Non-Hermitian Topological Sensors

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(Received 31 March 2020; accepted 23 September 2020; published 29 October 2020)

We introduce and study a novel class of sensors whose sensitivity grows exponentially with the size of the device. Remarkably, this drastic enhancement does not rely on any fine-tuning, but is found to be a stable phenomenon immune to local perturbations. Specifically, the physical mechanism behind this striking phenomenon is intimately connected to the anomalous sensitivity to boundary conditions observed in non-Hermitian topological systems. We outline concrete platforms for the practical implementation of these non-Hermitian topological sensors ranging from classical metamaterials to synthetic quantum materials.

DOI: [10.1103/PhysRevLett.125.180403](https://doi.org/10.1103/PhysRevLett.125.180403)

Introduction.—High-precision sensors represent a key technology that is of ubiquitous importance in both science and everyday life. In the quest for future sensors, a main challenge is to identify viable scenarios, where basic physical observables become highly susceptible to changes in the quantity to be detected, yet in a controllable fashion. Prominent examples along these lines include microcavity sensors [1–5], metal oxide semiconductor field-effect transistor (MOSFET)-based sensors [6], mechanical transducers [7], graphene sensors [8], superconducting quantum interference devices (SQUIDS) [9], and magnetometers [10].

Dissipative systems effectively described by non-Hermitian (NH) Hamiltonians can exhibit a high susceptibility to small perturbations of their complex energy spectrum that has no counterpart in closed Hermitian settings [11–18]. As a first step toward using the unique algebraic properties of NH matrices [11–13] for sensing [19], an enhancement of precision in sensors operating at NH spectral degeneracies known as exceptional points (EPs) has been claimed [20,21]. Furthermore, in NH lattice systems with many degrees of freedom, a striking spectral sensitivity has recently been found in the context of NH topological systems, where phase transitions driven by small changes in the boundary conditions have been predicted and observed [14–17,22–27].

Here, drawing intuition from NH topological phases [27–33], we identify and study a widely applicable mechanism for drastically enhancing the sensitivity of a novel

type of devices coined non-Hermitian topological sensors (NTOS). These systems are designed such that the physical quantity to be detected (measurand) effectively couples to the boundary conditions of an extended system of $2N - 1$ lattice sites [34], e.g., by modifying the coupling Γ between the ends of the NTOS with a tunneling barrier (see Fig. 1). Quite remarkably, in this scenario the energy E_0 of a characteristic bound state is then found to exhibit the exponential sensitivity

$$S = \frac{\partial E_0}{\partial \Gamma} = \kappa \exp(\alpha N), \quad (1)$$

FIG. 1. Left: Illustration of the non-Hermitian topological sensor (NTOS) setup consisting of a one-dimensional chain with $2N - 1$ sites in open ring geometry, described by an effective NH Hamiltonian. The measurand interferes with the coupling Γ between the ends of the open ring (sites 1 and $2N - 1$). The simplest measurement signal is the energy shift ΔE of the lowest eigenmode. Right: exponential scaling of $|\Delta E|$ with system size N of the NTOS based on the NH topological insulator model Eq. (5) for different values of Γ . Dots are exact numerical energies which are in striking agreement with the analytical perturbative expression Eq. (4) (solid lines). Parameters are $t_1 = t_2 = 2\gamma = 1$. In a wide parameter regime, the quotient of $|\Delta E|/\Gamma$ gives a good estimate for the sensitivity S of the NTOS.

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where α , κ are model-specific constants. Below, we demonstrate both analytically and numerically that the exponential amplification of the sensitivity corresponding to $\text{Re}[\alpha] > 0$ is a generic and robust phenomenon that does not rely on any fine-tuning or symmetry and is as such immune to local perturbations including random disorder in the NTOS. Furthermore, we show how Eq. (1) manifests in the response signal of an NTOS device, thus elevating the exponential sensitivity to an enhanced precision in noise-limited experiments.

To illustrate the broad range of physical platforms in which NTOS may be realized from existing building blocks, we outline several implementations in the classical as well as in the quantum realm, including a detailed analysis of a system of coupled optical ring resonators, as well as a discussion of topological electric circuits [23,24,36], a quantum condensed matter setting of coupled quantum dots [37–39], and a topological cold atom systems with dissipation [40–42].

Exponential amplification mechanism.—We now derive and explain with theory the exponential scaling in system size parameter N [cf. Eq. (1)], i.e., in the number of physical degrees of freedom of the NTOS. To this end, we discuss a generic class of NH topological insulator models that exhibit the following basic phenomenology, noting that a specific case study will be presented further below. We consider systems that for an odd number of sites $2N - 1$ exhibit a localized boundary state with energy $E_0(\Gamma)$, satisfying $E_0(0) = 0$, i.e., an exact zero-energy edge mode in the limiting case of open boundary conditions [14,43]. The corresponding unnormalized right and left eigenvectors are then of the general exponential form expressed in terms of the position (site) $j = 1, \dots, 2N - 1$ in the lattice with nonvanishing components

$$\langle j | \Psi_{R,0} \rangle = (r_R)^{(j+1)/2}, \quad \langle \Psi_{L,0} | j \rangle = (r_L)^{(j+1)/2}, \quad j \text{ odd}, \quad (2)$$

where $\mu = R, L$ denote right and left eigenstates localized around $j = 1$ if $|r_\mu| < 1$ and around $j = 2N - 1$ if $|r_\mu| > 1$, and where the amplitude of the boundary mode vanishes for even values of j [43]. It is worth emphasizing that, in stark contrast to the Hermitian realm where $|\Psi_{L,0}\rangle = |\Psi_{R,0}\rangle$, right and left eigenvectors generically differ in NH systems. In fact, the extreme scenario of an overlap

$$\mathcal{O}_{LR} = \langle \Psi_{L,0} | \Psi_{R,0} \rangle / (\|\Psi_{L,0}\| \|\Psi_{R,0}\|) \quad (3)$$

that is exponentially small in system size (N) represents a crucial ingredient for the exponential amplification at the heart of the proposed NTOS. An intuitive picture for this phenomenon is that in the considered NH scenario, a *single* zero-energy state may completely change its spatial location when switching from a right- to a left-eigenstate description.

To see this more quantitatively and derive Eq. (1), we now consider a coupling between the first ($j = 1$) and last ($j = 2N - 1$) sites with a matrix element Γ ,

i.e., $\Delta H = \Gamma(|1\rangle\langle 2N - 1| + |2N - 1\rangle\langle 1|)$. A straightforward calculation to leading order perturbation theory in Γ yields [43]

$$\begin{aligned} \Delta E &\approx \frac{\langle \Psi_{L,0} | \Delta H | \Psi_{R,0} \rangle}{\langle \Psi_{L,0} | \Psi_{R,0} \rangle} \\ &= \Gamma \frac{(r_L r_R - 1)(r_L^{N-1} + r_R^{N-1})}{(r_L r_R)^N - 1} \xrightarrow{N \gg 1} \Gamma \kappa e^{aN}, \end{aligned} \quad (4)$$

where the combination of the unperturbed right and left eigenstates entering the *biorthogonal* expectation values occurring in Eq. (4) is crucial, thus representing a natural generalization of perturbation theory to the non-Hermitian regime. The final expression in Eq. (4) describes the leading asymptotic behavior of ΔE in the limit of large system size that is readily derived by comparing the competing powers of r_L and r_R [43]. While $|\kappa(r_L, r_R)| > 0$, the real part of the crucial parameter $\alpha(r_L, r_R)$ changes its sign corresponding to different physical regimes: as long as both the left and right eigenstates are localized at the same end, i.e., $\text{sgn}[\log(|r_R|)] = \text{sgn}[\log(|r_L|)]$ [cf. Eq. (2)], $\text{Re}[\alpha]$ is negative and ΔE decays exponentially with system size, while $\mathcal{O}_{LR}$ remains of order 1. This is the behavior intuitively expected and familiar from Hermitian systems. However, as soon as the left and right eigenstates are localized at opposite boundaries, i.e., $\text{sgn}[\log(|r_R|)] \neq \text{sgn}[\log(|r_L|)]$, $\text{Re}[\alpha]$ becomes positive, driven by an exponentially small $\mathcal{O}_{LR}$ [see Eqs. (3) and (4)], thus hallmarking a response that grows exponentially with system size [cf. Eq. (1)].

Case study of microscopic model.—To illustrate this phenomenology with a concrete microscopic model, and to quantify the validity regime of the perturbative result (4), we consider a NH Su-Schrieffer-Heeger (SSH) chain [45,46] with alternating nearest neighbor hopping amplitudes $t_1 \pm \gamma$ and t_2 , respectively. However, we stress that our main findings are of relevance beyond this specific setting [47]. In reciprocal space, the model with periodic boundaries is described by the Bloch Hamiltonian

$$H_S(k) = (t_1 + t_2 \cos k, t_2 \sin k + i\gamma, 0) \cdot \boldsymbol{\sigma}, \quad (5)$$

where length is measured in units of the lattice spacing, and $\boldsymbol{\sigma}$ is the vector of Pauli matrices acting on a sublattice degree of freedom with sublattices consisting of the sites with odd (even) index j . For an odd total number of sites $2N - 1$ and open boundary conditions, there is an exact $E = 0$ state of the form (2) with $r_R = -(t_1 - \gamma)/t_2$ and $r_L = -(t_1 + \gamma)/t_2$ [14,43]. To the open chain, we again add the coupling ΔH between the end sites.

In Fig. 1, we plot the absolute value of the energy shift $\Delta E = E_0(\Gamma) - E_0(0)$ due to the presence of a finite coupling Γ of the mode with smallest absolute value of energy for various values of N and Γ . We have chosen the parameters $t_1 = t_2 = 2\gamma = 1$, noting that all qualitative results are robust against changing these parameters

(see discussion below). The exact numerical energies (dots) are found to be essentially identical to the approximate analytical result (4) for a wide range of N and Γ , yielding an exponentially amplified signal over many orders of magnitude. This quantitatively corroborates to striking accuracy our above general analysis.

The perturbative energy shift in Eq. (4) is real valued for the NH SSH model, and the numerically obtained exact energies share this property to a remarkable precision until the energy shift reaches the scale set by the bulk gap. This is reflected in Fig. 1, where deviations from the perturbative results (solid lines) occur at roughly similar ΔE independent of the coupling strength Γ .

Generally, we find an exponential enhancement of the sensitivity $\mathcal{S}$ whenever $||t_1| - |t_2|| < |\gamma| < ||t_1| + |t_2||$. Interestingly, this desirable regime precisely corresponds to the parameter range with a nontrivial spectral winding number [28]

$$\nu = \frac{1}{2\pi i} \oint_{-\pi}^{\pi} dk \frac{\partial}{\partial k} \log(\det[H_S(k)]). \quad (6)$$

The genuinely non-Hermitian integer topological invariant ν characterizes the bulk band structure of the NTOS by measuring how often the complex phase of the determinant of $H_S(k)$ winds around the origin of the complex plane. Specifically, in Eqs. (1) and (4), we obtain $\text{Re}[\alpha] > 0$, i.e., the desired exponential amplification of the sensitivity $\mathcal{S}$, if $|\nu| = 1$, and $\text{Re}[\alpha] < 0$ implying exponential damping of $\mathcal{S}$ if $\nu = 0$. We note that winding numbers around energies shifted from the origin of the complex plane may readily be defined [28]. For the particularly experimentally relevant case of a constant imaginary shift to all energies corresponding to a passive setting with loss only [48], this redefinition does not affect the spatial localization and existence of boundary modes.

While the qualitative change in the response corresponds to the bulk topological invariant ν —and thus occurs at parameters where the Bloch bands touch—it does not entail gap closings in the open boundary system. In fact, from the perspective of energy gaps there is a well-documented breakdown of the conventional bulk-boundary correspondence: In the open boundary system, gap closings occur at $|t_1^2 - \gamma^2| = |t_2|^2$ [14,17,35], instead of the above condition for the change of ν . The origin of this disparity can be traced back to the zero modes in Eq. (2): the response phase transition (sign change of $\text{Re}[\alpha]$) occurs when the right *or* left eigenstates change their localization, i.e., when $|r_R| = 1$ or $|r_L| = 1$, whereas the spectral phase transitions are tied to changes in the biorthogonal localization occurring at $|r_L r_R| = 1$ [14].

Stability against local perturbations.—We now demonstrate the robustness of our main findings against local perturbations. To this end, we add (complex) Gaussian on-site disorder to our above SSH model Hamiltonian of the

NTOS. The disorder strength is quantified by $w > 0$. In a wide parameter range, in particular, as long as w is significantly smaller than the absolute value of bulk energy gap, we find that such local random perturbations do not have any qualitative effect on the exponential amplification of the sensitivity $\mathcal{S}$ with system size. Notably, even though the considered disorder breaks the chiral symmetry of the above NH SSH model resulting in a shift of the lowest (in absolute value) energy eigenvalue away from zero, the difference in energy $\Delta E = E_0(\Gamma) - E_0(0)$ induced by a change in boundary conditions is still found to exhibit a similar exponential sensitivity as in the absence of disorder. In Fig. 2, we corroborate and exemplify this result: even at the level of individual disorder realizations, the exponential growth of the sensitivity $\mathcal{S}$ is very clear, unsurprisingly with visible fluctuations. However, we emphasize that these fluctuations are irrelevant for the operation principle of a given NTOS device for which the system size N as well the disorder realization are fixed. In this sense, our data shows that the expected value for the sensitivity of an individual device follows the same rules as without disorder, which becomes clear when averaging over sufficiently many disorder realizations for every system size N .

Platforms for implementation.—Making available the key ingredient for the proposed NTOS, the general physics of anomalous boundary modes in NH topological systems has recently been observed in a range of experimental settings, including mechanical metamaterials [22], electrical

FIG. 2. Scaling of the boundary-condition induced energy shift $|\Delta E|$ with system size $2N - 1$ in the presence of complex disorder, for various values of Γ and system sizes up to 119 sites. Data for a single disorder realization (solid dots) are compared to a disorder average (circles) over $M = 10^4$ disorder realizations at every N [49]. The disorder strength is $w = 0.3$ in both plots. Solid lines indicate the perturbative result [see Eq. (4)] in the absence of disorder.

circuits [23,24], and photonic quantum walks [25]. For mechanical metamaterials, a detailed dynamical response theory has been developed and shown to possess sharp transitions in the response to external excitations [18]. Furthermore, coupling the boundaries of NH systems has been shown to lead to anomalous dynamics [50]. Motivated by these exciting developments, we now outline several scenarios for the implementation of NTOS.

Our main focus is on an array of coupled optical ring resonators [51], as inspired by experiments on NH EP sensors in such systems [20] as well as optical implementations of topological materials [52,53]. Combining existing technology [20,54–56], the NH SSH model (5) for our NTOS may be realized as follows [43]: the sublattice degree of freedom is represented by the clockwise-anticlockwise mode of each ring resonator. The asymmetric coupling between these modes, parametrized by γ in Eq. (5), has been experimentally achieved in Refs. [20,55] by introducing two scatterers into the mode volume of a ring resonator. While realizing an EP sensor requires fine-tuning of γ to compensate the symmetric coupling strength t_1 , e.g., by adjusting the angle that the scatterers form with the center of the resonator [20], we stress that the proposed NTOS works in a broad parameter range of finite asymmetry γ [43], thus highlighting the topological robustness of our approach. The final ingredient, namely the chirality-selective coupling between neighboring ring resonators [t_2 in Eq. (5)], may be achieved by chiral couplers (see Ref. [56] for a review). Again, far from perfect chirality of the coupling is acceptable to enter the right topological phase of the NTOS, characterized by a nonvanishing winding number ν [cf. Eq. (6)] [43]. In this scenario, the NTOS may be seen as an extension of an EP sensor, alleviating the requirement of fine-tuning and enhancing the attainable precision exponentially in the number coupled ring resonators [cf. Eq. (1)]. Regarding power consumption and response times, the microscopic similarity of these two devices, in both cases limited by a comparable overall energy scale of t_1 , γ (~ 10 MHz in Ref. [20]), allows for their direct comparison. In particular, this implies their operation at similar time scales and similar dissipated energy per resonator, respectively.

Beyond this optical realization, in various classical metamaterials, NTOS devices may be constructed from the available toolbox. The probably simplest platform is provided by topological electrical circuits [23,24], where basically any topological insulator model in arbitrary geometry may be realized, since arbitrary connectivity of the circuit can readily be achieved. The weak link Γ can be designed at any point of a circuit in ring geometry, and a measurand which either couples to an inductance or a capacitor of the device may be detected. By downsizing this architecture to the nanoscale, it may complement the existing toolbox of microelectronic devices including MOSFET sensors.

The proposed NTOS may also be realized in synthetic quantum materials, such as ultracold atoms in optical lattices

[40–42] or coupled arrays of quantum dots [37–39]. In this quantum many-body context, NH Hamiltonians may emerge as an effective description of an underlying quantum master equation [57,58]. In particular, the spectral sensitivity of NH matrices has recently been shown [59] to carry over to a full quantum master-equation description corresponding to NH models similar to the above NH SSH model [see Eq. (5)]. These insights pave the way toward implementing NTOS in chains of cold atoms or coupled quantum dots, where the weak link Γ coupling to a measurand is simply provided by a high tunnel barrier between two neighboring sites. For the example of quantum dot arrays, such a tunnel barrier may consist of an electrostatic potential, which renders charge measurements a natural application of this quantum version of the proposed NTOS.

Discussion.—Harnessing the susceptibility of NH topological phases to changes in the boundary conditions, we have identified a robust and widely applicable mechanism for high-precision sensors. In particular, the proposed NTOS devices, designed in an open ring geometry (cf. Fig. 1), are capable of measuring any observable affecting the coupling between their ends with a sensitivity that increases exponentially with the number of degrees of freedom of the sensor. We have demonstrated this central result by a simple perturbative analysis that is accurately confirmed by the numerically exact solution of a microscopic model, where our findings are largely insensitive to random perturbations in the form of disorder. Furthermore, several platforms for realizing NTOS systems with state of the art experimental methods have been outlined.

On a related note, in a number of recent studies on NH systems, the possibility and implementation of sensors operating at exceptional points, i.e., parameter points where a NH matrix becomes nondiagonalizable, has been extensively discussed [19–21]. There, the energy splitting around an EP of order m at energy E_e scales as $\Delta E \sim \epsilon^{1/m}$, with ϵ denoting the deviation from the EP, thus also enhancing the sensitivity with respect to ϵ exponentially strongly in m . However, at least two crucial differences of the EP scenario compared to our present proposal are worth mentioning: first, to prepare an EP of order m , an extensive number of $(2m - 2)$ real parameters [60,61] need to be fine-tuned. By contrast, the NTOS systems studied in our present work are largely insensitive to changes in system parameters, except the coupling Γ between the boundaries, which is designed to be sensitive to the measurand. Second, the spectral sensitivity of EP sensors has been criticized for not being a suitable quantity to assess the precision of an EP sensor in a noise-limited measurement [62,63]. In particular, the simultaneous ϵ dependence of the coalescing eigenstates and ΔE may conspire to yield a linear field response in ϵ , despite the sublinear ΔE [62]. By contrast, the sensitivity of the proposed NTOS is not stymied by such an interplay between eigenstates and eigenvalues. This is quite intuitive since our scheme relies on the frequency shift of an

energetically isolated mode rather than the coalescence of two modes at an EP. This intuition is supported by a microscopic analysis of the NH SSH model [43] along the lines of [62], clearly showing that the field response of the NTOS system exhibits a similar exponential enhancement with system size as the frequency shift itself. In this sense, the spectral sensitivity of the NTOS is a suitable quantity to assess the signal to noise ratio that can be experimentally achieved in principle. This point in principle being made, we note that due to the applicability of the NTOS mechanism to a wide range of physical systems, a detailed analysis of the relevant sources of noise for each individual implementation is beyond the scope of this first study, and thus remains a subject of more specialized future work.

The spectral sensitivity at the heart of the NTOS is also intriguing from a viewpoint of boundary modes in topological materials. Normally, in Hermitian systems, hybridization of the edge modes localized at the ends of a topological insulator leads to a minigap that *decays* exponentially with system size. In the considered NH SSH model instead, coupling the ends of the system leads to a shift in the energy of the boundary mode that *increases* exponentially in system size in the parameter regime where the complex bulk energy spectrum of the NTOS winds around the origin of the complex plane. In this sense, non-Hermitian topological sensors are based on a reversed minigap mechanism.

We would like to thank Flore Kunst, Alexander Palatnik, Hannes Pichler, and Carsten Timm for discussions. J. C. B. acknowledges financial support from the German Research Foundation (DFG) through the Collaborative Research Centre SFB 1143 and the Cluster of Excellence ct.qmat. E. J. B. is supported by the Swedish Research Council (VR) and the Wallenberg Academy Fellows program of the Knut and Alice Wallenberg Foundation.

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