Generalized Eigenstate Thermalization Hypothesis in 2D Conformal Field Theories

Anatoly Dymarsky^{1,2} and Kirill Pavlenko^{2,3}¹*Department of Physics and Astronomy, University of Kentucky, Lexington, Kentucky 40506, USA*²*Skolkovo Institute of Science and Technology, Skolkovo Innovation Center, Moscow 143026, Russia*³*Moscow Institute of Physics and Technology, Dolgoprudny 141700, Russia*

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Infinite-dimensional conformal symmetry in two dimensions leads to integrability of 2D conformal field theories (CFTs) by giving rise to an infinite tower of local conserved quantum Korteweg–de Vries (qKdV) charges in involution. We discuss how the presence of conserved charges constrains equilibration in 2D CFTs. We propose that in the thermodynamic limit large central charge 2D CFTs satisfy generalized eigenstate thermalization, with the values of qKdV charges forming a complete set of thermodynamically relevant quantities, which unambiguously determine expectation values of all local observables from the vacuum family. Equivalence of ensembles further provides that local properties of an eigenstate can be described by the generalized Gibbs ensemble that includes only qKdV charges. In the case of a general initial state, upon equilibration, the emerging generalized Gibbs ensemble will necessarily include negative chemical potentials and holographically will be described by a quasiclassical black hole with quantum soft hair.

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The topic of thermalization—and, more generally, equilibration of isolated many-body quantum systems—has been an active area of research during the past decade. In the case of nonintegrable systems, i.e., those without an extensive number of local conserved quantities, the emergence of thermal equilibrium has been traced to eigenstate thermalization hypothesis (ETH), which postulates thermal properties of individual energy eigenstates [1–3]. In the simplest form it requires the expectation value of some appropriate (often taken to be local) observable $\mathcal{O}$ in a many-body eigenstate $|E_i\rangle$ to be a smooth function of energy,

$$\langle E_i | \mathcal{O} | E_i \rangle = f_{\mathcal{O}}(E_i). \quad (1)$$

Qualitatively, Eq. (1) postulates that energy is the only thermodynamically relevant quantity, which completely specifies the local properties of an eigenstate. Condition (1) may apply to all or most eigenstates, in which case it is referred to as strong or weak ETH. The eigenstate thermalization ensures equivalence between the expectation value in the eigenensemble, $f_{\mathcal{O}}(E_i)$, and the thermal expectation value of $\mathcal{O}$ in the Gibbs ensemble, $f_{\mathcal{O}}(E_i) = \text{Tr}(e^{-\beta H} \mathcal{O})/Z$, where the effective temperature β is fixed through the energy balance relation, $E_i = \text{Tr}(e^{-\beta H} \mathcal{O})/Z$ [4].

When the system is integrable, with an extensive number of conserved charges Q_i , ETH does not apply. Accordingly emerging equilibrium can be different from the Gibbs state. In this case the equilibrium can be described by the generalized Gibbs ensemble (GGE), a generalization of the grand canonical ensemble that includes an infinite tower of conserved charges [5]. Validity of the GGE has been related to generalized eigenstate thermalization [6–8], which generalizes (1) to include an infinite number of conserved quantities,

$$\langle E_i | \mathcal{O} | E_i \rangle = f_{\mathcal{O}}(Q_k(E_i)). \quad (2)$$

Here $|E_i\rangle$ is a mutual eigenstate of the Hamiltonian, charges Q_k , $Q_k(E_i)$ are the eigenvalues of Q_k associated with $|E_i\rangle$, and the function $f_{\mathcal{O}}$ is assumed to be a smooth function of all of its arguments. As in Eq. (1), at the qualitative level, Eq. (2) postulates that charges Q_k form a complete set of thermodynamically relevant quantities which fully specify local properties of an eigenstate. Provided that Eq. (2) applies to most states, it ensures equivalence between the generalized microcanonical ensemble and the GGE, establishing validity for the latter to describe the emerging equilibrium, e.g., following a quantum quench [6].

In this Letter we discuss thermalization of two-dimensional conformal field theories (CFTs), a rich topic with multiple connections ranging from the cold atom experiments [9] to the physics of quantum gravity [10]. It has been shown that following a quantum quench 2D conformal theories equilibrate and reach a steady state, which in many cases can be described in terms of the Gibbs

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ensemble [11,12]. At the same time emergence of thermal equilibrium is not universal. Previous studies have shown that the values of different local quantities measured in the individual energy eigenstates differ from their thermal counterparts, unless the central charge is taken to be infinite [13–21]. Furthermore, conformal symmetry in two dimensions gives rise to an infinite tower of local mutually commuting conserved quantum Korteweg–de Vries (qKdV) charges Q_{2k-1} , with the CFT Hamiltonian for the left movers being $Q_1 \equiv H$, a part of the integrable structure of the 2D CFTs [22–24]. The question we are concerned with is how the presence of these charges affects equilibration. By analogy with the integrable lattice models, it is natural to expect that locally equilibrium states can be described in terms of the GGE, which includes all local qKdV charges. Indeed, emergence of exactly such a qKdV GGE was analytically shown for a special family of so-called Cardy-Calabrese initial states [25,26].

In the context of integrable systems, the question of which quantities should be included in the GGE is far from trivial. Early studies in the context of XXZ and Lieb-Liniger models have shown that a full set of extensive local charges does not specify local properties of eigenstates, signaling failure of the generalized ETH [27,28]. These works raised an important question of the validity of the GGE to describe an emerging equilibrium following a quantum quench [29–31]. A resolution comes from the fact that, besides local conserved quantities, these models give rise to quasilocal conserved charges [32]. Taking them into account restores validity of the GGE [33]. Following studies in the context of integrable field theoretic models, both free and interacting ones, have decisively established that adding quasilocal charges is necessary to accurately describe the after-quench equilibrium state [34–39]. These findings raise an important question emphasized in Ref. [26] if the set of local qKdV charges is generally sufficient to describe equilibrium in large c 2D CFTs, or it should be extended by nonlocal or perhaps some new local charges [40]. In this Letter we show that, at large c , 2D CFTs satisfy generalized eigenstate thermalization (2) with the local qKdV charges forming a complete set and ambiguously specifying local properties of the eigenstates.

Our consideration below applies to all large central charge theories with $c \gg 1$. In the case in which theory admits a gravity dual, many aspects can be interpreted holographically. Below we provide holographic interpretation when available without requiring the holographic description to exist.

Two-dimensional conformal field theories admit a split into noninteracting sectors of left and right movers. For simplicity we discuss only one sector explicitly, while all results automatically extend to the full theory. We consider 2D CFT on a circle of the circumference ℓ in a mutual eigenstate of all charges Q_{2k-1} ,

$$|E\rangle = |\{m_i\}, \Delta\rangle, \quad E = Q_1 = \left(\Delta + \sum m_i\right)/\ell, \quad (3)$$

labeled by the primary state Δ and the set of integers $\{m_i\}$ [41]. The set $\{m_i\}$ is convenient to parametrize using free boson representation where an integer n_k for $k = 1, 2, \dots$ specifies the number of times integer k appears in the set $\{m_i\}$. In the thermodynamic limit $\ell \rightarrow \infty$ “energy” Q_1 and all other qKdV charges are assumed to scale with the system size to yield finite charge densities $q_{2r-1} = Q_{2r-1}/\ell$. In terms of Δ , n_k this implies the scaling

$$\Delta \sim \ell^2, \quad \sum_k n_k k^{2r-1} \sim \ell^{2r}. \quad (4)$$

In what follows we restrict the discussion to the eigenstates (3) with the density charges $q_{2r-1} = \langle E | Q_{2r-1} | E \rangle / \ell$ which additionally satisfy

$$\frac{q_{2r-1}}{q_1^r} = 1 + O(1/c). \quad (5)$$

Here and below the CFT central charge c is assumed to be large. Holographically, this regime corresponds to a quasiclassical black hole in three-dimensional anti-de Sitter space (AdS₃), where 1 on the rhs of Eq. (5) corresponds to classical gravity, while the $O(1/c)$ term is due to quantum corrections [41–44]. In terms of Δ , n_k an exponential majority of states in the generalized microcanonical ensemble specified by q_{2k-1} subject to Eq. (5) will satisfy

$$\frac{\sum_k n_k k^{2r-1}}{\Delta^r} = O(1/c^r). \quad (6)$$

The typicality of Eq. (6) is justified in the Supplemental Material [45]. In fact Eq. (6) may apply to all states in the generalized microcanonical ensemble (5), yielding a strong version of the generalized ETH in 2D CFTs. To verify that one would need to know the full spectrum of qKdV charges, going beyond currently known leading $1/c$ expansion.

In the regime of “quasiclassical gravity” (6), $c \gg 1$, expectation values of qKdV charges can be calculated explicitly [46],

$$\ell^2 q_1 = \Delta + \sum_k n_k k, \quad (7)$$

$$\begin{aligned} \ell^4 q_3 &= \Delta^2 + \sum_k n_k \left(6\Delta k + \frac{ck^3}{6}\right) + O(c^0), \\ \dots \end{aligned} \quad (8)$$

$$\ell^{2r} q_{2r-1} = \Delta^r + \sum_k n_k p_{2r-1}(c, \Delta, k) + O(c^{r-2}), \quad (9)$$

where $p_{2r-1}(c, \Delta, k)$ are some known polynomials of degree $2r-1$ which include only odd powers of k .

Because of translational invariance the expectation value of a full derivative $\mathcal{O} = \partial\mathcal{O}'$ in the energy eigenstate will vanish. Hence it suffices to consider expectation values $\langle E|\mathcal{O}|E\rangle$ only when $\mathcal{O}$ is a quasiprimary operator. Below we consider the case in which $\mathcal{O}$ belongs to the vacuum family, i.e., it is a Virasoro descendant of the identity. To streamline the notations, we introduce $\langle\mathcal{O}\rangle \equiv \langle E|\mathcal{O}|E\rangle$. It is convenient to parametrize $\mathcal{O}$ by its dimension (level). On levels 2 and 4 there are unique quasiprimary operators in the vacuum family,

$$\mathcal{O}_2 = T, \quad \mathcal{O}_4 = T^2 - \frac{3}{10}\partial^2 T. \quad (10)$$

Thus expectation values of $\mathcal{O}_{2,4}$ are identically equal to charge densities q_1, q_3 [47]. On level 6 there are two quasiprimary operators (we always choose quasiprimaries in the basis which diagonalizes Zamolodchikov metric),

$$\mathcal{O}_6^{(1)} = T^3 - \frac{9}{10}(T\partial^2 T) + \frac{4}{35}\partial^4 T + \frac{93}{70c+29}\mathcal{O}_6^{(2)}, \quad (11)$$

$$\mathcal{O}_6^{(2)} = (\partial T \partial T) - \frac{4}{5}(T\partial^2 T) + \frac{23}{210}\partial^4 T. \quad (12)$$

The expectation value of the combination $\mathcal{O}_6^{(1)} + \frac{5}{9}(c/12)\mathcal{O}_6^{(2)}$ is identically equal to q_5 . As in Eqs. (7)–(9), at leading order the expectation value of $\mathcal{O}_6^{(2)}$ has the form of a polynomial in Δ and odd powers of k ,

$$\langle\mathcal{O}_6^{(2)}\rangle = \frac{9}{5}\sum_k n_k \left(\frac{c}{6}k^5 + 4\Delta k^3 \right) + O(c^0). \quad (13)$$

It is possible to use Eqs. (7)–(9) to express any term of the form $\sum_k n_k k^{2r-1}$ via q_{2j-1} , $j \leq r$, but *a priori* the result would also depend on Δ . Thus, at leading order in $1/c$, expectation values of $\mathcal{O}_6^{(i)}$ are some functions of Δ and q_{2r-1} . Remarkably, because of the nontrivial cancellations, the final result is Δ independent and can be expressed solely in terms of q_{2r-1} . To simplify the answer, we introduce dimensionless ratio $q_{2k-1} = q_{2k-1}/q_1^k$ such that $\delta q_{2k-1} \equiv q_{2k-1} - 1$ is of order $1/c$. Then $\mathcal{O}_6^{(i)}$ measured in units of energy density q_1 is given by

$$q_1^{-3}\langle\mathcal{O}_6^{(1)}\rangle = 1 + 3\delta q_3 + O(1/c^2), \quad (14)$$

$$q_1^{-3}\langle\mathcal{O}_6^{(2)}\rangle = \frac{9}{5}(12/c)(\delta q_5 - 3\delta q_3) + O(1/c^3). \quad (15)$$

As we see, different quasiprimary operators have different scalings with c . Our calculation applies to leading $1/c$ behavior of each quasiprimary operator, except for a special

one which includes maximal power of T without derivatives. The expectation value of that quasiprimary operator starts with $O(c^0)$, and our result applies to the first two terms in $1/c$ expansion.

The possibility of expressing eigenstate expectation value $\langle\mathcal{O}\rangle$ as a polynomial in q_{2j-1} extends to all higher levels. For an operator of dimension $2r$ the answer depends only on q_{2j-1} for $j \leq r$. We write down explicit expressions for all operators up to level 10 in terms of q_{2j-1} in the Supplemental Material [45]. Our results establish generalized eigenstate thermalization for vacuum block observables in large c CFTs.

That expectation value $\langle\mathcal{O}\rangle$ of an operator of dimension $2r$ includes only qKdV charges q_{2j-1} up to the same dimension $j \leq r$ can be interpreted as a manifestation of locality. It is analogous to the observation in the context of integrable lattice models that, to describe equilibrium state locally at length scales not exceeding some distance a , it is necessary to include only local and quasilocal charges in the GGE with the support within a [48,49].

Our results capture only leading order in $1/c$. It remains an open question to find exact expressions for the eigenstate expectation values in terms of the qKdV charges, as was recently done for a particular family of integrable spin models [50,51].

Generalized eigenstate thermalization implies the validity of the qKdV generalized Gibbs ensemble

$$\rho = \exp \left\{ - \sum_k \mu_{2k-1} Q_{2k-1} \right\} / Z, \quad \mu_1 \equiv \beta, \quad (16)$$

to describe the local properties of individual energy eigenstates provided that chemical potentials μ_{2k-1} are tuned to match values of the eigenstate charges

$$\ell q_{2k-1} = \langle E_i | Q_{2k-1} | E_i \rangle = \text{Tr}(\rho Q_{2k-1}). \quad (17)$$

Provided that q_{2k-1} , chosen to represent the charge densities of some nonequilibrium initial state $|\Psi\rangle$, a standard argument would consequently equate the GGE expectation values of local operators with those in the diagonal ensemble of $|\Psi\rangle$, written in eigenbasis (3). In most cases the latter would be equal to the expectation values in state $|\Psi\rangle$ upon equilibration. It should be noted though that left and right Hamiltonians $Q_1, \bar{Q}_1$ are highly degenerate, and therefore the validity of the diagonal ensemble to describe local physics upon equilibration may be violated.

It remains an open question to establish the existence of μ_{2k-1} , which would solve Eq. (17) for any given set of q_{2k-1} . Using an explicit form of the generalized partition function in the large c limit [46], we can find, up to the $O(1/c^2)$ corrections,

$$\begin{aligned}
 \delta q_{2k-1} &= \frac{q_{2k-1}}{q_1^k} - 1 \\
 &= \frac{24k}{c} \int_0^\infty \frac{d\kappa \kappa [(2k-1)_2 F_1(1, 1-k, 3/2, -\kappa^2) - 1]}{e^{2\pi\kappa\gamma} - 1}, \\
 \gamma &= \sum_{j=1}^\infty \tilde{\mu}_{2j-1} j(2j-1) \sigma^{j-1/2} {}_2F_1(1, 1-j, 3/2, -\kappa^2),
 \end{aligned} \tag{18}$$

where $\tilde{\mu}_{2k-1} = (\sqrt{6}/\pi) c^{k-1} \mu_{2k-1}$, and where $\sigma(\tilde{\mu}_{2k-1})$ is positive and satisfies

$$\sum_{k=1}^\infty k \tilde{\mu}_{2k-1} \sigma^{k-1/2} = 1. \tag{19}$$

From here it follows that, when all chemical potentials are positive, q_{2k-1} satisfies an infinite series of inequalities (see the Supplemental Material [45])

$$\begin{aligned}
 \frac{q_3}{q_1^2} - 1 &\leq \frac{22}{5c} + O(1/c^2), & \frac{q_5}{q_1^3} - 1 &\leq \frac{302}{21c} + O(1/c^2), \\
 \dots
 \end{aligned} \tag{20}$$

Thus the GGE emerging after equilibration of some general initial state will have to include negative chemical potentials, unless all inequalities (20) are satisfied.

To match the GGE to a primary state all qKdV densities should be related to each other via $q_{2k-1} = q_1^k$ [47]. This is possible only if the integral in Eq. (18) vanishes, which requires γ to be infinite. This is consistent with the observation of Ref. [41] that an ensemble with any finite number of nonzero μ_{2k-1} cannot describe primary states. This is because in full generality $q_{2k-1} \geq q_1^k$, and hence primary states are at the boundary of the phase space of q_{2k-1} 's. It is nevertheless possible to describe them in the limit, via a GGE with at least some coefficients approaching infinity. The simplest scenario is to consider $\mu_3 > 0$ and arbitrary $\beta \equiv \mu_1$, while all other chemical potentials are identically zero. Then in the limit $\tau = \beta(6/\pi^2 c \mu_3)^{1/3} \rightarrow -\infty$, for all k , $q_{2k-1}/q_1^k - 1$ will vanish as $\sim |\tau|^{-3}$, as is shown for $k = 2, 3$ in Fig. 1.

With just two chemical potentials β, μ_3 being nonzero, the values of $q_{2k-1}/q_1^k - 1$ is confined to be between zero and their thermal (Gibbs ensemble) values. This constraint is removed already after turning on one additional chemical potential. For example by taking $\beta, \mu_5 > 0$ and $\mu_3 < 0$, one can fine-tune function γ to become arbitrarily small for some positive value of κ , leading to the divergence of the integral in Eq. (18) and violating quasiclassical regime (5).

From the holographic point of view equilibration in field theory is associated with the formation of a black hole in AdS_3 , a background dual to the GGE (16). Conserved qKdV charges correspond to the black hole soft hair, which are visible only at the quantum level. At the level of

FIG. 1. Plot of $q_{2k-1}/q_1^k - 1$ in the units of $1/c$ as a function of $\tau = \beta(\pi^2/(6c\mu_3))^{1/3}$ for $k = 2, 3$. It approaches zero as $|\tau|^{-3}$ for all k when $\tau \rightarrow -\infty$. The opposite limit $\tau \rightarrow \infty$ corresponds to the Gibbs ensemble, $q_1 \sim \beta^{-1}$, $\mu_3 \rightarrow 0$, and $c(q_{2k-1}/q_1^k - 1)$, for $k = 2, 3$ approaching $22/5$ and $302/21$, correspondingly.

classical gravity $c \rightarrow \infty$, all qKdV charges are related, $q_{2k-1} = q_1^k$. Accordingly there is a unique classical Bañados-Teitelboim-Zanelli black hole family of solutions parametrized by $q_1, \tilde{q}_1$ [42,43]. It is an important question to understand the regime $q_{2k-1} \neq q_1^k$ holographically, by taking quantum gravity corrections into consideration. This, in particular, should provide holographic interpretation to negative temperature and other chemical potentials, which will necessarily appear starting from a general initial state.

In this Letter we have considered only local probes $\mathcal{O}$ from the vacuum block. In case $\mathcal{O}$ is a nontrivial Virasoro primary, or its descendant, it will have zero expectation value in the GGE (16) for any values of μ_{2k-1} . This is because in the thermodynamic limit $\ell \rightarrow \infty$ geometry degenerates into a cylinder which is conformally flat. Thus, to satisfy any version of eigenstate thermalization, the eigenstate expectation value $\langle E|\mathcal{O}|E \rangle$ must simply vanish. In terms of the CFT data, this means most or all heavy-heavy-light operator product expansion coefficients must approach zero when the dimension of heavy operators grows to infinity. If that is the case, generalized eigenstate thermalization will be trivially satisfied. It remains an outstanding problem to establish whether large central charge chaotic CFTs, in particular those with gravity duals, exhibit this behavior.

In this Letter we have established that large central charge 2D CFTs in the thermodynamic limit satisfy generalized eigenstate thermalization with the tower of local qKdV charges forming a complete set of thermodynamically relevant quantities. Our analysis has established the universal validity of a generalized Gibbs ensemble that includes all qKdV charges to describe individual energy eigenstates, and hence in most cases, asymptotic equilibrium states in such theories. It would be important to extend

the analysis to the next order in $1/c$, which would reveal whether the eigenstate thermalization is strong—i.e., it applies to all finite energy density eigenstates—or weak—i.e., it applies to most such states.

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