n} \frac{n_\gamma^2}{\prod_\gamma}, \quad (8)$$

which turns out to be an amplitude for external gravitons—this is the double copy procedure [24]. This presentation of the double copy relies heavily on the fact that external particles are in the plane wave basis and on explicit momentum conservation. Both of these require a generalization in order to arrive at a double copy for celestial amplitudes.

Celestial double copy.—Consider a representation of the amplitudes obtained from position space Feynman diagrams where spacetime integrals like

$$\delta^{(4)} \left(\sum_{j=1}^n k_j^\mu \right) = \int \frac{d^4 X}{(2\pi)^4} e^{\sum_{j=1}^n i k_j \cdot X} \quad (9)$$

are left undone. Rewriting the Yang-Mills amplitude (7) and the gravitational amplitude (8) using the integral representation (9) gives rise to the most natural representation in which to generalize the double copy, as first discussed in Ref. [51].

To compute celestial gluon and graviton amplitudes, we use as external states the conformal wave functions (3) and (4), namely,

$$V_\mu(X) = T^a \epsilon_\mu \phi(X), \quad V_{\mu\nu}(X) = \epsilon_\mu \epsilon_\nu \phi(X), \quad (10)$$

where we dropped various labels and added a color index. In analogy to how spacetime derivatives act on wave functions in a momentum basis, we define the generalized celestial momentum K_μ via $\partial_\mu \phi(X) = K_\mu(X) \phi(X)$ as

$$K_\mu(X) = \frac{\Delta q_\mu}{(-q \cdot X)}. \quad (11)$$

In the plane wave basis we would proceed by squaring the Yang-Mills integrand and use momentum conservation $\sum_j k_j^\mu = 0$, as well as $\epsilon_j \cdot k_j = 0$, to show equality with the gravity integrand. In the celestial case, the conformal wave functions still obey $\epsilon_j \cdot K_j = 0$ due to our choice of gauge, but, because the celestial momenta are spacetime dependent, momentum conservation is no longer manifest in (14) and (15) [63]. A naive double copy of the celestial Yang-Mills numerator $(\tilde{n}_{\text{YM}})^2 = \frac{1}{4} [\epsilon_1 \cdot (K_2 - K_3) \epsilon_2 \cdot \epsilon_3 + \text{cyclic}]^2$ gives terms proportional to $(\epsilon_i \cdot \epsilon_j)^2$ as well as cross terms which one might hope to reexpress as the corresponding terms in the gravity integrand (15) via

The explicit dependence of K_μ on spacetime is to be contrasted with the spacetime-independent k_μ of the plane wave basis and is a precursor for the failure of the naive squaring procedure for celestial amplitudes.

To compute celestial three-point amplitudes we extract from the Yang-Mills Lagrangian the three-point vertex

$$\tilde{\mathcal{A}}_3^{\text{YM}} = \frac{1}{2} \text{Tr} \int \frac{d^4 X}{(2\pi)^4} (V_1^\mu V_2^\nu \partial_\nu V_{3\mu} - V_2^\mu V_1^\nu \partial_\nu V_{3\mu} + \text{cyclic}), \quad (12)$$

and for the gravitational amplitude we use

$$\tilde{\mathcal{A}}_3^G = \frac{1}{2} \int \frac{d^4 X}{(2\pi)^4} (V_1^{\mu\nu} \partial_\mu V_{2\rho\sigma} \partial_\nu V_3^{\rho\sigma} - 2V_1^{\rho\nu} \partial_\mu V_{2\rho\sigma} \partial_\nu V_3^{\mu\sigma} + \text{permutations}) \quad (13)$$

as three-point interaction evaluated in the conformal basis. This is most easily obtained from the action introduced in Refs. [61,62], which was checked in the plane wave basis in Ref. [51] to match the three-point interaction from the Einstein-Hilbert Lagrangian. Inserting the external states (10) into (12) and (13), we obtain the celestial three-gluon amplitude

$$\tilde{\mathcal{A}}_3^{\text{YM}} = f^{a_1 a_2 a_3} \frac{1}{2} \int \frac{d^4 X}{(2\pi)^4} [\epsilon_1 \cdot (K_2 - K_3) \epsilon_2 \cdot \epsilon_3 + \text{cyclic}] \prod_{j=1}^3 \phi_j \quad (14)$$

and the celestial three-graviton amplitude

$$\tilde{\mathcal{A}}_3^G = \frac{1}{2} \int \frac{d^4 X}{(2\pi)^4} [(\epsilon_2 \cdot \epsilon_3)^2 \epsilon_1 \cdot K_2 \epsilon_1 \cdot K_3 - 2\epsilon_1 \cdot \epsilon_2 \epsilon_2 \cdot \epsilon_3 \epsilon_3 \cdot K_2 \epsilon_1 \cdot K_3 + \text{perm.}] \prod_{j=1}^3 \phi_j. \quad (15)$$

integration by parts. However, due to the spacetime dependence of the celestial momenta, performing integration by parts on terms quadratic in the same momenta produces extra terms which are not present in the gravity amplitude, resulting in an obstruction to the naive double copy.

Our proposal is to introduce formal differential operators $\mathcal{K}_i$ that replace the generalized celestial momenta K_i . These operators are defined to act on the conformal wave functions ϕ_i as

$$\mathcal{K}_i^\mu \phi_j(X) = K_i^\mu(X) \phi_i(X) \delta_{ij}. \quad (16)$$

Notice that these operators keep track of particle labels and, thus, are *not* equivalent to the usual partial derivatives. Instead, they admit a representation as [64]

$$\mathcal{K}_i^\mu = q_i^\mu e^{\partial_{\Delta_i}}, \quad (17)$$

which is the translation generator of the Poincaré algebra acting on the i th particle [65]. Promoting the celestial momenta K_i in (14) to the operators $\mathcal{K}_i$ and squaring gives

$$(\mathcal{N}_{\text{YM}})^2 = \frac{1}{4}[\epsilon_1 \cdot (\mathcal{K}_2 - \mathcal{K}_3)(\epsilon_2 \cdot \epsilon_3) + \text{cyclic}]^2, \quad (18)$$

which acts on the external wave functions in the three-point amplitude. Evaluating this action explicitly yields terms that differ from the naive double copy by factors of $1 + (1/\Delta_i)$ in terms with the same celestial momenta. These arise from

$$\mathcal{K}_i^\mu \mathcal{K}_i^\nu \phi_i(X) = \left(1 + \frac{1}{\Delta_i}\right) K_i^\mu(X) K_i^\nu(X) \phi_i(X) \quad (19)$$

and cancel precisely the additional terms generated when integrating by parts (18), matching it to the gravitational integrand. The final result is

$$\begin{aligned} (\mathcal{N}_{\text{YM}})^2 \prod_{i=1}^3 \phi_i = & -[(\epsilon_2 \cdot \epsilon_3)^2 \epsilon_1 \cdot K_2 \epsilon_1 \cdot K_3 \\ & + 2\epsilon_1 \cdot \epsilon_2 \epsilon_2 \cdot \epsilon_3 \epsilon_3 \cdot K_2 \epsilon_1 \cdot K_3 + \text{cyclic}] \prod_{i=1}^3 \phi_i, \end{aligned} \quad (20)$$

which is, up to a sign, exactly the gravitational numerator in (15). Hence, the celestial three-gluon amplitude double copies into the celestial three-graviton amplitude.

For four-point amplitudes the new ingredients are the presence of the four-point contact term and the exchange diagrams

$$\tilde{\mathcal{A}}_4^{\text{YM}} = \tilde{\mathcal{A}}_4^s + \tilde{\mathcal{A}}_4^t + \tilde{\mathcal{A}}_4^u + \tilde{\mathcal{A}}_4^{\text{contact}}. \quad (21)$$

As in the plane wave basis, we split the contact term into three contributions according to the color factors, using that the propagator

$$G_{\mu\nu}(X, Y) = \eta_{\mu\nu} G(X, Y) = \eta_{\mu\nu} \int d^4k \frac{e^{ik \cdot (X-Y)}}{k^2} \quad (22)$$

satisfies $\square_X G(X, Y) = (2\pi)^4 \delta^{(4)}(X - Y) = \square_Y G(X, Y)$ as well as the fact that the celestial momenta are null, $K_i^2(X) = 0$. This yields

$$\begin{aligned} \tilde{\mathcal{A}}_4^{\text{YM}} = & \int \frac{d^4X}{(2\pi)^4} \frac{d^4Y}{(2\pi)^4} G(X, Y) \\ & \times (c_s \tilde{n}_s \tilde{\Phi}_s + c_t \tilde{n}_t \tilde{\Phi}_t + c_u \tilde{n}_u \tilde{\Phi}_u), \end{aligned} \quad (23)$$

where

$$\begin{aligned} c_s = f^{a_1 a_2 b} f^{a_3 a_4 b}, \quad \tilde{\Phi}_s &= \phi_1 \phi_2(X) \phi_3 \phi_4(Y), \\ c_t = f^{a_1 a_3 b} f^{a_2 a_4 b}, \quad \tilde{\Phi}_t &= \phi_1 \phi_3(X) \phi_2 \phi_4(Y), \\ c_u = f^{a_1 a_4 b} f^{a_2 a_3 b}, \quad \tilde{\Phi}_u &= \phi_1 \phi_4(X) \phi_2 \phi_3(Y), \end{aligned} \quad (24)$$

and the numerators are given by

$$\begin{aligned} \tilde{n}_s = & [\epsilon_1 \cdot \epsilon_2 (K_1 - K_2)^\mu + 2\epsilon_1 \cdot K_2 \epsilon_2^\mu - 2\epsilon_2 \cdot K_1 \epsilon_1^\mu](X) \eta_{\mu\nu} \\ & \times [\epsilon_3 \cdot \epsilon_4 (K_4 - K_3)^\nu - 2\epsilon_3 \cdot K_4 \epsilon_4^\nu + 2\epsilon_4 \cdot K_3 \epsilon_3^\nu](Y) \\ & + (\epsilon_1 \cdot \epsilon_3 \epsilon_2 \cdot \epsilon_4 - \epsilon_1 \cdot \epsilon_4 \epsilon_2 \cdot \epsilon_3) [K_1 \cdot K_2(X) + K_3 \cdot K_4(Y)], \end{aligned} \quad (25)$$

with appropriate permutations for the other channels.

Following the three-point case, we promote all celestial momenta K_i to operators acting on the scalar wave functions

$$\begin{aligned} \mathcal{N}_s = & [\epsilon_1 \cdot \epsilon_2 (\mathcal{K}_1 - \mathcal{K}_2)^\mu + 2\epsilon_1 \cdot \mathcal{K}_2 \epsilon_2^\mu - 2\epsilon_2 \cdot \mathcal{K}_1 \epsilon_1^\mu] \eta_{\mu\nu} \\ & \times [\epsilon_3 \cdot \epsilon_4 (\mathcal{K}_4 - \mathcal{K}_3)^\nu - 2\epsilon_3 \cdot \mathcal{K}_4 \epsilon_4^\nu + 2\epsilon_4 \cdot \mathcal{K}_3 \epsilon_3^\nu] \\ & + (\epsilon_1 \cdot \epsilon_3 \epsilon_2 \cdot \epsilon_4 - \epsilon_1 \cdot \epsilon_4 \epsilon_2 \cdot \epsilon_3) (\mathcal{K}_1 \cdot \mathcal{K}_2 + \mathcal{K}_3 \cdot \mathcal{K}_4). \end{aligned} \quad (26)$$

The gravity amplitude is obtained by substituting each color factor by its corresponding kinematical operator

$$\begin{aligned} \tilde{\mathcal{A}}_4^G = & \int \frac{d^4X}{(2\pi)^4} \frac{d^4Y}{(2\pi)^4} G(X, Y) \\ & \times [(\mathcal{N}_s)^2 \tilde{\Phi}_s + (\mathcal{N}_t)^2 \tilde{\Phi}_t + (\mathcal{N}_u)^2 \tilde{\Phi}_u]. \end{aligned} \quad (27)$$

To check it, we rely on the fact that any set of numerators of a trivalent graph decomposition of a four-point Yang-Mills amplitude in the plane wave basis,

$$\mathcal{A}_4^{\text{YM}} = \int \frac{d^4X}{(2\pi)^4} \frac{d^4Y}{(2\pi)^4} G(X, Y) (c_s n_s \Phi_s + c_t n_t \Phi_t + c_u n_u \Phi_u), \quad (28)$$

obeys color kinematics and, hence, the Yang-Mills numerators square to a gravitational amplitude

$$\begin{aligned} \mathcal{A}_4^G = & \int \frac{d^4X}{(2\pi)^4} \frac{d^4Y}{(2\pi)^4} G(X, Y) \\ & \times [(n_s)^2 \Phi_s + (n_t)^2 \Phi_t + (n_u)^2 \Phi_u]. \end{aligned} \quad (29)$$

Here n_s and Φ_s are given by expressions analogous to, respectively, Eq. (24) and (25) with K_j replaced by k_j and ϕ_j replaced by $e^{ik_j \cdot X}$. To show that Eq. (27) is indeed the Mellin transform of the gravitational amplitude (29), note that it depends polynomially on the energy of each particle k_i^0 . There are two cases to consider: The numerator is linear in the energy

$$\int d\omega_i \omega_i^{\Delta_i-1} (ik_i^\mu) e^{ik_i \cdot X} = K_i^\mu \phi_i(X), \quad (30)$$

and the numerator is quadratic in the same energy

$$\int d\omega_i \omega_i^{\Delta_i-1} (ik_i^\mu)(ik_i^\nu) e^{ik_i \cdot X} = \left(1 + \frac{1}{\Delta_i}\right) K_i^\mu K_i^\nu \phi_i(X). \quad (31)$$

Since this mirrors the action of the $\mathcal{K}_i$ operators on the conformal wave functions, Mellin transforming Eq. (29) gives precisely Eq. (27) after the action of the $\mathcal{K}_i$ operators. Hence, the celestial four-gluon amplitude double copies into the celestial four-graviton amplitude.

Discussion.—The n -point generalization is straightforward. Given a set of color kinematics satisfying numerators, promote them to operator-valued numerators as delineated above. These are then squared as *operators* and act on the respective scalar amplitude. This gives the representations

$$\tilde{\mathcal{A}}_n^{\text{YM}} = \sum_{\gamma \in \Gamma} c_\gamma \mathcal{N}_\gamma S_\gamma \rightarrow \tilde{\mathcal{A}}_n^G = \sum_{\gamma \in \Gamma} (\mathcal{N}_\gamma)^2 S_\gamma, \quad (32)$$

where Γ is the set of trivalent graphs and S_γ is the scalar amplitude for the trivalent graph γ . It is clear that this is the correct gravity amplitude, since the construction above corresponds to the Mellin transform of a double copied amplitude by the same arguments used at four points. Thus, our prescription is valid to arbitrary multiplicity.

To illustrate this, we write down explicitly the four-point amplitudes in this representation:

$$\begin{aligned} \tilde{\mathcal{A}}_4^{\text{YM}} &= c_s \mathcal{N}_s \mathfrak{s} + c_t \mathcal{N}_t \mathfrak{t} + c_u \mathcal{N}_u \mathfrak{u}, \\ \tilde{\mathcal{A}}_4^G &= (\mathcal{N}_s)^2 \mathfrak{s} + (\mathcal{N}_t)^2 \mathfrak{t} + (\mathcal{N}_u)^2 \mathfrak{u}, \end{aligned} \quad (33)$$

where the s -channel contribution to the cubic four-point scalar amplitude is

$$\mathfrak{s} = \int \frac{d^4 X}{(2\pi)^4} \frac{d^4 Y}{(2\pi)^4} G(X, Y) \prod_{i=1}^2 \phi_i(X) \prod_{j=3}^4 \phi_j(Y), \quad (34)$$

with $\mathfrak{t}$ and $\mathfrak{u}$ obtained by the corresponding permutations. It would be interesting to understand how to obtain these numerators directly in the celestial framework instead of building on their plane wave representations, and what the celestial analog is of the Jacobi relations obeyed by

color-kinematics satisfying numerators. This could also hint at a geometric interpretation of this kinematical algebra as found in Ref. [66] for the maximally helicity violating sector.

The way we presented celestial amplitudes is reminiscent of recent results on ambitwistor strings in Anti-de Sitter spacetime [67,68] which also has operator-valued kinematical numerators. The difference here is that propagators are generated by the usual Feynman rules, while the ambitwistor string uses the scattering equations to do so. Since ambitwistor string theories also provide formulas with manifest double copy structure [69], it would be natural to study them in the celestial context, and, in fact, they have already been useful in studying some properties of celestial amplitudes [13,70,71].

We expect that the natural way to implement the double copy in *curved* spacetimes will be through operator-valued numerators. Our work thus presents an important stepping stone in this direction. It would also be interesting to see how our proposal for the celestial double copy translates into a statement about the two-dimensional correlators directly in the putative celestial CFT.

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