Current Driven Tricritical Point in Large- N_c Gauge Theory

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We discover a new tricritical point realized only in nonequilibrium steady states, using the AdS/CFT correspondence. Our system is a $(3 + 1)$ -dimensional strongly coupled large- N_c gauge theory. The tricritical point is associated with a chiral symmetry breaking under the presence of an electric current and a magnetic field. The critical exponents agree with those of the Landau theory of equilibrium phase transitions. This suggests that the presence of a Landau-like phenomenological theory behind our nonequilibrium phase transitions.

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Critical phenomena are important since they show universal behaviors irrespective of the microscopic details of the systems. However, critical phenomena in nonequilibrium steady states (NESSs) have not yet been investigated comprehensively. In NESSs, we have an additional control parameter that is absent from equilibrium systems. For example, the electric current can be a new variable that parameterizes the phase diagrams of the systems of current-driven NESSs. However, the phase diagrams and the critical phenomena associated with the current have not yet been completely understood. One reason is the lack of theoretical approaches especially for the nonlinear regime.

In order to investigate the phase structure of the current-driven NESSs, we employ the anti-de Sitter/conformal field theory (AdS/CFT) correspondence. The AdS/CFT correspondence is a duality between a classical gravity theory and a strongly coupled quantum gauge field theory [1–3]. One of the important applications of this correspondence is to study nonequilibrium physics (for example, applications of AdS/CFT correspondence to nonequilibrium physics are reviewed in [4,5]). By using the duality, one can analyze the gauge field theory beyond the linear response regime in terms of the gravity theory. For example, the nonlinear conductivity was computed in a NESS in a probe brane model [6]. It has been reported that the probe brane system exhibits interesting behaviors such as the negative differential conductivity [7] and the nonequilibrium phase transition [8]. It is suggested that critical phenomena of this nonequilibrium phase transition have remarkable similarity to those of the Landau theory of equilibrium phase transitions [9]. A similar nonequilibrium

phase transition was also observed in the presence of an external magnetic field [10].

In this Letter, we investigate a spontaneous symmetry breaking of $U(1)$ chiral symmetry of a strongly coupled large- N_c gauge theory under the presence of an electric current density J and a magnetic field B perpendicular to the current. We assume that the system is homogeneous and the size of the system is infinite. The gauge theory we employ is $SU(N_c)$ $\mathcal{N} = 4$ supersymmetric Yang-Mills (SYM) theory in $(3 + 1)$ dimensions with $\mathcal{N} = 2$ hypermultiplet. The charged particles belong to the hypermultiplet whereas the particles that form the heat bath belongs to the sector of $\mathcal{N} = 4$ SYM. The charged particles in our system are massless. This corresponds that we are dealing with gapless systems in condensed matter physics when the system has chiral symmetry. We find that the system exhibits first-order or second-order phase transitions of the chiral symmetry breaking. A tricritical point (TCP) exists between the line of the first-order phase transition and the second-order critical line. We stress that the newly discovered TCP is located at finite J in the NESS regime on the phase diagram, hence we call it as nonequilibrium tricritical point. We present numerical results for critical exponents at the critical points. Our results imply that the phase transitions of the present system are described by a Landau-like phenomenological model.

We would like to stress that our aim is not to investigate the nature of the above mentioned particular microscopic theory of SYM. The reason why we employ the SYM is to make the duality between the gravity theory and the microscopic theory clear. Our aim is to study the universal properties of nonequilibrium phase transitions that fall into the same category with the system presented here.

Holographic setup.—The gravity dual of our system is the D3-D7 model [11] with an electromagnetic field [6,12]. We briefly review the setup of the gravity dual to make the Letter self-contained. The dual geometry is a direct product

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of a five-dimensional AdS-Schwarzschild black hole and an S^5 :

$$ds^2 = \frac{dz^2}{z^2} - \frac{1}{z^2} \frac{(1 - z^4/z_H^4)^2}{1 + z^4/z_H^4} dt^2 + \frac{(1 + z^4/z_H^4)}{z^2} d\vec{x}^2 + d\Omega_5^2, \quad (1)$$

where $z(0 \leq z \leq z_H)$ is the radial direction, t and $\vec{x} = (x^1, x^2, x^3)$ are the coordinates for the $3 + 1$ dimensions of the corresponding gauge theory. The black hole horizon is located at $z = z_H$ and the boundary is located at $z = 0$. The Hawking temperature is given by $T = \sqrt{2}/(\pi z_H)$, which is identified with the temperature of the heat bath in the gauge theory side. The line element of the S^5 is given by $d\Omega_5^2 = d\theta^2 + \sin^2 \theta d\psi^2 + \cos^2 \theta d\Omega_3^2$, where $d\Omega_3^2$ is the line element of the S^3 part on which the D7-brane wraps. The configuration of the D7-brane is given by $\theta(z)(0 \leq \theta \leq \pi/2)$ and ψ . We take $\psi = 0$ without loss of generality for our purpose. For simplicity, we set the AdS radius and the radius of S^5 to 1. The dynamics of the D7-brane is described by the Dirac-Born-Infeld (DBI) action $S_{\text{DBI}} = -T_{D7} \int d^8 \xi \mathcal{L}_{D7}$ with $\mathcal{L}_{D7} = \sqrt{-\det[g_{ab} + (2\pi l_s^2)F_{ab}]}$ where g_{ab} is the induced metric and F_{ab} is the field strength of the U(1) gauge field on the D7-brane. l_s is the string length and T_{D7} is the tension of the D7-brane given by $T_{D7}^{-1} = (2\pi)^7 l_s^8 g_s$, where g_s is the string coupling constant. Since θ depends only on z , the nontrivial element of the induced metric is $g_{zz} = 1/z^2 + \theta'(z)^2$, where the prime denotes the differentiation with respect to z .

Let us employ the following ansatz for the gauge fields: $A_1(t, z) = -Et + a_1(z)$, $A_2(x^1) = Bx^1$ and A_a for $a \neq 1$, $a \neq 2$ are zero. Then the nonvanishing components of the field strength F_{ab} are $F_{01} = -F_{10} = -E$, $F_{12} = -F_{21} = B$, and $F_{1z} = -F_{z1} = a'_1(z)$. From now, we simply denote x^1 by x , and we set $2\pi l_s^2 = 1$ for simplicity. In our setup, the system is neutral and we have current density only in the x direction. The expectation value of the current density, which is denoted by J , is given by $J = \partial \mathcal{L}_{D7} / \partial a'_1(z)$ [6]. In order to make J to be a control parameter, let us perform the Legendre transformation $\tilde{\mathcal{L}}_{D7} = \mathcal{L}_{D7} - J \partial \mathcal{L}_{D7} / \partial a'_1$. For on shell, we have $\tilde{\mathcal{L}}_{D7} = \sqrt{-(g_{zz}/g_{tt}g_{xx})} \sqrt{\chi \xi}$, where $\chi = J^2 + \mathcal{N}^2 g_{tt} g_{xx} \cos^6 \theta$ and $\xi = g_{tt} g_{xx} + g_{tt} B^2 + g_{xx} E^2$. Here, $\mathcal{N} = \lambda N_c / (2\pi)^4$. In order to guarantee the DBI action to be real, the functions χ and ξ in the square-root must change the sign at the same location on the z axis, say $z = z_*$, simultaneously [13]. Thus, $\xi = 0$ and $\chi = 0$ are satisfied at $z = z_*$, which we call *effective horizon*. The first condition gives us the location of the effective horizon explicitly:

$$z_*^4 = \left(F(e, b) - \sqrt{[F(e, b) - 1]^2 - 1} - 1 \right) z_H^4, \quad (2)$$

where $F(e, b) = 1 + e^2 - b^2 + \sqrt{(e^2 - b^2)^2 + 2(e^2 + b^2) + 1}$. Here e and b are defined as dimensionless quantities: $e = 2E/(\pi\sqrt{\lambda}T^2)$ and $b = 2B/(\pi\sqrt{\lambda}T^2)$, and λ is the 't Hooft coupling of the gauge theory. The second condition provides the explicit form of the current density $J^2 = -\mathcal{N}^2 g_{tt} g_{xx} \cos^6 \theta(z_*)$. The D7-brane embedding function $\theta(z)$ in the vicinity of the boundary can be expanded as $\theta(z) = \theta_0 z + \theta_2 z^3 + \dots$. Here, θ_0 and θ_2 are, respectively, related to the mass of the charge carriers m and the chiral condensate $\langle \bar{q}q \rangle$ as $m = \theta_0$ and $\langle \bar{q}q \rangle = -\mathcal{N}(2\theta_2 - m^3/3)$.

Nonequilibrium phase transition.—We set $z_H = 1$ for simplicity [in our convention, the unit of B and E is $[(\sqrt{2}T/\pi)^2]$ and that of J and $\langle \bar{q}q \rangle$ is $[\mathcal{N}(\sqrt{2}T/\pi)^3]$ in the following figures]. In order to solve the equation of motion (EOM) for $\theta(z)$, we have to specify the boundary condition at the effective horizon. The explicit form of the boundary condition is given by $\theta'(z_*) = (C - \sqrt{C^2 + D^2})/(z_* D)$, where $C = 3 + 2z_*^4 + 3z_*^8$ and $D = 3(z_*^8 - 1) \tan \theta(z_*)$ [7,14]. $\theta(z_*)$ is a parameter to be set and z_* is determined by Eq. (2) for given E and B . We solve the EOM numerically to obtain the solutions $\theta(z)$ in $\varepsilon \leq z \leq z_* - \varepsilon_{IR}$, where ε and ε_{IR} are cutoffs introduced to avoid numerical divergence at the boundary and the effective horizon, respectively.

Once $\theta(z)$ is obtained for each parameter, one can compute m and $\langle \bar{q}q \rangle$. Inversely, the configurations of the D7-brane and the parameters can be found for the fixed m . In this study, since we are interested in the system of massless charge carriers and the spontaneous chiral symmetry breaking, we take solutions for $m = 0$ by using the shooting method. We show the configurations of the D7-brane in Fig. 1. There exist two possible branches: the flat branch and the bending branch as shown in Fig. 1. The flat branch is a trivial solution: $\theta(z) = 0$. The bending branch is a nontrivial solution: $\theta(z) \neq 0$. These branches can be distinguished by the chiral condensate: $\langle \bar{q}q \rangle = 0$ in the flat branch, and $\langle \bar{q}q \rangle \neq 0$ in the bending branch. In other words, the chiral symmetry is preserved in the flat

FIG. 1. The D7-brane configurations of two possible branches for $E = 1$ and $B = 14$. The black solid line represents the black hole horizon. The coordinate is changed following by $1/z^2 = r^2 + \sqrt{r^4 - 1}$. The black hole horizon is located at $r = 1$, the effective horizon is located at $r = r_* > 1$, and the AdS boundary is located at $r = \infty$.

FIG. 2. The behaviors of the chiral condensate $\langle \bar{q}q \rangle$ as a function of the current density J for $B = 19$, $B = 20$, and $B = 21$ from left to right.

branch, whereas it is spontaneously broken in the bending branch. To be more specific, the chiral symmetry is the $U(1)_R$ symmetry of the SYM theory [15]. Compared to the previous studies on the chiral symmetry breaking in the AdS/CFT correspondence [15–19], the crucial difference in our study is that we consider chiral symmetry breaking in the NESS regime where $J \cdot E \neq 0$.

Figure 2 shows the behaviors of the chiral condensate $\langle \bar{q}q \rangle$ and the current density J for various B . Note that if we choose the current density as a control parameter, there is a multivalued region where $\langle \bar{q}q \rangle$ has two or three possible values for a given J . In this region, we expect that only the most stable solution is realized.

In the following, we assume that the most stable solution has the lowest *thermodynamic potential* which is defined by the renormalized Hamiltonian of the D7-brane per unit volume with appropriate Legendre transformation [8,9] See also [20] for a similar proposal. (The Hamiltonian at $J \rightarrow 0$ gives the free energy in equilibrium. The free energy employed in Ref. [21–23] can be understood as the Legendre transform of the Hamiltonian to make the electric field a control parameter.) The Hamiltonian density is given by

$$\tilde{\mathcal{H}}_{D7} = g_{xx} \sqrt{-g_{tt} g_{zz}} \sqrt{\frac{J^2 + \mathcal{N}^2 g_{tt} g_{xx}^2 \cos^6 \theta}{g_{tt} g_{xx}^2 + g_{tt} B^2 + g_{xx} E^2}}. \quad (3)$$

Then the thermodynamic potential (per unit volume) is defined as

$$\tilde{F}_{D7}(B, J) = \lim_{\epsilon \rightarrow 0} \left[\int_{\epsilon}^{z_*} dz \tilde{\mathcal{H}}_{D7} - L_{\text{count}}(\epsilon) \right], \quad (4)$$

where $L_{\text{count}}(\epsilon)$ is the counterterm to renormalize the divergence at the boundary $z = 0$. The explicit form is given in [6] except for the finite counterterm we propose (see also [24], for the holographic renormalization of probe D-branes). The details on the finite counterterm is shown in the Supplemental Material [25].

If we compute the thermodynamic potential in each plot of Fig. 2, we find that as the magnetic field increases, the thermodynamic potential of the bending branch becomes smaller and it eventually becomes more stable than the flat branch. As a result, there is a transition between these two branches. We regard this transition as a *nonequilibrium phase transition*. More interestingly, when B becomes larger, the chiral condensate of the bending branch changes from a multivalued function to a single-valued function, and thus the discontinuous jump of the chiral condensate becomes continuous transition. Hence, the *first-order phase transition* changes to the *second-order phase transition* by increasing B . In this Letter, we choose the chiral condensate $\langle \bar{q}q \rangle$ as an order parameter.

Phase diagram.—To summarize the foregoing discussions, we show the phase diagram of the nonequilibrium phase transitions in Fig. 3. The thick blue curve is the first-order phase transition line and the red curve is the critical line where the second-order phase transition occurs. Note that there are two specific points in the phase diagram: B_{gap} and B_{TCP} . For $B < B_{\text{gap}}$, stable solutions are only the flat branches. However, for $B > B_{\text{gap}}$, the bending branch can be stable. The other specific point B_{TCP} is the *tricritical point* (TCP) that is defined as a point where three coexisting phases become identical simultaneously. In order to see this in our system, it is necessary to consider another parameter dimension: the mass of the charged particles m which breaks the chiral symmetry explicitly. In analogy with the QCD phase diagram (for example, see [26]), we find that the (B, J) phase diagram with a finite mass contains a first-order phase transition line which terminates at a critical point. This can be understood by checking the behaviors of the chiral condensate as a function of the current density for small mass as shown in Fig. 4. Critical points for different masses form a critical line in the (B, J, m) phase diagram and the critical line terminates at B_{TCP} that is located on the

FIG. 3. Phase diagram of the present system on the slice of $m = 0$. There are two specific points denoted by B_{gap} and B_{TCP} . The first-order phase transitions occur along the thick blue curve and the second-order phase transitions occur along the red curve. These curves completely separate the phase with chiral symmetry restoration (χSR) and that with chiral symmetry breaking (χSB).

FIG. 4. The behavior of the chiral condensate $\langle \bar{q}q \rangle$ at $m = 0.01$. When the magnetic field increases, the transition of the chiral condensate appears to be changed from first order (solid line) to second order (dashed line) and crossover (dotted line).

slice of $m = 0$. We can extend the phase diagram into the negative mass region by performing a U(1) chiral transformation. One finds that the extended phase diagram is symmetric under $m \rightarrow -m$. Hence, the three critical lines end at a single point B_{TCP} . This is why B_{TCP} is regarded as the tricritical point.

Critical exponent.—In the previous section, we found second-order phase transitions in our nonequilibrium system. In order to understand the critical phenomena, we analyze the critical exponents. Critical exponents are defined from the behavior of the order parameter near the critical point. In our analysis, we define the critical exponent β by [27] $\langle \bar{q}q \rangle \propto (J_c - J)^\beta$ along the line of $m = 0$ [28], where J_c is the current density at the critical point. Figure 5 shows the behaviors of $\langle \bar{q}q \rangle$ for $B > B_{\text{TCP}}$, $B = B_{\text{TCP}}$, and $B < B_c$. As shown in the inset of Fig. 5, we find $\beta = 1/2$ for $B > B_{\text{TCP}}$ and $\beta = 1/4$ for $B = B_{\text{TCP}}$ within the numerical error by linear fitting.

FIG. 5. The relationship between the chiral condensate $\langle \bar{q}q \rangle$ as a function of the current density $J_c - J$. The numerical plots and the lines obtained by nonlinear fitting to the form of (6) for $B = 19.8$ (blue), $B = B_{\text{TCP}}$ (red), and $B = 20.4$ (green) are shown. The inset shows numerical plots and linear fittings for $B = B_{\text{TCP}}$ and $B = 30$.

Analytical approach.—We show that the values of the critical exponents can be derived analytically as follows. Let us focus on the case for $B \geq B_{\text{TCP}}$. In the language of the D7-brane configuration, $\theta(z)$ continuously changes from $\theta(z) = 0$ to $\theta(z) \neq 0$ when the chiral symmetry is broken in the second-order phase transition. This means that $\theta(z_*) \ll 1$ in the vicinity of the critical point. Since $J = \sqrt{-g_{tt}g_{xx}}|_{z_*} \cos^3 \theta(z_*)$ is an even function of $\theta(z_*)$, J can be expanded as

$$J = J_c + a_2 \theta(z_*)^2 + a_4 \theta(z_*)^4 + \dots, \quad (5)$$

where $a_i (i = 2, 4, \dots)$ are coefficients that depend on z_* . Here we have used the fact that $J = J_c$ at the critical point where $\theta(z_*) = 0$. Note that J_c depend only on B .

Next, we argue that the chiral condensate is an odd function of $\theta(z_*)$. When we flip the sign of $\theta(z_*)$, the function $\theta(z)$ also flips the overall sign. For massless case, $\theta(z)$ in the vicinity of the boundary is expanded as $\theta(z) = -\frac{1}{2} \langle \bar{q}q \rangle z^3 + O(z^5)$. Then we find that the chiral condensate flips the sign, too. Therefore, the chiral condensate is an odd function of $\theta(z_*)$ which may be expanded as $\langle \bar{q}q \rangle = b_1 \theta(z_*) + b_3 \theta(z_*)^3 + \dots$, where $b_i (i = 1, 3, \dots)$ are coefficients that depend on z_* .

Suppose that $b_1 \neq 0$. Then the chiral condensate is *linear* in $\theta(z_*)$ in the vicinity of the critical point, and (5) gives

$$J_c - J = c_2 \langle \bar{q}q \rangle^2 + c_4 \langle \bar{q}q \rangle^4 + \dots, \quad (6)$$

where c_2 and c_4 are some coefficients. The numerical results given in Fig. 5 suggest that $c_2 = \kappa(B - B_{\text{TCP}})$ with $\kappa > 0$ and $c_4 > 0$, in the vicinity of $B = B_{\text{TCP}}$. We have checked numerically that c_2 is indeed proportional to $B - B_{\text{TCP}}$ and c_4 is a small positive value in the vicinity of the critical point. Then we obtain $\langle \bar{q}q \rangle \sim (J_c - J)^{1/2}$ for $B > B_{\text{TCP}}$ and $\langle \bar{q}q \rangle \sim (J_c - J)^{1/4}$ for $B = B_{\text{TCP}}$, respectively. The above mentioned numerical results for the critical exponents justify the above consideration. Therefore, we conclude that $\beta = 1/2$ for $B > B_{\text{TCP}}$ and $\beta = 1/4$ for $B = B_{\text{TCP}}$.

Conductivity.—In [8,9], it has been found that the conductivity $\sigma = J/E$ also plays a role of order parameter at least for the purpose of detection of the critical phenomena. From experimental point of view, measurement of conductivity is much more easier than that of chiral condensate. Therefore, it is worth while studying the phase transitions from the viewpoint of conductivity. Figure 6 shows the J - E characteristics for various B . In terms of the conductivity, our nonequilibrium phase transitions are regarded as the transition between the negative differential conductivity (NDC) phase and the positive differential conductivity (PDC) phase, which are corresponding to two branches, the bending branch and the flat branch, respectively. As shown in Fig. 6, the transition between the PDC phase and the NDC phase is also the second-order

FIG. 6. The J - E characteristics for (a) $B = 19$, (b) $B = 20$, and (c) $B = 21$.

phase transition along the critical line and the first-order phase transition below B_{TCP} . At the tricritical point, $\partial E / \partial J \rightarrow \infty$, which is a similar behavior to that of the chiral condensate in Fig. 2. This shows that the chiral symmetry breaking is closely related to the transition between the PDC phase and the NDC phase. In other words, our results suggest that our nonequilibrium phase transitions and the tricritical point will be experimentally observed as transitions between the NDC phase and the PDC phase in a strongly coupled system with a constant current.

Conclusion.—We studied the phase structure associated with the chiral symmetry of the current-driven NESS by using the AdS/CFT correspondence. We have discovered the current-driven tricritical point. If we define the critical exponent β as $\langle \bar{q}q \rangle \propto (J_c - J)^\beta$, we obtain $\beta = 1/2$ on the critical line and $\beta = 1/4$ at the tricritical point. These values of the critical exponents agree with those of the Landau theory for equilibrium systems. We infer that this agreement is due to the large- N_c limit in the AdS/CFT correspondence. Our results suggest the existence of an effective theory for our nonequilibrium phase transitions whose “Landau

free energy” $f(\langle \bar{q}q \rangle)$ is given by $f(\langle \bar{q}q \rangle) = a(J - J_c)\langle \bar{q}q \rangle^2 + b(B - B_{\text{TCP}})\langle \bar{q}q \rangle^4 + c\langle \bar{q}q \rangle^6 - d m \langle \bar{q}q \rangle$, where a, b, c are positive constants and d is a nonzero constant at a fixed T . It is interesting to investigate the validity of the foregoing “Landau free energy” and if other critical exponents also agree with those of the Landau theory or not. These are left for future exploration. We propose that this type of nonequilibrium phase transitions and the current-driven tricritical point may be detected in a system consisted of gapless chiral fermions, such as a Weyl semimetal, with a constant current. We hope the results presented here provide clues to reveal universal properties of some class of NESSs.

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