Erratum: Landau Phonon-Roton Theory Revisited for Superfluid ^4He and Fermi Gases [Phys. Rev. Lett. 119, 260402 (2017)]

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The results of the Letter on the effective ϕ - γ scattering amplitude and associated damping rate, based on the roton Hamiltonian [Eqs. (1) and (4) of the Letter], do not account for the phonon-phonon interaction, which is described, to lowest order in q , by the Hamiltonian [1]:

$$\hat{H}_{\phi-\phi}^{(3)} = \sum_{\mathbf{q}_1, \mathbf{q}_2, \mathbf{q}_3} \delta_{\mathbf{q}_1 + \mathbf{q}_2, \mathbf{q}_3} \frac{\mathcal{A}_{\phi-\phi}^{2\leftrightarrow 1}(\mathbf{q}_1, \mathbf{q}_2; \mathbf{q}_3)}{\gamma^{1/2}} (\hat{b}_{\mathbf{q}_1}^\dagger \hat{b}_{\mathbf{q}_2}^\dagger \hat{b}_{\mathbf{q}_3} + \text{H.c.}) \quad (1)$$

with the $\phi + \phi \leftrightarrow \phi$ amplitude [2]:

$$\mathcal{A}_{\phi-\phi}^{2\leftrightarrow 1}(\mathbf{q}_1, \mathbf{q}_2; \mathbf{q}_3) = \frac{mc^2}{\rho^{1/2}} \sqrt{\frac{\hbar^3 q_1 q_2 q_3}{32m^3 c^3}} \left(2 \frac{d \ln c}{d \ln \rho} - 1 + \frac{\mathbf{q}_1 \cdot \mathbf{q}_2}{q_1 q_2} + \frac{\mathbf{q}_1 \cdot \mathbf{q}_3}{q_1 q_3} + \frac{\mathbf{q}_2 \cdot \mathbf{q}_3}{q_2 q_3} \right) \quad (2)$$

We have used here the Grüneisen parameter $(d \ln c / d \ln \rho) = (\rho \mu'' / \mu' + 1) / 2$. To the leading order, this adds two diagrams mediated by $\hat{H}_{\phi-\phi}^{(3)}$ to Eq. (7) of the Letter:

$$\begin{aligned} \mathcal{A}_2^{\text{eff}}(\mathbf{k}, \mathbf{q}; \mathbf{k}', \mathbf{q}') &= \text{diagram 1} + \text{diagram 2} + \text{diagram 3} + \text{diagram 4} + \text{diagram 5} \\ &= \mathcal{A}_2(\mathbf{k}, \mathbf{q}; \mathbf{k}', \mathbf{q}') + \frac{\mathcal{A}_1(\mathbf{k}, \mathbf{q}; \mathbf{k} + \mathbf{q}) \mathcal{A}_1(\mathbf{k}', \mathbf{q}'; \mathbf{k}' + \mathbf{q}')}{\hbar \omega_{\mathbf{q}} + \epsilon_{\mathbf{k}} - \epsilon_{\mathbf{k} + \mathbf{q}}} + \frac{\mathcal{A}_1(\mathbf{k} - \mathbf{q}', \mathbf{q}'; \mathbf{k}) \mathcal{A}_1(\mathbf{k} - \mathbf{q}', \mathbf{q}; \mathbf{k}')}{\epsilon_{\mathbf{k}} - \hbar \omega_{\mathbf{q}'} - \epsilon_{\mathbf{k} - \mathbf{q}'}} \\ &\quad + \frac{2 \mathcal{A}_{\phi-\phi}^{2\leftrightarrow 1}(\mathbf{q}', \mathbf{q} - \mathbf{q}'; \mathbf{q}) \mathcal{A}_1(\mathbf{k}, \mathbf{q} - \mathbf{q}'; \mathbf{k}')}{\hbar(\omega_{\mathbf{q}} - \omega_{\mathbf{q} - \mathbf{q}'} - \omega_{\mathbf{q}'})} + \frac{2 \mathcal{A}_{\phi-\phi}^{2\leftrightarrow 1}(\mathbf{q}, \mathbf{q}' - \mathbf{q}; \mathbf{q}') \mathcal{A}_1(\mathbf{k}', \mathbf{q}' - \mathbf{q}; \mathbf{k})}{\epsilon_{\mathbf{k}} - \epsilon_{\mathbf{k}'} - \hbar \omega_{\mathbf{q} - \mathbf{q}'}} \end{aligned} \quad (3)$$

The energy denominators of the two new diagrams are equal to $-\hbar \omega_{\mathbf{q} - \mathbf{q}'}$ to leading order (see note [12] of the Letter). One should thus add $-(\hbar q / m c \rho)(\rho \Delta' / 2)(2 d \ln c / d \ln \rho - 1 + w)$ to Eq. (10) of the Letter:

$$\begin{aligned} \mathcal{A}_2^{\text{eff}}(\mathbf{k}, \mathbf{q}; \mathbf{k}', \mathbf{q}') &\underset{T \rightarrow 0}{\sim} \frac{\hbar q}{m c \rho} \left(\frac{1}{2} \rho^2 \Delta'' - \frac{\rho \Delta'}{2} \left(2 \frac{d \ln c}{d \ln \rho} - 1 \right) + \frac{(\hbar \rho k_0')^2}{2 m_*} + \frac{\hbar^2 k_0^2}{2 m_*} \left\{ \left(\frac{\rho \Delta'}{\hbar c k_0} \right)^2 u u' \right. \right. \\ &\quad \left. \left. + \frac{\rho \Delta'}{\hbar c k_0} \left[(u + u') \left(u u' - \frac{\rho k_0'}{k_0} \right) + \frac{m_* c}{\hbar k_0} w \right] + \frac{m_* c}{\hbar k_0} (u + u') w + u^2 u'^2 - \frac{\rho k_0'}{k_0} (u^2 + u'^2) \right\} \right). \end{aligned} \quad (4)$$

This also changes the angular integral given in note [14] of the Letter: $I / (4\pi)^2 = (\hbar^2 k_0^2 / 2 m_* m c^2)^2 \{ \frac{1}{25} - (4\alpha/15) + \frac{28}{45} \alpha^2 + (2\beta^2/9) + A[\frac{2}{9} - (4\alpha/3)] + A^2 + 4\beta B[\frac{1}{15} - (\alpha/9)] + B^2[\frac{2}{15} - (4\alpha/9) + (2\alpha^2/3) + (\beta^2/3)] + (2\beta/9)B^3 + (B^4/9) \}$, and the definition of $A = [m_* / (\hbar k_0)^2][\rho^2 \Delta'' + \rho \Delta'(1 - 2 d \ln c / d \ln \rho)] + \alpha^2$. Accordingly, the black dashed curves in Figs. 1, 2(a), and 2(b) are multiplied, respectively, by 0.46, 0.78, and 0.85. Note that the Grüneisen parameter also appears in Ref. [3]; our coefficients of the $u u'$ and w monomials of $\mathcal{A}_2^{\text{eff}}$ are however still in disagreement with that reference.

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- [3] A. Nicolis and R. Penco, Mutual interactions of phonons, rotons, and gravity, *Phys. Rev. B* **97**, 134516 (2018).