Precision Test of $\text{AdS}_6/\text{CFT}_5$ in Type IIB String Theory

Martin Fluder^{*}

Walter Burke Institute for Theoretical Physics, California Institute of Technology, Pasadena, California 91125, USA

Christoph F. Uhlemann[†]

*Mani L. Bhaumik Institute for Theoretical Physics Department of Physics and Astronomy,
University of California, Los Angeles, California 90095, USA*

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Superconformal field theories (SCFTs) in dimensions greater than 4 have become an integral part in the general understanding of quantum field theory, with many interesting implications in lower dimensions. They are hard to define using traditional methods, but can be engineered in string theory. Recently, a large class of AdS/CFT dualities has been constructed for five-dimensional SCFTs, which further supports their existence and allows for quantitative studies. We confront these dualities with a decisive test. We obtain the partition functions and central charges in gauge theory deformations of the SCFTs and extrapolate the results to the conformal fixed points. In the appropriate large N limits, this precisely matches the AdS/CFT predictions, providing strong support for the proposed dualities.

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Introduction.—Superconformal field theories (SCFTs) in dimensions greater than 4 have become an integral part in the understanding of quantum field theory (QFT) in general. In particular, upon compactification, they often reveal new perspectives and nonperturbative insights into four-dimensional theories. Defining interacting QFTs in dimensions greater than 4 is challenging, as conventional gauge theories are perturbatively nonrenormalizable. Nevertheless, a large body of evidence, based on string theory and QFT, suggests that interacting SCFTs in five and six dimensions do indeed exist. A particularly interesting aspect of five-dimensional SCFTs, of which large classes can be realized using (p, q) 5-brane webs in type IIB string theory [1–5], is that many have relevant deformations that flow to conventional, although nonrenormalizable, five-dimensional gauge theories in the infrared (IR), rendering these gauge theories asymptotically safe. The ultraviolet (UV) fixed points themselves, however, have no conventional Lagrangian description.

In the absence of a Lagrangian description, AdS/CFT dualities are particularly useful for quantitative studies of the five-dimensional SCFTs. A prerequisite for this approach is the availability of AdS_6 supergravity solutions and a precise identification with dual field theories. A warped AdS_6 solution in massive type IIA supergravity has

been known for some time [6–8] and has been studied extensively [6–19]. More recently, large classes of warped AdS_6 solutions were constructed in type IIB supergravity [20–23]. These solutions provide holographic duals for large classes of five-dimensional SCFTs, and there are compelling arguments for their identification with 5-brane webs in type IIB string theory. Several aspects of these proposed dualities have since been studied [24–28].

The aim of this Letter is to provide a decisive test of the aforementioned identification of type IIB supergravity solutions with five-dimensional SCFTs at the level of the partition functions. We will obtain the partition functions of the SCFTs on S^5 , as well as the conformal central charges C_T , which are related to the partition functions on squashed spheres [19,29], using field theory analyses. The strategy will be to study gauge theory deformations of the SCFTs and use supersymmetric localization [30–35] to compute the partition functions at strong coupling. The resulting matrix models will be evaluated numerically using saddle point approximations, which become exact in suitable large N limits. From the results we can thus reliably extract the leading-order terms in the SCFT partition functions and central charges at large N and compare to the results obtained from holographic analyses of the same quantities.

In the next section, we introduce two representative classes of five-dimensional SCFTs and their gauge theory deformations. In the section that follows, we compute the partition functions using supersymmetric localization and extract their large N behavior. After that, we introduce the dual supergravity solutions and present the holographic results. We close with a comparison and discussion.

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FIG. 1. Five-brane webs for the T_N theory with $N = 3$ and for the $\#_{N,M}$ theory with $N = 5$, $M = 4$. The brane webs show relevant deformations of the SCFTs.

The T_N and $\#_{N,M}$ theories.—We study two classes of five-dimensional SCFTs: first, the five-dimensional T_N theories [36,37], which reduce to four-dimensional T_N theories [38] upon compactification on S^1 and are realized by junctions of N $D5$, N $NS5$, and N $(1,1)$ 5-branes, and second, the theories realized on intersections of N $D5$ and M $NS5$ -branes [5], which we will refer to as $\#_{N,M}$ theories. Example 5-brane webs are shown in Fig. 1.

A gauge theory description for the five-dimensional T_N theories is given by the linear quiver [39,40]

$$[2] - (2) - (3) - \cdots - (N-1) - [N], \quad (1)$$

where (k) denotes an $SU(k)$ gauge group node and $[n]$ denotes n hypermultiplets in the fundamental representation of the gauge node they are attached to. Associated with each link between adjacent gauge group nodes is a bifundamental hypermultiplet. Each link gives rise to a $U(1)_b$ flavor symmetry, and each gauge node gives rise to a $U(1)_I$ instanton symmetry. The resulting global symmetry of the gauge theory is $SO(4) \times U(1)_I^{N-2} \times U(1)_b^{N-3} \times U(N)$, where the first and last factors are the flavor symmetries associated with the fundamental fields. The Chern-Simons levels are zero for all nodes [39]. The UV SCFT has enhanced global symmetry $SU(N)^3$.

The $\#_{N,M}$ theory is realized by an intersection of N $D5$ - and M $NS5$ -branes. A relevant deformation flowing to a gauge theory in the IR yields the linear quiver

$$[N] - (N) - \cdots - (N) - [N], \quad (2)$$

where the $SU(N)$ node appears $M-1$ times. The Chern-Simons levels are zero for all nodes. The gauge theory exhibits the flavor symmetries arising from the fundamentals, the $U(1)_b$ associated with the bifundamentals, and the instanton $U(1)_I$ symmetries, $U(N) \times U(1)_b^{M-2} \times U(1)_I^{M-1} \times U(N)$. This symmetry is enhanced in the UV SCFT to $SU(N)^2 \times SU(M)^2 \times U(1)$. The S-dual quiver, corresponding to an intersection of M $D5$ - and N $NS5$ -branes, is

$$[M] - (M) - \cdots - (M) - [M], \quad (3)$$

where the $SU(M)$ node appears $N-1$ times. The “large N ” limits in the following will refer to large N and large M for the $\#_{N,M}$ theories.

Field theory partition functions.—The IR description of the SCFTs in terms of a Lagrangian gauge theory allows for localization computations. Generically, the partition functions are sensitive to higher derivative terms in the Lagrangians and receive instanton contributions. However, following the arguments in [9,32,41], the higher derivative terms are expected to be $\mathcal{Q}$ exact, where $\mathcal{Q}$ denotes the supercharge used for the localization argument, and instanton contributions are expected to be exponentially suppressed at large N . Both are therefore irrelevant for the comparison to holographic computations.

The zero instanton part of the partition function is expressed conveniently as

$$\mathcal{Z}_0 = \int_{-\infty}^{\infty} \prod_{i,j} d\lambda_i^{(j)} \exp(-\mathcal{F}), \quad (4)$$

where $\lambda_i^{(j)}$ are the Coulomb branch parameters, with $i = 1, \dots, j-1$ and $j = 1, \dots, N-1$ for the T_N theory, and $j = 1, \dots, M-2$ and $i = 2, \dots, N-1$ for the $\#_{N,M}$ theory.

For the T_N theory,

$$\begin{aligned} \mathcal{F}^{T_N} = & \sum_{j=2}^{N-1} \ln j! - \frac{(N-2)(N-1)}{2} \ln \frac{S'_3(0)}{2\pi} \\ & + 2 \sum_{i=1}^2 \ln S_3 \left(i\lambda_i^{(2)} + \frac{\omega_{\text{tot}}}{2} \right) \\ & + N \sum_{i=1}^{N-1} \ln S_3 \left(i\lambda_i^{(N-1)} + \frac{\omega_{\text{tot}}}{2} \right) \\ & + \sum_{j=2}^{N-1} \sum_{\ell=1}^j \sum_{m=1}^{j+1} \ln S_3 \left(i[\lambda_\ell^{(j)} - \lambda_m^{(j+1)}] + \frac{\omega_{\text{tot}}}{2} \right) \\ & - \sum_{j=2}^{N-1} \sum_{\substack{\ell,m=1 \\ \ell \neq m}}^j \frac{1}{2} \ln S_3(\pm i[\lambda_\ell^{(j)} - \lambda_m^{(j)}]), \end{aligned} \quad (5)$$

where $\omega_{\text{tot}} \equiv \omega_1 + \omega_2 + \omega_3$, ω_i are the squashing parameters, and $S_3(z) \equiv S_3(z|\omega_1, \omega_2, \omega_3)$ is the triple sine function. The Coulomb branch parameters satisfy $\sum_{i=1}^j \lambda_i^{(j)} = 0$ for all j . For the SCFT partition function, the Yang-Mills coupling is taken to infinity, $g_{\text{YM}}^{-2} = 0$.

For the $\#_{N,M}$ theory, the Coulomb branch parameters satisfy $\sum_{i=1}^N \lambda_i^{(j)} = 0$ for all j , and

$$\begin{aligned}
 \mathcal{F}^{\#_{N,M}} = & \sum_{j=1}^{M-2} \sum_{\ell,m=1}^N \ln S_3 \left(i[\lambda_\ell^{(j)} - \lambda_m^{(j+1)}] + \frac{\omega_{\text{tot}}}{2} \right) \\
 & - (N-1)(M-1) \ln \frac{S'_3(0)}{2\pi} \\
 & - \sum_{j=1}^{M-1} \sum_{\substack{\ell,m=1 \\ \ell \neq m}}^N \frac{1}{2} \ln S_3(\pm i[\lambda_\ell^{(j)} - \lambda_m^{(j)}]) \\
 & + N \sum_{i=1}^N \ln S_3 \left(i\lambda_i^{(1)} + \frac{\omega_{\text{tot}}}{2} \right) \\
 & + N \sum_{i=1}^N \ln S_3 \left(i\lambda_i^{(M-1)} + \frac{\omega_{\text{tot}}}{2} \right). \quad (6)
 \end{aligned}$$

The partition functions are obtained by explicitly evaluating the Coulomb branch integral in (4) with (5) and (6), using a numerical saddle point method [42]. The saddle point equations are

$$\frac{\partial \mathcal{F}}{\partial \lambda_i^{(j)}} = 0. \quad (7)$$

To solve these equations numerically, the problem is rephrased in terms of a system of $(N-2)(N-1)/2$ particles for the T_N theory and $(N-1)(M-1)$ particles for the $\#_{N,M}$ theory, with time-dependent coordinates $\lambda_i^{(j)}(t)$ in a potential given by (5) and (6), respectively. The corresponding equations of motion read

$$\frac{d\lambda_i^{(j)}(t)}{dt} = -\frac{\partial \mathcal{F}}{\partial \lambda_i^{(j)}(t)}. \quad (8)$$

As $t \rightarrow \infty$, the equilibrium configurations describe solutions to the saddle point equations.

With these solutions the Coulomb branch integrals for the S^5 partition functions can be evaluated. The central charges C_T can be computed using the same techniques for the squashing corrections to the matrix models.

Results: We have obtained the S^5 partition functions for the T_N theories with $3 \leq N \leq 52$ and for the $\#_{N,M}$ theories with $2 \leq N, M \leq 20$, as well as the conformal central charges for the T_N theories with $3 \leq N \leq 22$ and for the $\#_{N,M}$ theories with $2 \leq N, M \leq 15$ (see the Supplemental Material [43]).

The data for the T_N theory show striking agreement with a quartic scaling ansatz; a least-squares fit for the free energy $F = -\log \mathcal{Z}$ to a degree-four polynomial in N yields

$$\begin{aligned}
 -F_{\text{fit}}^{T_N} = & 0.41118N^4 - 0.25712N^3 - 3.0686N^2 \\
 & + 13.572N - 35.854. \quad (9)
 \end{aligned}$$

A similar fit for C_T gives

$$\begin{aligned}
 C_{T,\text{fit}}^{T_N} = & 26.665N^4 - 16.274N^3 - 87.065N^2 \\
 & 56.342N + 2.9766. \quad (10)
 \end{aligned}$$

Both provide remarkably accurate approximations to the numerical data, with relative errors of $\mathcal{O}(10^{-3})$ or below for all $N > 10$.

For the $\#_{N,M}$ theories, the results for the quiver (2) and S-dual quiver (3) are expected to agree. The difference in the free energies, $F^{\#_{N,M}} - F^{\#_{M,N}}$, thus provides a quantitative indicator for the validity of the employed approximations. For $16 \leq M, N \leq 30$, the discrepancy is less than 1%, and we use these data to extract the free energy at large N . For fixed N , $F^{\#_{N,M}}$ exhibits a clear quadratic scaling with M , and for fixed M a clear quadratic scaling with N . The excellent agreement between the results for the quiver and the dual quiver motivates a fit to a quadratic polynomial in the $SL(2, \mathbb{Z})$ -invariant combination NM [44], which yields

$$-F_{\text{fit}}^{\#_{N,M}} = 1.4383(NM)^2 - 15.746NM + 1556.7. \quad (11)$$

An analogous fit for C_T with $11 \leq N, M \leq 15$ yields

$$C_{T,\text{fit}}^{\#_{N,M}} = 93.463(NM)^2 - 597.75NM + 24288. \quad (12)$$

Both again capture the results accurately, with maximal relative errors less than 1%. We emphasize that, within the approximations we have employed, the leading terms can be predicted reliably, while the subleading terms may be subject to corrections.

Partition functions from supergravity.—The geometry of the supergravity solutions in [20] is a warped product of $\text{AdS}_6 \times S^2$ over a Riemann surface Σ , and they involve nontrivial axion-dilaton and two-form fields. The solutions are parametrized by two locally holomorphic functions $\mathcal{A}_\pm$ on Σ . The explicit expressions for the supergravity fields can be found in [20]. Physically regular solutions that are naturally associated with 5-brane webs were constructed in [21,22]. For these solutions, Σ is the upper half plane with complex coordinate w , and

$$\mathcal{A}_\pm = \mathcal{A}_\pm^0 + \sum_{\ell=1}^L Z_\pm^\ell \ln(w - p_\ell). \quad (13)$$

The differentials $\partial_w \mathcal{A}_\pm$ have $L \geq 3$ poles on the boundary $\partial\Sigma$, with residues given by $Z_\pm^\ell$. At these poles, the external (p, q) 5-branes of the associated brane web emerge, with

$$Z_+^m = \frac{3}{4} \alpha'(p + iq). \quad (14)$$

For a given choice of residues, the remaining parameters are determined by regularity conditions [22].

The type IIB supergravity on shell action, for global Euclidean AdS_6 such that the dual SCFT is defined on S^5 , was computed in [24]. For a solution with three poles,

FIG. 2. (a) T_N theory partition functions obtained from localization as orange points and the supergravity result as dashed line in a log-log plot. (b) Results for the $\#_{N,M}$ theories. The analytic supergravity and numerical field theory computations show excellent agreement on the large N scaling exponents and on the leading coefficients.

$$S_{\text{IIB}} = \frac{64\pi^4}{3G_N} \zeta(3) (Z_+^1 Z_-^2 - Z_+^2 Z_-^1)^2, \quad (15)$$

with $16\pi G_N = (2\pi)^7 \alpha'^4$. For a solution with four poles and pairwise opposite-equal residues,

$$S_{\text{IIB}} = \frac{224\pi^4}{3G_N} \zeta(3) (Z_+^1 Z_-^2 - Z_+^2 Z_-^1)^2. \quad (16)$$

The holographic duals for the T_N theories are realized by three-pole solutions, $L = 3$, with residues

$$Z_+^1 = \frac{3}{4} \alpha' N = -i Z_+^2, \quad Z_+^3 = -\frac{3}{4} \alpha' (1+i) N, \quad (17)$$

with $p_1 = -p_3 = 1$, $p_2 = 0$, and $\mathcal{A}_+^0 = -\bar{\mathcal{A}}_-^0 = (3i/4) \alpha' N \ln 2$. The free energy computed from the on shell action is

$$F_{\text{sugra}}^{T_N} = -\frac{27}{8\pi^2} \zeta(3) N^4. \quad (18)$$

The holographic duals for the $\#_{N,M}$ theories are realized by four-pole solutions with residues

$$-Z_+^1 = Z_+^3 = \frac{3}{4} i \alpha' N, \quad Z_+^2 = -Z_+^4 = \frac{3}{4} \alpha' M, \quad (19)$$

and $p_1 = 1$, $p_2 = 2/3$, $p_3 = 1/2$, $p_4 = 0$, and $\mathcal{A}_+^0 = -\bar{\mathcal{A}}_-^0 = Z_+^2 \ln 3 - Z_+^1 \ln 2$. The free energy is

$$F_{\text{sugra}}^{\#_{N,M}} = -\frac{189}{16\pi^2} \zeta(3) N^2 M^2. \quad (20)$$

The conformal central charges C_T can be obtained from the results of [26], and are given by

$$C_{T,\text{sugra}}^{T_N} = \frac{2160}{\pi^4} \zeta(3) N^4, \quad (21)$$

$$C_{T,\text{sugra}}^{\#_{N,M}} = \frac{7560}{\pi^4} \zeta(3) N^2 M^2. \quad (22)$$

Discussion.—We have extracted the large N behavior of the S^5 partition functions and conformal central charges for two representative classes of five-dimensional SCFTs engineered by (p, q) 5-brane webs using field theory analyses. The results for the T_N theories are in (9) and (10), and those for the $\#_{N,M}$ theories are in (11) and (12). The corresponding results obtained from the proposed holographic duals in type IIB supergravity are given in (18) and (21) and (20) and (22). We find excellent agreement on the large N scaling behavior of the partition functions (N^4 for the T_N theories and $N^2 M^2$ for the $\#_{N,M}$ theories), as well as on the numerical coefficients of the leading terms, which agree to $\mathcal{O}(10^{-5})$. The results for the S^5 partition functions are illustrated graphically in Fig. 2, and the corresponding plots for the central charges C_T are fully analogous.

The excellent agreement between holographic and field theory analyses lends strong support to the proposed AdS/CFT dualities and establishes them at the level of rigor at which holographic dualities can typically be established. For the analysis presented here, we have selected two particular classes of five-dimensional SCFTs. The results, however, confirm the general arguments presented in [21] and support the entire class of holographic dualities proposed there.

With a tested and reliable interpretation of supergravity computations as field theory results, extensive quantitative studies of five-dimensional SCFTs are a natural next step. Questions of interest concern the spectrum of local and extended operators, correlation functions, renormalization group flows, and many more.

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*fluder@caltech.edu

†uhlemann@physics.ucla.edu

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- [43] See Supplemental Material at <http://link.aps.org/supplemental/10.1103/PhysRevLett.121.171603> for the explicit numerical results.
- [44] Completing the parameters N and M of the field theory to charge vectors $c_1 = (0, M)$ and $c_2 = (N, 0)$ on which $SL(2, \mathbb{Z})$ acts linearly, as suggested by the string theory realization, the combination NM corresponds to the invariant $\det(c_1, c_2)$.