Comment on “Hiding the Cosmological Constant”

In a recent Letter [1], Carlip argues that the fluctuations in the metric at Planck scales (Wheeler’s spacetime foam) make it possible to hide the effect of a large cosmological constant. We certainly agree with him that the fluctuations in the metric must be taken into account, and have previously suggested how this might come about [2–4]. However, unfortunately his proposal suffers from some problems.

His proposal is that the cosmological constant can be hidden because one can find a spacetime foliation by slices of vanishing average expansion. Technically, he starts from an initial spatial slice Σ with $\langle K \rangle = 0$ and chooses $N, \dot{N}, \ddot{N}$, etc., on Σ to make all the time derivatives $[(d^n \langle K \rangle)/dt^n]_{|\Sigma} = 0$. In this way, he finds a family of spatial slices $\{t = \text{constants}\}$ for which $\langle K \rangle \equiv 0$ (at least for short times).

However, $\langle K \rangle \equiv 0$ does not mean that the macroscopic volume $V_U = \int_U \sqrt{g} d^3x$ he looks at stays constant. Direct calculation gives

$$\left. \frac{dV_U}{dt} \right|_{\Sigma} = \int_U NK \sqrt{g} d^3x \neq (\langle K \rangle V_U)|_{\Sigma} = 0, \quad (1)$$

$$\begin{aligned} \left. \frac{d^2 V_U}{dt^2} \right|_{\Sigma} &= \int_U (\dot{N}K + N^2(-R + 3\Lambda) + ND^i D_i N) \sqrt{g} d^3x \\ &\neq \left(\frac{d\langle K \rangle}{dt} V_U \right) \Big|_{\Sigma} = 0, \end{aligned} \quad (2)$$

which mean that V_U is certainly not constant with t .

Moreover, a choice of N corresponds to a choice of coordinates, and no physics can depend purely on the choice of coordinates. If distances between geodesics, or more importantly, the wavelengths of fields, grow with time (usually proper time, not coordinate time) they will do so in all coordinate systems, and cannot be hidden by a coordinate choice. One should not try to hide the cosmological constant by choosing N in the first place.

Let us first take a look at how he chooses N . We can see from Eq. (7) in [1] that for the points in $A = \{x \in \Sigma : R(x) < 3\Lambda\}$ we have $\mathcal{L}_n(K\sqrt{g}) > 0$, which means that they are accelerating toward expansion; while for the points in $B = \{x \in \Sigma : R(x) > 3\Lambda\}$ we have $\mathcal{L}_n(K\sqrt{g}) < 0$, which means they are accelerating toward contraction [5]. Since the average spatial curvature $\langle R \rangle = 0$, the volume of A is larger (smaller) than the volume of B if $\Lambda > 0$ ($\Lambda < 0$). In order to make $d\langle K \rangle/dt = 0$, one has to choose N to be

small (large) at points in A and large (small) at points in B . Then for the same amount of coordinate time change δt , the proper time changes $\delta\tau = N\delta t$ are small (large) at points in A and large (small) at points in B . The physical progress of time is measured by proper time instead of coordinate time, so what Carlip is really doing is trying to keep $\langle K \rangle = 0$ by slowing down (speeding up) the progress of time in places that are accelerating toward expansion and speeding up (slowing down) the progress of time in places that are accelerating toward contraction. To say that this hides the effect of Λ is like saying that if you delay the test in places where there are infections, then the spread of the coronavirus is slowed down. Of course it does not work. What is really happening is that the overall space expands (contracts) at an accelerated rate.

This can also be seen from a different perspective. Note that V_U represents the volume occupied by the particles enclosed in the region U whose world lines are orthogonal to the slices $\{t = \text{constants}\}$. If we choose $N = 1$, the world lines orthogonal to these slices are geodesics with proper time $\tau = t$ and V_U represents the volume occupied by these geodesic particles. Using the second line of Eq. (7) in [1] we obtain

$$\left. \frac{d\langle K \rangle}{d\tau} \right|_{\Sigma} = 3\Lambda, \quad (3)$$

where we have used the requirement that $\langle R \rangle = 0$. Then (1) and (2) become

$$\left. \frac{dV_U}{d\tau} \right|_{\Sigma} = (\langle K \rangle V_U)|_{\Sigma} = 0 \quad (4)$$

$$\left. \frac{d^2 V_U}{d\tau^2} \right|_{\Sigma} = \left(\frac{d\langle K \rangle}{d\tau} V_U \right) \Big|_{\Sigma} = 3\Lambda V_U|_{\Sigma}. \quad (5)$$

Therefore, the geodesic volume V_U will disastrously explode if $\Lambda > 0$ or collapse if $\Lambda < 0$.

The behavior of geodesic particles who feels only gravity reveals the gravitational property of Λ . If N is position dependent, the particles following world lines orthogonal to the slices $\{t = \text{constants}\}$ are nongeodesics and there are nongravitational forces $\mathbf{F} = m\mathbf{D}N/N$ acting on them (see Eq. (3.17) in [6]). Carlip thinks that the nongeodesic volume who satisfies (1), (2) does not explode or collapse violently like the geodesic volume who satisfies (4), (5) because the effect of the large cosmological constant Λ is hidden. However, it is simply because those nongravitational forces are trying to balance the effect of Λ .

In summary, the effect of Λ is not hidden in the way proposed in [1]. Fortunately, this is not the end of the story. It can be hidden in Planck scale curvature fluctuations in a different way [3,4,7].

Again, we emphasize that we agree completely with Carlip that one must take the fluctuations of curvature at small scales seriously, and that simply assuming a homogeneous and isotropic spacetime is not sufficient.

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