Scalars Gliding through an Expanding Universe

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In this Letter, we investigate the effects of single derivative mixing in massive bosonic fields. In the regime of large mixing, we show that this leads to striking changes of the field dynamics, delaying the onset of classical oscillations and decreasing, or even eliminating, the friction due to Hubble expansion. We highlight this phenomenon with a few examples. In the first example, we show how an axionlike particle can have its number abundance parametrically enhanced. In the second example, we demonstrate that the QCD axion can have its number abundance enhanced allowing for misalignment driven axion dark matter all the way down to f_a of order astrophysical bounds. In the third example, we show that the delayed oscillation of the scalar field can also sustain a period of inflation. In the last example, we present a situation where an oscillating scalar field is completely frictionless and does not dilute away in time.

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Introduction.—Light scalar fields are present in many scenarios of physics beyond the standard model. In the early Universe, scalar fields are generically displaced from the minimum of their potential, and therefore lead to a new form of energy that evolves dynamically during the history of the Universe. They might play many different roles in cosmology, such as dark matter [1–3], dark energy [4], moduli fields determining the value of standard model parameters [5,6], and as dynamical solutions to the hierarchy or cosmological constant problems [7,8].

The dynamics of scalar fields in the early Universe is largely determined by the form of their potential and by the Hubble expansion. In this Letter, we study how their evolution can be dramatically changed by introducing mixing with another massive field via a single derivative interaction (see Refs. [9–12] for other work studying single derivative interactions). As we will show, such a mixing leads to two striking features: it delays the beginning of the oscillations of the scalar field, and it also changes the dilution of its energy density due to the expansion of the Universe.

We focus most of our discussion on examples where a scalar mixes with a massive spin-1 field. As a case study we present a scenario where the QCD axion and a massive dark photon mix in a background magnetic field. This can lead to a significant enhancement of the final axion density, enlarging the parameter space for which axions make up all

of the observed dark matter density. We also briefly discuss an example with only scalar fields, in which the energy density in the scalar field remains constant even after the field starts oscillating.

Simple example.—We are interested in mixing scalars with other (bosonic) degrees of freedom via mixing terms that involve a single derivative. This requires the presence of a background field that breaks Lorentz invariance. To concentrate on the basic idea we will first study an example with a scalar ϕ , a massive dark photon $A_{d,\mu}$, a massless photon A_μ , and we ignore the effects of gravity. The Lagrangian we consider is (see also Refs. [13–15], where the same Lagrangian was considered in a different regime)

$$\mathcal{L} = \mathcal{L}_{\text{kinetic}} - \frac{1}{2}m^2\phi^2 + \frac{1}{2}M^2 A_d^\mu A_{d,\mu} + \frac{\phi}{2f} F_d \tilde{F}. \quad (1)$$

We take A to have a time-independent, uniform magnetic field B in the $\hat{x}$ direction. As we take the B field to be constant in space we can focus on solutions that are also constant in space, and can ignore spatial derivatives (we discuss the changes to this scenario due to an inhomogeneous magnetic field in the Supplemental Material [16]) [17]. When the solutions are spatially homogeneous, it can be easily shown that the scalar ϕ and the massive vector A_d do not source the massless vector field, i.e., $\delta A = 0$ (where δA are the fluctuations of A on top of the B field) at all times and its dynamics can be ignored. The remaining equations of motion in this background are

$$\ddot{\phi} + \Gamma_\phi \dot{\phi} = -m^2\phi + \frac{B}{f} \dot{A}_{d,x}, \quad (2)$$

$$\ddot{A}_{d,x} + \Gamma_{A_d} \dot{A}_{d,x} = -M^2 A_{d,x} - \frac{B}{f} \dot{\phi}, \quad (3)$$

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where we have included Γ_ϕ and Γ_{A_d} to represent generic friction terms.

Focusing on the small friction regime, in which the system is underdamped, we first solve Eqs. (2) and (3) exactly (ignoring friction) and find the oscillation frequencies,

$$\omega_s^2 = \frac{\alpha - \sqrt{\alpha^2 - 4m^2M^2}}{2} \approx \frac{m^2M^2}{B^2/f^2 + M^2}, \quad (4)$$

$$\omega_f^2 = \frac{\alpha + \sqrt{\alpha^2 - 4m^2M^2}}{2} \approx \frac{B^2}{f^2} + M^2, \quad (5)$$

$$\alpha = \frac{B^2}{f^2} + m^2 + M^2, \quad (6)$$

where the approximate sign shows the small m limit. Including friction as a perturbation, we find that the effective damping terms are

$$\Gamma_s \approx \frac{1}{B^2/f^2 + M^2 + m^2} [\Gamma_\phi(M^2 - \omega_s^2) + \Gamma_{A_d}(m^2 - \omega_s^2)],$$

$$\Gamma_f \approx \Gamma_\phi + \Gamma_{A_d}.$$

This shows that the effect of mixing is to decrease (increase) both the oscillation frequency and the friction depending on which of the modes is being examined. The change is only modest if $M > B/f$ but can be substantial for $B/f \gg M$.

Solving for the slow mode we find

$$A_{d,x} = -i \frac{\omega_s B/f}{M^2 - \omega_s^2} \phi \approx -i \frac{m}{M} \frac{B/f}{\sqrt{M^2 + B^2/f^2}} \phi. \quad (7)$$

From this one sees that if $m \ll M$, the slow mode is approximately pure scalar field. Therefore, if at early times only the scalar field is displaced from its minimum, only the slow mode is significantly excited.

Furthermore, in the $B/f \gg M$ regime, there is almost no kinetic energy associated with the slow mode because the oscillation frequency is highly suppressed. In fact, instead of having an energy density that oscillates between being purely potential energy to purely kinetic energy, the large mixing causes the energy of the slow mode to oscillate between the scalar potential energy $m^2\phi^2$ and the vector potential energy $M^2A_{d,x}^2$.

Note that even though the equations in this section can be solved exactly, in the small m limit they can be greatly simplified by dropping the $\ddot{A}_{d,x}$ and $\Gamma_{A_d}\dot{A}_{d,x}$ terms, as they are subdominant to the mass term in Eq. (3). Ignoring these terms, A_d can be explicitly written in terms of ϕ and the equation for ϕ becomes much simpler, leading to the small m limit results very directly. This trick will prove very

useful in the next section when the equations cannot be solved exactly.

Gliding in an expanding universe.—Using the intuition from the previous section, we proceed to study the effects of mixing in the model of Eq. (1) in an expanding universe. The equations of motion for the homogeneous fields become

$$\ddot{\phi} + 3H\dot{\phi} + m^2\phi = \frac{B(t)}{af} \dot{A}_d, \quad (8)$$

$$\ddot{A}_d + H\dot{A}_d + M^2A_d = -\frac{aB(t)}{f} \dot{\phi}, \quad (9)$$

where we assumed $B(t)$ to be a homogeneous background magnetic field (no longer necessarily constant in time), A_d to be the component of the massive gauge field $A_{d,i}$ in the direction of the magnetic field, H the Hubble expansion, and a the scale factor.

When H is larger than all other factors $[m, M, B(t)/f]$, the fields are effectively frozen. Since we are interested in the low frequency mode, and with the expectation from last section that $\omega_s < m, M$, we will study the system dynamics when $H < m, M$, while $m \ll B(t)/f$ and M . As mentioned before, for $m \ll M$, the $\ddot{A}_d$ and $H\dot{A}_d$ terms are subdominant compared to the mass term of A_d and can be neglected. In this limit the slow mode Eq. (8) can be simplified into a second-order equation for ϕ :

$$\ddot{\phi} + \left((3 - 2\lambda)H + \lambda \frac{\dot{B}(t)}{B} \right) \dot{\phi} = -\frac{m^2M^2}{B(t)^2/f^2 + M^2} \phi,$$

$$\lambda = \frac{B(t)^2/f^2}{B(t)^2/f^2 + M^2}. \quad (10)$$

Most of the important effects can be directly read from the above equation. The (time-dependent) frequency of the mode is the small m limit of ω_s in Eq. (4). It also explicitly shows how the friction term interpolates between $3H$ in the $M \gg B/f$ limit to H in the $B/f \gg M$ limit, which leads to the associated energy density diluting as a^{-1} instead of a^{-3} in the large B regime [we show that the $\dot{B}(t)/B(t)$ term does not lead to any additional friction effect in what follows].

We can solve Eq. (10) in the limit $H < \omega_s$ using the adiabatic approximation:

$$\phi(t) = \varphi(t) e^{i \int_{t_0}^t dt' \omega_s(t')}, \quad (11)$$

where $\varphi(t)$ varies slowly compared to the effective frequency ω_s (i.e., $|\dot{\varphi}| \ll \omega_s |\varphi|$ and $|\ddot{\varphi}| \ll \omega_s |\dot{\varphi}|$). With this approximation one finds

$$\varphi(t) = \varphi(t_0) e^{-\frac{1}{2} \int_{t_0}^t dt' H [1 + 2M^2/(M^2 + B^2/f^2)]}, \quad (12)$$

FIG. 1. A comparison between the full numerical solution of Eq. (8) (black line) and the approximation from Eq. (13) using $\omega_s^2 \approx m^2 M^2 / [B(t)^2 / f^2 + M^2]$ (red dotted line). Here, $B(t) = B_i a^{-2}$. The gray curve shows the solution of $B_i = 0$, which oscillates at a larger frequency and larger damping as compared to the $B_i \neq 0$ case.

which satisfies the slow varying condition for φ since $H \ll \omega_s$ and shows that the amplitude of ϕ scales as $a^{-1/2}$ while $B/f > M$ and as $a^{-3/2}$ when $M > B/f$. It also confirms that the term containing $\dot{B}$ in Eq. (10) does not affect the amplitude of ϕ .

Let us briefly summarize the cosmological history of the scalar ϕ in the limit where $m \ll M$ and $B(t)/f \gg M$ initially. From Eq. (10) we see that the field remains frozen until $\omega(t) \sim H$. Once $H \lesssim \omega(t)$, we can solve the equation in the small H limit and find

$$\phi(t) \approx \phi_0 \sqrt{\frac{a_0}{a(t)}} \exp\left(i \int_{t_0}^t \omega_s(t') dt'\right), \quad (13)$$

where a_0 is the scale factor when oscillations start. Eventually $B(t)/f \sim M$ and the approximation of large $B(t)/f$ breaks down. At this point, the scalar behaves like a normal scalar field, feeling the full effect of Hubble friction, and oscillating at the usual frequency m . Note that this approximation of the cosmological history completely neglects the transition regions where $H \sim \omega(t)$ and $B(t)/f \sim M$; however, we have numerically checked the solution and found these approximations to be a reasonable fit, as shown in Fig. 1.

This effect can also be generalized to cases when the masses of ϕ and A_d are time dependent. The solutions to the general case are presented in the Supplemental Material [16].

Axionlike particle example.—As an example of how the above mechanism can impact the cosmology of models beyond the standard model, we first show how to increase the number abundance of an axionlike particle (ALP) that obtains its abundance from the misalignment mechanism. A large variety of light ALPs are expected from string theory considerations [18], which has motivated a rich

experimental program searching for their signatures (see, e.g., Refs. [19,20]). For a given field range of the ALP, there is a direct relation between its mass and its cosmological abundance. We will show that through single derivative mixing such a tight connection can be broken, enlarging the range of parameter space for which ALPs can naturally constitute dark matter.

We will take a Friedmann–Robertson–Walker universe that is radiation dominated and approximate the ALP potential by its mass term. In the standard scenario an ALP ϕ would begin to oscillate at a temperature T_{osc} when $3H(T_{\text{osc}}) \approx m$. Before that it behaves as a cosmological constant and afterward it behaves like cold dark matter. Once the scalar starts oscillating its comoving number density is approximately conserved and its final abundance (normalized by the photon entropy density) is

$$Y = \frac{n_\phi}{s_\gamma} \sim \frac{m\phi_0^2}{T_{\text{osc}}^3} \sim \frac{\phi_0^2}{\sqrt{m}M_p^{3/2}}, \quad (14)$$

where ϕ_0 is the initial misalignment of the scalar. This shows that for a given mass and field range, the ALP abundance is approximately fixed (assuming that $\phi_0 \sim f$, where the latter stands for the ALP field range).

We now modify the situation by utilizing a B field combined with mixing as in the previous section. We will take the A_μ to have a homogeneous B field that scales as $B(T) = \beta T^2$, and for simplicity we will take $M \gg m$ and assume that $B/f \gg M$ when oscillations begin. Because the frequency of oscillation is greatly suppressed by the mixing, the scalar will now start to oscillate at a lower temperature T'_{osc} when $H(T'_{\text{osc}}) \approx \omega \approx mMf/B(T'_{\text{osc}})$. This effect makes the energy density in the scalar remain constant for a longer time. In addition, Hubble friction is effectively reduced in this limit so that the ALP energy density only dilutes away as a^{-1} after oscillations start. Both effects enhance the final abundance of ϕ . Normal Hubble friction turns on once $B(T_B)/f \sim M$, and from this point on the scalar dilutes away like cold matter.

After accounting for these additional effects, the final abundance of the scalar is

$$Y \sim \frac{m\phi_0^2}{T_B^2 T'_{\text{osc}}} \sim \frac{m^{3/4} \phi_0^2 \beta^{5/4}}{f^{5/4} M^{5/4} M_p^{1/4}}, \quad (15)$$

giving a total enhancement factor of

$$\frac{Y(\beta)}{Y(\beta=0)} \sim \left(\frac{mM_p\beta}{Mf}\right)^{5/4} \sim \left(\frac{T_{\text{osc}}}{T'_{\text{osc}}}\right)^5. \quad (16)$$

So that as long as the initial B field is sufficiently large, the abundance of the scalar can rise drastically.

Enhancing the QCD axion number abundance.—The QCD axion is one of the best motivated dark matter candidates as it can solve the strong CP problem

[21–24] and be dark matter at the same time [1–3]. Unlike an ALP, the axion mass and field range are tied together, making its final number abundance only function of its decay constant f_a . The correct dark matter abundance is only obtained when $f_a \sim 10^{11}$ – 10^{12} GeV in the minimal setup. There have been several attempts to increase the abundance of the QCD axion for the smaller f_a values, so that it can play the roll of dark matter even in the small f_a limit [25–33]. In this section, we show that our gliding scalar mechanism provides a simple mechanism for this and can extend the parameter space ($10^9 \lesssim f_a \lesssim 10^{12}$ GeV) over which the QCD axion can be dark matter.

The QCD axion Lagrangian that we will consider is

$$\mathcal{L} = \mathcal{L}_{\text{kinetic}} + \frac{M^2}{2} A_d^\mu A_{d,\mu} + \frac{\phi}{2f_\gamma} F_d \tilde{F} + \frac{\alpha}{8\pi} \frac{\phi}{f_a} G\tilde{G}. \quad (17)$$

The only difference between the QCD axion and the ALP is that due to the gluon $G\tilde{G}$ coupling, its mass has temperature dependence. In the following section, we approximate the axion mass by a simple time-dependent form:

$$m_a(T) = m_a(T=0) \times \begin{cases} \left(\frac{T_0}{T}\right)^4 & \text{for } T > T_0 \\ 1 & \text{for } T \leq T_0, \end{cases} \quad (18)$$

where $T_0 \sim 100$ MeV [34]. As with the ALP, the axion starts to oscillate when

$$3H(T_{\text{osc}}) \approx m_a(T_{\text{osc}}); \quad (19)$$

afterward the axion number density $m_a(T)\phi^2$ dilutes away as a^{-3} . Note that while the mass is increasing, comoving axion number density conservation implies that the axion amplitude is changing faster than $a^{-3/2}$. The end result is an abundance:

$$Y \sim \frac{\phi_0^2}{T_{\text{osc}} M_p}. \quad (20)$$

In the presence of a B field and a heavy dark photon A_d , with $M > m_a$, the axion number abundance from misalignment can be enhanced. As before, we take the B field to be $B = \beta T^2$. The axion starts to oscillate when

$$H(T'_{\text{osc}}) = \omega_s(T'_{\text{osc}}) \approx \frac{m_a(T'_{\text{osc}}) M f_\gamma}{B(T'_{\text{osc}})}. \quad (21)$$

Afterward its evolution is well described by (see Supplemental Material for details [16])

$$\phi(t) \approx \phi_0 \sqrt{\frac{m_a(T'_{\text{osc}})}{m_a(T)}} \frac{T}{T'_{\text{osc}}} \exp\left(i \int \omega(t) dt\right), \quad (22)$$

until the magnetic field has decayed sufficiently such that

$$\frac{B(T_f)}{f_\gamma} = M. \quad (23)$$

Finally, after this point, the axion dilutes away as normal. The final number abundance of the axion is

$$Y \sim \frac{m_a \phi_0^2 T_0^4}{T_f^2 T_{\text{osc}}^5}, \quad (24)$$

which gets an enhancement comparing to the case with no B field up to a factor

$$\frac{Y(\beta)}{Y(\beta=0)} \sim \left(\frac{T_{\text{osc}}}{T'_{\text{osc}}}\right)^{13} \lesssim \left(\frac{M_p f_a^2}{f_\gamma^3}\right)^{13/24}. \quad (25)$$

We have used the fact that the enhancement is maximized when $M \sim m_a \sim T_0^2/f_a$. This leads to an enhancement factor that allows for QCD axion dark matter to come entirely from the misalignment mechanism all the way down to where supernova constraints become important, $f_a \sim 10^9$ GeV [35,36]. More details about this particular scenario will be given in Ref. [37].

Inflating on a nonslow roll potential.—In this section, we show that small magnetic fields can even allow for a short period of inflation on potentials that normally do not support slow roll inflation. The Lagrangian we will consider is

$$\mathcal{L} = \mathcal{L}_{\text{kinetic}} - \frac{m^2}{2} \phi^2 + \frac{M^2}{2} A_d^\mu A_{d,\mu} + \frac{\phi}{2f} F_d \tilde{F}. \quad (26)$$

We will take as initial conditions, $\phi \approx \phi_0$, $\dot{\phi} \approx 0$, and $B_0^2 \ll m^2 \phi_0^2$, as well as taking $M > m$. The energy density of the Universe is initially dominantly in ϕ , with a small component in the magnetic field. To illustrate the effect, we will take $m > H$ or, equivalently, $\phi_0 < M_p$, so that this potential does not support slow roll inflation.

The scalar ϕ oscillates with a frequency $\omega_s(t) = m M f / B(t) \ll m$, so it behaves as a cosmological constant until $H \sim \omega_s(t)$, after which inflation ends. We will be concerned with how many e -foldings can inflation last. Inflation ends when $B_f / f \sim M$, where we take B_0 to be the initial B field and B_f to be the final B field and assume the usual field dilution $B \propto a^{-2}$.

$$N_e = \frac{1}{2} \log \frac{B_0}{B_f} \sim \frac{1}{2} \log \frac{c m \phi_0}{M f} < \frac{1}{2} \log \frac{c \phi_0}{f}, \quad (27)$$

where $c < 1$ encodes the initial fraction of the energy density of the universe stored in the magnetic field, $B_0 = c m \phi_0$. To maximize the number of e -foldings, we expect $M \sim m$ and $\phi_0 \gg f$. For the effective field theory to be consistent, we require

$$m^2\phi_0^2 \sim T_{\text{RH}}^4 < f^4, \quad (28)$$

where T_{RH} is the reheat temperature of the universe assuming instantaneous reheating.

Thus we see that we have the bound on the number of e -foldings of inflation to be

$$N_e < \frac{1}{2} \log \frac{cM_p}{T_{\text{RH}}} \lesssim 23, \quad (29)$$

where we have required that the reheat temperature is above a MeV and that the fraction of initial energy density stored in the magnetic field is $\mathcal{O}(1\%)$. Thus we find the amusing result that a B field can cause 20 odd e -foldings of inflation on a potential that normally does not support slow roll inflation. At the end of inflation, the energy in the magnetic field is 10^{40} times smaller than the energy in the inflaton, demonstrating that even an extremely tiny magnetic field can have rather drastic consequences on the evolution of the inflaton. It would be interesting if there were models in which all of inflation could be due to this effect, which could potentially lead to interesting observational consequences due to anisotropic contributions due to the magnetic field [38].

Completely frictionless scalar.—In the scenarios we studied so far, the energy density oscillates between the potential energy of a scalar field and the potential energy of a massive vector field. Since the energy density of the vector field redshifts even when the field amplitude A_d does not decay, the energy of the coupled system still dilutes with the Hubble expansion. In this section, we entertain the idea of getting a completely frictionless system by focusing on a scenario with only scalars.

In analogy with the previous example, one could construct a five scalar model with a mixing term $\phi_1 \partial_\mu \phi_2 \partial_\nu \phi_3 \partial_\rho \phi_4 \partial_\sigma \phi_5 \epsilon^{\mu\nu\rho\sigma}$. If three scalars have nontrivial spatially dependent backgrounds, the end result is a scalar that does not dilute away in time. Rather than studying this complicated and admittedly implausible example, we consider instead a three scalar system with a mixing term:

$$\mathcal{L} \supset -\frac{m^2}{2} \phi^2 - \frac{M^2}{2} \Phi^2 + \frac{\phi}{f} \partial_\mu \Phi \partial^\mu \Psi. \quad (30)$$

Instead of a B field, we will take Ψ to have a large kinetic term (velocity). In order to make this two time derivative mixing act like a single time derivative mixing, we take $\dot{\Psi}$ to dominate the energy density of the universe so that we can treat it as a background that dilutes away as $\dot{\Psi} \propto a^{-3}$. The equations of motion are

$$\ddot{\phi} + 3H\dot{\phi} = -m^2\phi + \frac{\dot{\Psi}}{f}\dot{\Phi}, \quad (31)$$

$$\ddot{\Phi} + 3H\dot{\Phi} = -M^2\Phi - \frac{\dot{\Psi}}{f}\dot{\phi}. \quad (32)$$

It can be easily shown that we can neglect the effect of ϕ and Φ in the dynamics of Ψ as long as $\dot{\Psi}^2 \gg m^2\phi_0^2$. Following the analysis done in the previous section, it is easy to see that the scalar ϕ does not dilute away in time as long as $\dot{\Psi}/f \gtrsim m$, M and $\dot{\Psi}^2 \gg m^2\phi_0^2$.

Conclusion.—In this Letter, we highlighted the effects of single derivative mixing between particles, as opposed to the usual mass mixing or kinetic mixing. In the regime of large mixing, the time evolution of the system is dictated by a single time derivative mixing term, similar to what happens in slow-roll inflation where the dynamics is dictated by the single time derivative Hubble friction term. In both of these cases, the evolution of the scalar and its associated energy density is drastically modified. The single derivative interaction we considered has two effects. The first is that it leads to a significant reduction of the oscillation frequency for massive scalars, somewhat similar to how Hubble friction can completely prevent the oscillation of a scalar. However, because our single derivative term is a mixing term rather than a friction, the total energy density in the system oscillates between the potential energy of two fields with negligible contributions to their respective kinetic energies. This has the unique features of reducing the frictional effects and delaying the onset of oscillations.

These effects present a myriad of interesting opportunities, only a few of which were briefly explored in this Letter. Utilizing B fields, the number abundance of QCD axions could be enhanced allowing for the axion to be dark matter for f_a all the way down to the supernova bound, a notoriously difficult feat. Single derivative mixing with B fields may also have interesting applications during inflation, as it can support short periods of inflation on potentials that do not normally allow for inflation. It will be exciting to see what opportunities single derivative mixing brings.

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