Carlip Replies: In [1], I suggested that a large cosmological constant might be “hidden” in Planck-scale fluctuations of geometry and topology. The thrust of the Letter was to show the existence of a large class of initial data for which the expansion and shear, averaged over small regions, vanished even in the presence of an arbitrary cosmological constant. Whether this behavior was preserved by evolution was, I argued, fundamentally a question for quantum gravity. As a first small step, though, I showed that, for short times, a classical time slicing could be chosen for which the average expansion $\langle K \rangle$ remained zero. (Angle brackets $\langle \cdot \rangle$ will denote spatial averages over regions larger than Planck size but small compared to observed sizes.) The “short times” condition was necessary—the geometries considered in [1] contain high densities of marginally outer trapped surfaces, and the classical evolution quickly becomes singular [2].

The present Comment [3] focuses on the question of classical evolution but now concentrating on long-time behavior. This is an interesting problem, of course, particularly if one has a way to resolve the singularities. For $\Lambda < 0$, Wang and Unruh make a proposal for the treatment of certain “cosmological” singularities in [4], based on a construction rather different from [1].

Wang and Unruh first observe, correctly, that, in the slicings of [1] for which $\langle K \rangle$ vanishes, the volume V_U of a fixed region remains time dependent. It is not obvious which criterion is more relevant: red shift observations measure something closer to expansion rate than volume, and it is unclear how one would demarcate a “fixed region of space” to compare volumes at different times. But it is straightforward to repeat the construction of [1] to find a slicing in which V_U is constant. Indeed, starting with Eqs. (1) and (2) of [3] and taking further derivatives, one finds that

$$\frac{d^{n+1}V_U}{dt^{n+1}} = \int_U \left(K \frac{d^n N}{dt^n} + \dots \right) \sqrt{g} d^3x. \quad (1)$$

Since the derivatives of the lapse N can be chosen independently on the initial hypersurface, it is easy to make the right-hand side of (1) vanish. Of course, as in [1], this is only a short-time condition, since singularities will inevitably develop.

Wang and Unruh next argue that the time slicing of [1] “slow[s] down the progress of time” in some areas and speeds it up in others, where by “time” they mean the proper time of a congruence of geodesic observers who

start at rest on the initial hypersurface. (It is not obvious why these observers are important, since the choice of initial hypersurface in [1] is arbitrary.) But this is true of almost every slicing used in general relativity: constant mean curvature slicing, for instance, or maximal slicing, or the static slicing for any inhomogeneous static spacetime. The authors argue for a special role for a slicing corresponding Gaussian normal coordinates (lapse $N = 1$, shift $N^i = 0$). But while this choice can be useful, it can also be misleading. For the exterior Schwarzschild solution, for instance, such “Novikov coordinates” [5] lead to time-dependent metric components, disguising the staticity. This behavior occurs more generally for static inhomogeneous spacetimes, where it reflects the fact that proper time depends on the gravitational potential. Even in cosmology, where a Gaussian normal slicing is common, it is typically only used for average properties, in an approximation in which local variations of the gravitational field are unimportant.

In the final section, Wang and Unruh show that, under certain special circumstances, volumes in a spacetime with positive Λ “disastrously expand.” Their conclusion is again correct, but the proof requires two conditions on the time slicing: that it is Gaussian normal, and that the average spatial scalar curvature $\langle {}^{(3)}R \rangle$ vanishes. These conditions greatly overdetermine the slicing; no such choice exists for many spacetimes, and there is no reason to expect it to exist for the geometries discussed in [1]. For those geometries, it may be that there is *some* slicing in which $\langle {}^{(3)}R \rangle = 0$ —this was left as an open question—but there is no reason to think such a slicing should be Gaussian normal.

The asymptotic behavior of vacuum spacetimes with a cosmological constant is an important open question, but it is far from being settled. There are a few rigorous partial results in the literature [6–9], but these all require a Cauchy surface that either has constant mean curvature or is at least “everywhere expanding.” The spacetimes of [1] do not satisfy the first condition [10], and it seems quite unlikely that they satisfy the second. There is, I believe, good reason to expect the answer to be complicated. We know that a lattice of black holes in de Sitter space acts on a large scale very much like homogeneous pressureless matter [11], and that a stochastic background of gravitational radiation acts very much like a homogeneous radiation fluid [12]. But a universe with a cosmological constant, matter, and radiation can exhibit many different behaviors: it can expand, collapse, “loiter,” or, for finely tuned initial data, remain static. When one adds the complication of quantum fluctuations, which may (or may not) reproduce the rich structure of the initial data of [1], there is no reason to expect a simple result.

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