Einstein Rings in Holography

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Clarifying conditions for the existence of a gravitational picture for a given quantum field theory (QFT) is one of the fundamental problems in the AdS/CFT correspondence. We propose a direct way to demonstrate the existence of the dual black holes: imaging an Einstein ring. We consider a response function of the thermal QFT on a two-dimensional sphere under a time-periodic localized source. The dual gravity picture, if it exists, is a black hole in an asymptotic global AdS₄ and a bulk probe field with a localized source on the AdS boundary. The response function corresponds to the asymptotic data of the bulk field propagating in the black hole spacetime. We find a formula that converts the response function to the image of the dual black hole: The view of the sky of the AdS bulk from a point on the boundary. Using the formula, we demonstrate that, for a thermal state dual to the Schwarzschild-AdS₄ spacetime, the Einstein ring is constructed from the response function. The evaluated Einstein radius is found to be determined by the total energy of the dual QFT. Our theoretical proposal opens a door to gravitational phenomena on strongly correlated materials.

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Introduction.—One of the definitive goals of the research of the holographic principle, or the AdS/CFT correspondence [1–3], is to find what class of quantum field theories (QFTs) or quantum materials possesses their gravity dual. Is there any direct test for the existence of a gravity dual for a given material? Among various gravitational physics, one of the most peculiar astrophysical objects is the black hole. Gravitational lensing [4] is one of the fundamental phenomena of strong gravity. When there is a light source behind a gravitational body, observers will see the Einstein ring. If the gravitational body is a black hole, some light rays are so strongly bent that they can go around the black hole many times, and especially infinite times on the photon sphere. As a result, multiple Einstein rings corresponding to winding numbers of the light ray orbits emerge and infinitely concentrate on the photon sphere. Recently, the Event Horizon Telescope (EHT) [5], which is an observational project for imaging black holes, has captured the first image of the supermassive black hole in M87. (See the left panel of Fig. 1. We include the image of M87 for motivational purposes; the physical origin of it, which may not be the same as that of ours, is yet to be confirmed, though in Ref. [5] it was claimed that it is consistent

theoretically with a black hole shadow surrounded by the brightest photon sphere.) In this Letter, we propose a direct method to check the existence of a gravity dual from measurements in a given thermal QFT—imaging the dual black hole as an Einstein ring.

We demonstrate explicitly construction of holographic “images” of the dual black hole from the response function of the boundary QFT with external sources, as follows. As the simplest example, we consider a $(2 + 1)$ -dimensional boundary conformal field theory on a 2-sphere S^2 at a finite temperature, and study a one-point function of a scalar operator $\mathcal{O}$, under a time-dependent localized Gaussian source $J_{\mathcal{O}}$ with the frequency ω . We measure the local response function $e^{-i\omega t} \langle \mathcal{O}(\vec{x}) \rangle$. This QFT setup may allow a gravity dual (see Fig. 2), which is a black hole in the global AdS₄ and a probe massless bulk scalar field in the spacetime. The time-periodic source $J_{\mathcal{O}}$ amounts to a

FIG. 1. (Left) Image of the black hole in M87 (This figure is taken from Ref. [5].) (Right) Image of the AdS black hole constructed from the response function.

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FIG. 2. Our setup for imaging a dual black hole, the Schwarzschild-AdS₄ spacetime. An oscillating Gaussian source J_O is applied at a point on the AdS boundary. Its response $\langle \mathcal{O}(x) \rangle$ is observed at another point on the boundary.

dynamical AdS boundary condition for the scalar field, which injects a bulk scalar wave into the bulk from the AdS boundary. The scalar wave propagates inside the black hole spacetime and reaches other points on the S^2 of the AdS boundary. The scalar amplitude there corresponds to $\langle \mathcal{O}(\vec{x}) \rangle$ in the QFT.

Using wave optics, we find a formula which converts the response function $\langle \mathcal{O}(\vec{x}) \rangle$ to the image of the dual black hole $|\Psi_S(\vec{x}_S)|^2$ on a virtual screen:

$$\Psi_S(\vec{x}_S) = \int_{|\vec{x}| < d} d^2x \langle \mathcal{O}(\vec{x}) \rangle e^{-\frac{i\omega}{f} \vec{x} \cdot \vec{x}_S}, \quad (1)$$

where $\vec{x} = (x, y)$ and $\vec{x}_S = (x_S, y_S)$ are Cartesian-like coordinates on boundary S^2 and the virtual screen, respectively, and we have set the origin of the coordinates to a given observation point. This operation is mathematically a Fourier transformation of the response function on a small patch with the radius d around the observation point, that is, applying an appropriate window function. Note that f describes magnification of the image on the screen. In wave optics, we have virtually used a lens with the focal length f and the radius d to form the image. The right panel of Fig. 1 shows a typical image of the AdS black hole computed from the response function through our method. The AdS/CFT calculation clearly gives a ring similar to the observed image of the black hole by EHT. Equation (1) can be regarded as the dual quantity of the Einstein ring.

Several criteria for QFTs to have a gravity dual have been proposed in some previous works. A popular criterion is a large gap in the anomalous dimension spectra of CFT operators [6]. The strong redshift of the black hole has been also used as the condition for the existence of gravity dual [7–9]. However, in the literature, it has never been discussed whether and how we can holographically observe effects of the gravitational lensing from observables in QFT. We propose that the observation of the Einstein ring can be an alternative criterion for the existence of the gravity dual. The method is simple and can be applied to any QFT on a sphere, thus probing efficiently a black hole of its possible gravity dual. Once we have a strongly

correlated material on S^2 , we can apply a localized external source such as electromagnetic waves and measure its response in principle. Then, from Eq. (1), we would be able to construct the image of the dual black hole if it exists. The holographic image of black holes in a material, if observed by a tabletop experiment, may serve as a novel entrance to the world of quantum gravity.

Scalar field in Schwarzschild-AdS₄ spacetime.—We consider Schwarzschild-AdS₄ (Sch-AdS₄) with the spherical horizon

$$ds^2 = -F(r)dt^2 + \frac{dr^2}{F(r)} + r^2(d\theta^2 + \sin^2\theta d\varphi^2), \quad (2)$$

where $F(r) = r^2/R^2 + 1 - 2GM/r$, R is the AdS curvature radius, and G is the Newton constant. This is the black hole solution in the global AdS₄: The dual CFT is on $\mathbf{R}_t \times S^2$. The event horizon is located at $r = r_h$ defined by $F(r_h) = 0$. The mass M is given by $M = r_h(r_h^2 + R^2)/(GR^2)$, which relates to total energy of the system in the dual CFT. In what follows, we take the unit of $R = 1$.

We focus on dynamics of a massless scalar field $\Phi(t, r, \theta, \varphi)$ in the Sch-AdS₄ satisfying the Klein-Gordon equation $\square\Phi = 0$. Near the AdS boundary ($r = \infty$), the asymptotic solution of the scalar field becomes

$$\Phi = J_O - \frac{1}{2r^2}(\partial_t^2 - D^2)J_O + \frac{\langle \mathcal{O} \rangle}{r^3} + \dots, \quad (3)$$

where D^2 is the scalar Laplacian on unit S^2 . In the asymptotic expansion, we have two independent functions, J_O and $\langle \mathcal{O} \rangle$, which depend on boundary coordinates (t, θ, φ) . In the AdS/CFT, the leading term J_O corresponds to the scalar source in the boundary CFT. On the other hand, the subleading term $\langle \mathcal{O} \rangle$ corresponds to its response function [10].

We consider that an axisymmetric and monochromatically oscillating Gaussian source is localized at the south pole ($\theta = \pi$) of the boundary S^2 : $J_O(t, \theta, \varphi) = e^{-i\omega t} g(\theta)$, where $g(\theta) = \exp[-(\pi - \theta)^2/(2\sigma^2)]/(2\pi\sigma^2)$ and $\sigma \ll \pi$. In the bulk point of view, this source J_O is the boundary condition of the scalar field at the AdS boundary. We also impose the ingoing boundary condition on the horizon of the Sch-AdS₄. Imposing these boundary conditions, we solve the Klein-Gordon equation numerically and determine the solution of the scalar field in the bulk. We read off the response from the coefficient of $1/r^3$. Because of the symmetries of the source J_O , the response function does not depend on φ and its time dependence is just given by $e^{-i\omega t}$. Hence, we can write the response function as $\langle \mathcal{O}(t, \theta, \varphi) \rangle = e^{-i\omega t} \langle \mathcal{O}(\theta) \rangle$.

Imaging AdS black holes.—Figure 3 shows our procedure to obtain the image of the AdS black hole from the

FIG. 3. How to construct the image of the AdS black hole.

response function. The sphere of the AdS boundary is depicted at the left side of the figure. We show the absolute square of the response function $\langle \mathcal{O}(\theta) \rangle$ on the sphere as the color map. The brightest point is the north pole, i.e., the antipodal point of the Gaussian source. We can observe the interference pattern resulting from the diffraction of the scalar wave by the black hole. However, we cannot yet recognize this pattern directly as an “image of the black hole.” To obtain the image of the black hole, we need to look at the response function through a virtual “optical system” with a convex lens.

Let us define an “observation point” at $(\theta, \varphi) = (\theta_{\text{obs}}, 0)$ on the AdS boundary, where an observer looks up into the

AdS bulk. To make an optical system virtually, we consider the flat three-dimensional space (x, y, z) as shown in the right side of Fig. 3. We set the convex lens on the (x, y) plane. The focal length and radius of the lens will be denoted by f and d . We also prepare the spherical screen at $(x, y, z) = (x_S, y_S, z_S)$ with $x_S^2 + y_S^2 + z_S^2 = f^2$. We copy the response function around the observation point onto the lens as the incident wave function and observe the image formed on the screen.

We map the response function defined on S^2 onto the lens as follows. We introduce new polar coordinates (θ', φ') as $\sin \theta' \cos \varphi' + i \cos \theta' = e^{i\theta_{\text{obs}}} (\sin \theta \cos \varphi + i \cos \theta)$, such that the direction of the north pole is rotated to align with the observation point: $\theta' = 0 \Leftrightarrow (\theta, \varphi) = (\theta_{\text{obs}}, 0)$. Then, we define Cartesian coordinates as $(x, y) = (\theta' \cos \varphi', \theta' \sin \varphi')$ on the AdS boundary S^2 . In this coordinate system, we regard the response function around the observation point as the wave function on the lens: $\Psi(\vec{x}) = \langle \mathcal{O}(\theta) \rangle$. For $\omega d \gg 1$, it is known that the image on the screen $\Psi_S(\vec{x}_S)$ is obtained by the Fourier transformation of the incident wave $\Psi(\vec{x})$ within a finite domain on the lens [11]. (See also Refs. [12–14].) This leads us to Eq. (1) for imaging the AdS black hole.

We now summarize our results. Figure 4 shows images of the black hole observed at various observation points:

 FIG. 4. Image of the Sch-AdS black hole for $\omega = 80$, $\sigma = 0.01$, and $d = 0.5$. The horizon radius and the observation point are varied as $r_h = 0.6, 0.3, 0.1$, and $\theta_{\text{obs}} = 0^\circ, 30^\circ, 60^\circ, 90^\circ$.

$\theta_{\text{obs}} = 0^\circ, 30^\circ, 60^\circ, 90^\circ$. The horizon radius is varied as $r_h = 0.6, 0.3, 0.1$. We fix other parameters as $\omega = 80$, $\sigma = 0.01$, and $d = 0.5$. For $\theta_{\text{obs}} = 0^\circ$, a clear ring is observed. As we will see later, this ring corresponds to the light rays from the vicinity of the photon sphere of the Sch-AdS₄. Not only the brightest ring, we can also see some concentric striped patterns. They are caused by a diffraction effect with imaging, which is not directly related to properties of the black hole, because we find these patterns change depending on the lens radius d and frequency ω . As we change the angle of the observer, the ring is deformed. We observed similar images as those for the asymptotically flat black hole [12–14]. In particular, at $\theta_{\text{obs}} \sim 90^\circ$, we can observe a double image of the point source. They correspond to light rays which are moving clockwise and anticlockwise, winding around the black hole on the plane of $\varphi = 0, \pi$. The size of the ring becomes bigger as the horizon radius grows.

Einstein radius.—To elucidate the property of the brightest ring, we focus on the observation point at $\theta_{\text{obs}} = 0$ and search $x_S = x_{\text{ring}}$ at which $|\Psi_S(\vec{x}_S)|^2$ has maximum value. [Since the image has the rotational symmetry in (x_S, y_S) -plane for $\theta_{\text{obs}} = 0$, we only consider the x_S axis.] We will refer to the angle of the Einstein ring $\theta_{\text{ring}} = \sin^{-1}(x_{\text{ring}}/f)$ as the Einstein radius. Figure 5 shows the Einstein radius θ_{ring} by the purple points varying the horizon radius r_h . Although the Einstein radius fluctuates as the function of r_h , it has an increasing trend as r_h is enlarged.

From geometrical optics, there is an infinite number of null geodesics connecting antipodal points on the AdS boundary, which are labeled by the winding number N_w : the number of times a geodesic goes around the black hole. Each geodesic will form an Einstein ring whose radius is determined by the angle of incidence to the AdS boundary. Which geodesic in the geometrical optics corresponds to the ring found in the image in the wave optics? In the Sch-AdS₄ spacetime (2), there is the photon sphere ($N_w = \infty$) at $r = 3r_h(r_h^2 + 1)/2$. Solving the null geodesic equation from the photon sphere, we obtain the angle of incidence of the geodesic to the AdS boundary ϑ_i as

$$\sin \vartheta_i = \frac{r_h(r_h^2 + 1)}{(r_h^2 + 1/3)\sqrt{r_h^2 + 4/3}}. \quad (4)$$

This geodesic prediction for the Einstein radius of the photon sphere is shown by the green curve in Fig. 5. The curve seems consistent with the Einstein radius from the wave optics. This indicates that the major contribution to the brightest ring in the image is originated by the “light rays” from the vicinity of the photon sphere, which are infinitely accumulated. Although there are expected to be multiple Einstein rings corresponding to light rays with winding numbers $N_w = 0, 1, 2, \dots$, the contribution for the image from small N_w may be so small that we cannot resolve them within our numerical accuracy.

The deviation of the Einstein radius $\theta_{\text{ring}}(r_h)$ from the geodesic prediction can be considered as some wave effects. In the AdS cases, whether the geometrical optics can adapt to imaging of black holes is not so trivial even for a large value of ω . The Eikonal approximation, which supports the geometrical optics, will inevitably break down since a component of the metric rapidly varies near the AdS boundary as $\sim r^2$. Our results based on the wave optics imply that the geometrical optics is qualitatively valid but gives a non-negligible deviation even for a large ω .

When $\omega d \gg 1$, Eq. (1) can be rewritten as $\Psi_S \propto G_\ell(\omega)|_{\ell \simeq \omega \sin \theta_S}$ by using the retarded Green’s function $G_\ell(\omega)$ for frequency ω and azimuthal quantum number ℓ of the spherical harmonics on S^2 . (See the Supplemental Material [15] for more details.) It turns out that the poles of $G_\ell(\omega)$, which correspond to quasinormal modes (QNMs) for the bulk black hole, play an important role for the image formation. In particular, the Einstein radius θ_{ring} for a given ω implies that there should exist the quasinormal modes such that the real part of the frequency is close to ω with respect to $\ell = \omega \sin \theta_{\text{ring}}$. Since the QNMs are closely related to the unstable photon orbit in the Eikonal limit $\omega \simeq \ell \gg 1$, this is consistent with our results.

Furthermore, in the Supplemental Material [15], we estimate the “Einstein radius” for a weakly coupled ϕ^4 theory with mass m , coupling λ , and temperature T . At the one loop level, we have

$$\theta_{\text{ring}} \simeq \frac{\pi}{2} - \frac{m_T}{\omega}, \quad (5)$$

where $m_T^2 = m^2 + \mathcal{O}(\lambda T)$ is the effective mass with the thermal correction. The temperature dependence of θ_{ring} is suppressed for $\omega \rightarrow \infty$, as opposed to the strongly coupled cases associated with gravitate dual. This result suggests that, from the temperature dependence of the Einstein radius, we can diagnose if a given QFT has its gravity dual or not.

Discussion.—We have shown that, if the dual black hole exists, we can construct the image of the AdS black hole from the observable in the thermal QFT. In other words,

FIG. 5. The Einstein radius θ_{ring} as a function of the horizon radius r_h . The green curve expresses the Einstein radius of the photon sphere, which is obtained by the geodesic approximation.

being able to observe the image the AdS black hole in the thermal QFT can be regarded as a necessary condition for the existence of the dual black hole. Finding conditions for the existence of the dual gravity picture for a given quantum field theory is one of the most important problems in the AdS/CFT. We would be able to use the imaging of the AdS black hole as a test for it. One of the possible applications is superconductors. It is known that properties of high T_c superconductors can be captured by the black hole physics in AdS [22–24]. One of the other interesting applications is the Bose-Hubbard model, which at the quantum critical regime has been conjectured to have a gravity dual [25–27]. Experimental realization of such strongly correlated materials on a two-dimensional sphere is a possible entrance to the world of quantum gravity: Measurement of responses under localized sources on such materials will lead to a novel observation of Einstein rings by tabletop experiments.

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