Comment on “Role of Radial Charges on the Angular Momentum of Electromagnetic Fields: Spin-3/2 Light”

In a recent Letter [1] the authors claimed that they discovered electromagnetic fields with “spin angular momentum typical of spin-3/2 objects.” In this Comment I prove that this claim is based on a total misunderstanding. The factor 3/2 has been obtained as a numerical coefficient in the ratio $\gamma = J_z/W$ of the z component of the total angular momentum to the beam energy per unit length. This ratio has the dimension of time. Therefore, the numerical coefficient 3/2 in Eq. (14) is completely arbitrary since it depends on the unit of time. The authors have chosen arbitrarily $1/\omega$ as this unit, but the wave period $T = 2\pi/\omega$ would be equally good. In the absence of a distinguished time scale in Maxwell theory, all efforts to assign a physical meaning to the numerical coefficient in the dimensional quantity are pointless. Moreover, I prove that the value 3/2 is as good as any other real number.

In order to show that any value of γ can easily be obtained, I use the complex combination of field vectors named the Riemann-Silberstein vector [2],

$$\mathbf{F} \equiv \frac{\mathbf{D}}{\sqrt{2\epsilon_0}} + i \frac{\mathbf{B}}{\sqrt{2\mu_0}}. \quad (1)$$

Every solution of Maxwell equations in free space,

$$i\partial_t \mathbf{F} = c \nabla \times \mathbf{F}, \quad (2)$$

can be expressed in terms of two complex Fourier amplitudes $a_+(\mathbf{k})$ and $a_-(\mathbf{k})$,

$$\begin{aligned} \mathbf{F}(\mathbf{r}, t) &= \int \frac{d^3k}{(2\pi)^{3/2}} \mathbf{e}(\mathbf{k}) [a_+(\mathbf{k}) e^{i(\mathbf{k}\cdot\mathbf{r} - \omega t)} + a_-(\mathbf{k}) e^{-i(\mathbf{k}\cdot\mathbf{r} - \omega t)}], \end{aligned} \quad (3)$$

where the normalized polarization vector $\mathbf{e}(\mathbf{k})$ is

$$\mathbf{e}(\mathbf{k}) = \frac{1}{\sqrt{2k} \sqrt{k_x^2 + k_y^2}} \begin{bmatrix} -k_x k_z + i k k_y \\ -k_y k_z - i k k_x \\ k_x^2 + k_y^2 \end{bmatrix}. \quad (4)$$

The polarization vector is defined up to a phase factor, but the adopted choice makes the analysis of angular momentum simple [3]:

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The assumptions of monochromaticity and positive helicity, made in Ref. [1], reduce the formula Eq. (3) to a two-dimensional Fourier integral,

$$\mathbf{F}(\mathbf{r}, t) = e^{-ickt} \int \frac{dk_x dk_y}{2\pi} \mathbf{e}(\mathbf{k}) a(k_x, k_y) e^{i\mathbf{k}\cdot\mathbf{r}}, \quad (5)$$

where $k_z = \sqrt{k^2 - k_x^2 - k_y^2}$.

The formulas for J_z and W in terms of the amplitude $a(k_x, k_y)$ can be obtained from the two-dimensional integrals over the $z = 0$ plane, as defined in Ref. [1]:

$$W = \int dx dy \mathbf{F}^* \cdot \mathbf{F} = \int dk_x dk_y |a(k_x, k_y)|^2, \quad (6)$$

$$\begin{aligned} J_z &= \frac{1}{ic} \int dx dy [F_z^* (xF_x + yF_y) - F_z (xF_x^* + yF_y^*)] \\ &= \int \frac{dk_x dk_y}{ck} a(k_x, k_y)^* (-ik_x \partial_{k_y} + ik_y \partial_{k_x}) a(k_x, k_y). \end{aligned} \quad (7)$$

The general amplitude $a(k_x, k_y)$ expressed in polar variables $k_x = k_\perp \cos \varphi$, $k_y = k_\perp \sin \varphi$ can be expanded into the Fourier series,

$$a(k_\perp, \varphi) = \sum_{M=-\infty}^{\infty} a_M(k_\perp) e^{iM\varphi}. \quad (8)$$

Each term in this sum represents the contribution to the solution of Maxwell equations characterized by the integer quantum number M of the z component of the *total* angular momentum. The integrals over $k_\perp$,

$$c_M = \int_0^\infty dk_\perp k_\perp |a_M(k_\perp)|^2, \quad (9)$$

are the contributions of the component with the quantum number M to the total angular momentum J_z and to the total energy W . The final formula for γ is

$$\gamma = \frac{1}{\omega} \frac{\sum_{M=-\infty}^{\infty} M c_M}{\sum_{M=-\infty}^{\infty} c_M}. \quad (10)$$

In the quantum description one may say that c_M is the probability to find the photon with the projection of the total angular momentum on the z axis equal $\hbar M$, while the sum in the numerator is the average value of J_z . The values of J_z given by the integral [Eq. (7)] are *not quantized* because this integral is the average value: the sum of contributions from all states with definite values of the total angular momentum.

In conclusion, the concept of spin-3/2 light does not stand up to close scrutiny. The value 3/2 announced in Ref. [1] does not play any distinguished role. Every value of γ can be realized with equal ease and each value can be

obtained in infinitely many ways. The values of the ratio J_z/W form a continuum extending from $-\infty$ to ∞ , with no preferred value whatsoever.

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