Is Self-Interacting Dark Matter Undergoing Dark Fusion?

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We suggest that two-to-two dark matter fusion may be the relaxation process that resolves the small-scale structure problems of the cold collisionless dark matter paradigm. In order for the fusion cross section to scale correctly across many decades of astrophysical masses from dwarf galaxies to galaxy clusters, we require the fractional binding energy released to be greater than $v^n \sim (10^{-(2-3)})^n$, where $n = 1, 2$ depends on local dark sector chemistry. The size of the dark-sector interaction cross sections must be $\sigma \sim 0.1$ –1 barn, moderately larger than for standard model deuteron fusion, indicating a dark nuclear scale $\Lambda \sim \mathcal{O}(100 \text{ MeV})$. Dark fusion firmly predicts constant σv below the characteristic velocities of galaxy clusters. Observations of the inner structure of galaxy groups with velocity dispersion of several hundred kilometers per second, of which a handful have been identified, could differentiate dark fusion from a dark photon model.

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Introduction.—A dark, stable form of matter is overwhelmingly successful at explaining astrophysical phenomena on the largest observable scales of time and space. Nevertheless, the inner structure of some dark matter potential wells may indicate discrepancies with expectations derived from the simplest models of dark matter [1,2]. Whether these discrepancies are due to mismodeled but entirely standard physics or exotic, “beyond the standard model” particles and forces is one of the central questions for the present generation of physicists. It has recently been noted that dark matter that interacts with itself with a nuclear-scale cross section can compactly account for many of these observations [3,4]. Intriguingly, the required scattering cross section times velocity seems to be constant for objects across several decades of velocity dispersion.

In parallel, investigations of the early-Universe cosmology of models with dark-sector forces have led to some striking conclusions. A number of authors working with a wide variety of model assumptions have concluded that synthesis of bound states of dark matter particles is a generic consequence of the high temperature history of such hidden sectors [5–13]. This implies that dark matter particles with a range of “dark nucleon numbers” could be present and fusing in dark matter halos today.

In this Letter, we link these developments by demonstrating that late-time two-to-two fusion of dark matter

particles has a velocity-dependent cross section that can resolve small-scale structure “crises” from the scales of dwarf galaxies to galaxy clusters. The simple relations for two-to-two fusion discussed here provide a firm prediction of an *exactly flat* σv at intermediate and low velocities. This contrasts with the model presented in [3,14,15], which exhibits a peak in σv at velocities between galaxy and cluster scales. The reason fusion is a powerful explanatory mechanism is that dwarf galaxies probe much smaller velocities than galaxy clusters, but the relaxation rates of these halos seem to be similar [3,4]. It may be possible to distinguish these models by detailed observation of galaxy groups, which are virialized and have velocity dispersion in an interesting intermediate range.

This model is not in jeopardy from limits on unitarity of scattering [16] because the particles are relatively light and can be compound objects, and it evades observational limits on the standard model flux from products of annihilation [17,18] if the initial states do not annihilate or if the final states are low energy or stable against decay to standard model particles. In this sense, dark fusion is a “gentle” generalization of annihilation in which much less rest mass energy is released and antiparticles need not be present. Decoupling the annihilation and energy release mechanisms opens up a bevy of new model-building opportunities and naturally allows a large cross section with less severe consequences. The large cross sections discussed here are also not problematic for direct detection since they connect the dark sector to itself, and the dark sector may be arbitrarily secluded from the standard model. Dark fusion is thus able to explain a wide variety of observations, while evading nontrivial bounds, in a compelling way.

In Sec. I, we discuss how the required velocity dependence arises. We show that there are two distinct limits of the

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annihilation participant masses that can solve this problem, and in Sec. II, we discuss the phenomenological requirements on (and benefits of) such models. In Sec. III, we provide an outline of some complete models whose early Universe dynamics could satisfy these phenomenological requirements. Finally, we conclude.

Two-to-two scattering and cross section velocity dependence.—The scattering cross section for a two-to-two process is

$$\frac{d\sigma}{d\Omega} = \frac{|\overline{\mathcal{M}}|^2}{64\pi^2 s} \frac{|\vec{p}_3|}{|\vec{p}_1|}, \quad (1)$$

where the momenta are in the center of mass frame, with $|\vec{p}_1| = |\vec{p}_2|$ and $|\vec{p}_3| = |\vec{p}_4|$. These momenta may be expressed solely in terms of s , the center of mass energy squared, and the particle masses,

$$|\vec{p}_1| = \frac{\sqrt{[s - (m_1 + m_2)^2][s - (m_1 - m_2)^2]}}{2\sqrt{s}}. \quad (2)$$

For $|\vec{p}_3|$, we take $m_i \rightarrow m_{i+2}$ in Eq. (2). Without loss of generality, we choose $m_1 > m_2$ and $m_3 > m_4$ if the masses are unequal.

We focus on exothermic fusion $m_3 + m_4 < m_1 + m_2$ and define the fractional binding energy released as

$$b = 1 - \frac{m_3 + m_4}{m_1 + m_2}. \quad (3)$$

Kinematically unsuppressed annihilation is the limit $b \sim 1$, and semiannihilation [19,20] is $b \sim 0.5$. By analogy with standard model nuclear forces, and for observational reasons discussed in more detail below, we are interested in the regime $b \ll 1$. Furthermore, we assume that the early-Universe yields of dark nuclei lead to a few dominant dark species with similar dark nucleon numbers, which implies that the initial state masses are comparable. A UV complete model simultaneously displaying all necessary ingredients has not been explicitly demonstrated to exist, but in Sec. III, we discuss models that plausibly have both $0 < b \ll 1$ and appropriately heterogeneous initial state abundances. Finally, we define a velocity via $E_{i,\text{cm}} = m_i(1 - v_{i,\text{cm}}^2)^{-1/2}$, so that in the nonrelativistic limit $|\vec{p}_1| = m_1 v_{1,\text{cm}}$.

For now, consider the scenario where $b \ll 1$ and the outgoing dark nucleons have similar nucleon numbers, allowing us to write $s - (m_3 - m_4)^2 \simeq s - (m_1 - m_2)^2 \simeq 4m_1 m_2$. In the nonrelativistic limit and to leading order in b , the final-state momenta are

$$|\vec{p}_3|_{m_4 \simeq m_3} \simeq m_1 \left(\frac{2m_2}{m_1} b + v_{1,\text{cm}}^2 \right)^{1/2}. \quad (4)$$

The kinematic assumptions break down, and Eq. (4) is changed if the mass of the lighter outgoing particle is much smaller than the binding energy. Consider a model of dark nucleons, with dark nucleon mass m_d , dark nucleon number A_i , and binding energy $\mathcal{B}_i$. A light outgoing particle corresponds to $A_4/A_3 \ll b$. This occurs with emission of a light, non-nuclear partner particle, so that $A_1 + A_2 = A_3$. A familiar example is standard model deuteron production, during which a photon is emitted. For this case,

$$|\vec{p}_3|_{m_4/m_3 \ll b} \simeq m_1 \left(\frac{1 + m_1/m_2}{2} \right) \left(\frac{2m_2}{m_1} b + v_{1,\text{cm}}^2 \right). \quad (5)$$

In either limit, the factor $|\vec{p}_3|/|\vec{p}_1|$ goes like $\sim v_{1,\text{cm}}^{-1}$ at velocities below the square root of the fractional binding energy released. Thus, fusion is qualitatively different than elastic scattering, for which $|\vec{p}_3| = |\vec{p}_1|$ is exact. The underlying difference is that in the case of fusion $|\vec{p}_3|$ need not be small if significant binding energy is released, even with initial state particles at rest. In this way, rates for exothermic self-interactions can be constant at low velocities. Regardless of the hierarchy of m_4 and m_3 , the comparison $2bm_2/m_1 \lesseqgtr v_{1,\text{cm}}^2$ is the correct diagnostic for determining whether $|\vec{p}_3|$ has a velocity dependence. The dependence on velocity in the high-velocity limit is linear when $m_4/m_3 \sim \mathcal{O}(1)$ and quadratic when m_4/m_3 is very small.

Given the discussion above, we define a function $f(b, v_{1,\text{cm}}) \equiv |\vec{p}_3|/m_1$, which has the limiting forms

$$f(b, v_{1,\text{cm}}) = \begin{cases} \left(\frac{2m_2}{m_1} b + v_{1,\text{cm}}^2 \right)^{1/2} & \frac{m_4}{m_3} \gtrsim b \\ \frac{1+m_1/m_2}{2} \left(\frac{2m_2}{m_1} b + v_{1,\text{cm}}^2 \right) & \frac{m_4}{m_3} \ll b \end{cases} \quad (6)$$

Using Eqs. (1) and (6), we may then write the non-relativistic fusion cross section simply as

$$\sigma_f v_{1,\text{cm}} = f(b, v_{1,\text{cm}}) \bar{\sigma}(m_1, m_2), \quad (7)$$

where we define $\bar{\sigma}(m_1, m_2) \simeq [1/16\pi(m_1 + m_2)^2] \int (d\Omega/4\pi) |\overline{\mathcal{M}}|^2$. For nonrelativistic two-to-two scattering with a constant matrix element, $\sigma_f v_{1,\text{cm}}$ inherits velocity dependence only from the factor $f(b, v_{1,\text{cm}})$. Further velocity or energy dependence can be understood to arise from a term like $S(E)$ defined in [21].

For these reasons, it is necessary to understand the velocity dependence of $\bar{\sigma}$. The magnitude is parametrically set by a dark nucleon scale Λ_d . We speculatively write $\bar{\sigma} \equiv 1/\Lambda_d^2$, and we point out that it is natural to assume that elastic scattering that does not change a dark nucleon number is also generically present with a similar size $\sigma_{\text{el}} \sim \bar{\sigma}$, as in the black disk limit [22]. We include elastic processes in what follows. The scale Λ_d will be set by the

requirement that $\sigma_f v_{1,\text{cm}}$ matches observations and turns out to be explained with $\Lambda_d \sim m \sim \mathcal{O}(100 \text{ MeV})$.

Collecting these observations, the velocity dependence of $\sigma_f v_{1,\text{cm}}$ has two distinct regimes: at high velocities $\sigma_f v_{1,\text{cm}} \sim v_{1,\text{cm}}^{1,2}$, where the exponent depends on the mass ratio m_4/m_3 , while at low velocities there is no velocity dependence, regardless of the value of m_4/m_3 . Early-Universe nucleosynthesis occurs at temperatures corresponding to velocities $v^2 \sim \mathcal{O}(1/20)$, for which the high-velocity limit is suitable. The physical intuition is that if the mass difference is small compared to the kinetic energy the scattering is nearly elastic. On the other hand, inelasticity is crucial at low velocity. We see from Eq. (6) that the product $\sigma_f v_{1,\text{cm}}$ asymptotes to be constant with velocity $\sigma_f v_{1,\text{cm}}|_{\text{low } v} \propto v^0$. Because dark matter halos provide potential wells with virial velocities $v \sim \sqrt{GM/R} \sim 10^{-(3 \pm 1)}$, the case $\sigma_f v_{1,\text{cm}} \propto v_{1,\text{cm}}^0$ is appropriate for virialized particles in galaxies at late times as long as b is not arbitrarily small.

Dark fusion at late times.—We begin this section with a brief analogy. Some decades ago, Bethe and Longmire [23] showed that the low-velocity cross section for production of a deuteron along with emission of a hard photon is [24]

$$\sigma_H v \simeq 6.64 \times 10^{-20} \text{ cm}^3/\text{s}. \quad (8)$$

The size of this cross section is set by the mass scale of the nucleons $\sim 1 \text{ GeV}$ and the fractional binding energy of deuterium, $1 - (m_D/m_n + m_p) \sim 10^{-3}$. More recently, the

preferred cross section for explaining the cores of dark matter halos through self-interacting dark matter has been claimed to be [3,4]

$$\frac{\langle \sigma_s v \rangle}{m_X} \simeq \frac{40\text{--}200 \text{ cm}^2 \text{ km}}{g} \frac{1}{s} \simeq \frac{6\text{--}30 \times 10^{-18} \text{ cm}^3}{\text{GeV s}}. \quad (9)$$

Particles with this scattering cross section will interact roughly once in a Hubble time, thermalizing a core in their host halo [3]. The cross section in Eq. (9) is suggestively close to the range in Eq. (8), albeit somewhat larger. Critically, both Eqs. (8) and (9) are much larger than the thermal annihilation cross section. This is understandable if the early Universe abundances are set by an asymmetry rather than weak self-annihilation—but such an asymmetry would render the current population nonannihilating. This both resolves the problem and removes the solution of extreme annihilations at the center of dark matter halos [17]. Instead, we speculate that SIDM scattering is dark fusion mediated by a dark sector interaction with a scale slightly below the nuclear scale.

We show the fusion and elastic cross sections times velocity $\sigma_f v_{1,\text{cm}}$ and $\sigma_{\text{el}} v_{1,\text{cm}}$ in Fig. 1. We relate the fusion to the elastic scattering cross section by $\sigma_f v_{1,\text{cm}} = f(b, v_{1,\text{cm}}) \sigma_{\text{el}}$ under the assumption that all dark sector dynamics are at the same scale. We take $m_1 = m_2 = m$ for simplicity (and thus $v_{1,\text{cm}} = v_{2,\text{cm}} \equiv v$), and we fix the elastic scattering cross section $\sigma_{\text{el}} = \bar{\sigma} = (100 \text{ MeV})^{-2}$. Observables in the velocity range of interest are set by $\bar{\sigma} b^k/m$ (where $k = 1/2, 1$ in the similar-mass or the

FIG. 1. Dark fusion (thick solid) and elastic (thick dashed) scattering cross sections. We assume $m_1 = m_2 = 200 \text{ MeV}$, we fix the elastic scattering cross section $\sigma_{\text{el}} = (100 \text{ MeV})^{-2}$, and we set $\sigma_f v = f(b, v) \sigma_{\text{el}}$. In the left panel, we assume that all reactants have similar mass. In the right panel, we assume that one outgoing fusion product is massless. Points with error bars provide locations of interesting anomalies in the inner structure of dark matter halos (red points are dwarf galaxies, blue points are low surface brightness galaxies, and green are clusters), and the thin orange line is a best fit dark photon model to this data, as in [3]. The short, thick blue line in the upper right corner of each plot shows a rough bound of $\bar{\sigma} \leq 0.4 \text{ cm}^2/\text{g}$ from stacked observations of cluster mergers at $v = 1000 \pm 200 \text{ km/s}$ [25].

massless fusion partner scenario, respectively), so we fix $m = 200$ MeV and vary b as required. In the left panel of Fig. 1, we assume that all reactants have similar mass, and the velocity dependence of the fusion cross section goes like $\sigma_f v_{1,\text{cm}} \simeq (2b + v_{1,\text{cm}}^2)^{1/2} \bar{\sigma}$. At high velocity, the fusion and elastic cross sections coincide. In the right panel of Fig. 1, we assume that one outgoing fusion product is massless, so that the velocity dependence of fusion goes like $\sigma_f v_{1,\text{cm}} \simeq (2b + v_{1,\text{cm}}^2) \bar{\sigma}$. Fusion becomes negligible at high velocity, so elastic scattering determines the structure of larger astrophysical objects. In both panels, we display a limit $\bar{\sigma} \leq 0.4 \text{ cm}^2/\text{g}$ from stacked observations of cluster mergers at $v = 1000 \pm 200 \text{ km/s}$ [25], which is now thought to be stronger than the limit from mass loss in the Bullet Cluster [26]. Because observations favor $(\sigma v/m)|_{\text{cluster}} \simeq (\sigma v/m)|_{\text{dwarf}}$, we see that fusion must dominate at and below the cluster scale. This implies $b \gtrsim v_{\text{cluster}}^{1/k}/2$: in other words, $b \gtrsim 10^{-6}, 10^{-3}$ depending on the dark sector nuclear physics.

At low velocity, the same results are obtained for any rescaling of $\bar{\sigma}$, b , and m that keeps $\bar{\sigma} b^k/m$ fixed. If we take the limit $b \rightarrow 1$, “fusion” becomes annihilation [17]. The small- b limit worked out explicitly above is changed at the $\mathcal{O}(1)$ level, but a similar analysis follows. However, the scale of Eq. (9) is naïvely too large for annihilation, since antiparticles are exponentially depleted if the annihilation cross section is as large as Eq. (9) indicates. Model-building solutions exist [17] but are constrained by nonobservation of annihilation products [18]. Dark fusion provides a “gentler” alternative.

We emphasize that this model predicts measurements of σv with a very different dependence on host mass compared to the model of [3,14,15]. Dark fusion has an exactly flat σv until $v_{1,\text{cm}} \gtrsim (2bm_2/m_1)^k$, where $k = 1/2$ for similar-mass final states and $k = 1$ for a massless fusion partner. If the final state particles have similar masses, the fusion and elastic scattering cross sections become similar above this threshold. If one final state particle is very light, fusion becomes negligible above this threshold, and elastic scatterings dominate. If dark matter chemistry varies between local environments, it is possible that there will be a family of curves populating the $\langle \sigma v \rangle - v$ plot at high velocities. However, a constant $\sigma_f v$ down to arbitrarily small host mass is a robust prediction of this model. This is qualitatively different than the dark photon model of [3,14,15], which relies on elastic scattering only. Models with a dark photon of mass m_ϕ have a cross section that varies strongly with velocity: one finds $\sigma_s v \sim v$ in small hosts with low velocity dispersions because interactions are contactlike; then a maximum in objects with characteristic momentum transfer $mv \sim m_\phi$, where a cancellation occurs in the denominator of σ_s , and a scaling like $\sigma_s v \sim v^{-4}$ in large halos, when m_ϕ is much less than the momentum transfer. Dark fusion is also qualitatively different from an

inelastic process with a threshold, like exciting dark matter [27], which has a high-velocity threshold below which inelasticity turns off.

Detailed observations of the inner structure of a virialized object with $v \simeq 700 \pm 100 \text{ km/s}$ or $v \ll 30 \text{ km/s}$ could differentiate the dark fusion and dark photon models. One interesting candidate for this purpose is PGC 43296, identified in 2MASS Redshift Survey data [28]. This galaxy group has 191 members, mass $4.2 \times 10^{14} M_\odot \leq 5.2 \times 10^{14} M_\odot$, and a velocity dispersion $595 \text{ km/s} \leq \sigma \leq 800 \text{ km/s}$, with 3% systematic error bars [29]. Near this velocity, the dark photon model has a maximum in $\sigma_s v$, while dark fusion predicts the same $\sigma_f v_{1,\text{cm}}$ as in dwarf galaxies. The dark photon model would lead to a significantly larger core and lower stellar velocities inside of the radius where dark matter scattering takes place once per Hubble time.

Finally, we point out that dark fusion may be accompanied by indirect detection signals if the dark sector is coupled even weakly to the standard model [6]. Semiannihilation is the $b \sim 0.5$ limit of dark fusion with a massless partner, but a cross section as in Eq. (9) would lead to higher rates and lower mass scales than previously considered [19,20]. This could lead to interesting signals or strong limits on the decays of the outgoing fusion partner.

Early Universe cosmology of models with dark fusion.— Because the required cross section of Eq. (9) is so large, we do not discuss symmetric thermal relics that could annihilate away [17]. Instead, we consider two-to-two fusion (which can be thought of as very compressed annihilation without antiparticles) of an asymmetric species. We comment here on particle physics models that could provide the ingredients necessary for dark fusion to be operative in small-scale structures in the present epoch. The model requirements are that two-to-few reactions are dominant, unbound nucleons are abundant, and reactions are non-relativistic so that the $\sigma_f v_{1,\text{cm}} \propto v_{1,\text{cm}}^0$ scaling holds [30]. Because nucleosynthesis happens at temperatures with $v^2 \sim \mathcal{O}(1/20\text{--}1/200)$ [6] while galactic dark fusion happens with velocities $v \lesssim 10^{-2}$, it is plausible that different chemical reactions will dominate in the two epochs and that unspent fuel remains in galaxies today. Indeed, in the standard model, one finds that early Universe synthesis is unable to go all the way to the iron peak. Even with the existence of stable $A = 5, 8$ nuclides, synthesis would only complete up to carbon. A complete model that results in a nontrivial early Universe yield of fusion participants with a cross section of the size of Eq. (9) does not exist yet but is hopefully within model-building reach.

In the model of [5], “dark protons” are the only matter. They have a small charge under a dark Coulomb force as well as a stronger nuclear force that is the residual of a dark $\text{SU}(N)$. The binding energy per nucleon based on the liquid drop model [31,32] has a peak similar to the iron peak of the standard model. The early Universe yields of dark

nuclei will determine which kinds of processes are dominant in galaxies today. If “strongly exothermic” processes determine the early Universe yield and most particles wind up with $A \sim 20$, as hinted in [5], then it is reasonable to expect that dark fusion today will be in the weakly exothermic regime. The liquid drop model is best interpreted as an ansatz, however, and more complete calculations of the nuclear binding are desirable [11,12].

In [6], the spectrum of a strongly coupled two-color, two-flavor QCD-like model is presented. The authors find a light scalar π_d and a light vector ρ_d that can fuse into a nucleus D by emitting a light dark Higgs h_d . The particle masses are near the dark QCD scale with the exception of h_d , which is lighter; heavier three- and four-body states may be stable as well [33]. A non-negligible yield of all species is obtained for a wide range of dark sector cross sections, which serves as a nontrivial demonstration of the potential relevance of our mechanism for late times. Because the yield of dark sector particles leads to significant quantities of each species [6] but cross sections as large as Eq. (9) have not yet been explored, this model is highly motivated to more explicitly investigate in the context of small-scale structure.

Models with matter that is coupled to an attractive Yukawa force only [7,8,11,12] are also intriguing. These models lead to composite states of extremely large dark nucleon number, aggregating enormous quantities of particles in the early Universe. Whether the late-time cosmology of these “nuggets” can be dominated by the simple cross sections worked out here is unclear and deserves further study. In particular, it is unclear if local fusion rates, fractional binding energy release, and particle multiplicities can match on to the simple velocity dependence suggested here [30].

Conclusions.—In this paper, we have proposed that the cross section times velocity for dark matter self-interactions may have no apparent velocity dependence in the late Universe because the dark matter is undergoing two-to-two fusion. In Fig. 1, we show that this scenario would explain the evident flatness of dark matter cross section times velocity over many decades in velocity that was proposed as an explanation for cores in dark matter halos of widely varying masses [3,4]. It may also provide nonrelativistic kicks to fusion daughters that can evaporate small, loosely bound dark matter substructures [34].

This model provides a firm prediction: observed galaxy cores will point to the same value of σv for all velocities below cluster scales. There will be a linear rise in the required value of σv at higher velocities, either due to the high-velocity regime in Eq. (6) (for the similar-mass scenario) or due to dominance of elastic scatterings at high velocities (in the massless-fusion-partner case). This differs markedly from σv at the same scales in the dark photon model of [3]. Thus, improved astrophysical observations at low, intermediate, or high masses could shed light on the

details of the dark sector. We have noted the relevance of galaxy group PGC 43296 [29] for breaking the degeneracy between these models. It is also possible that astrophysical objects have different chemical compositions, with the linear rise of σv turning on at different values of the velocity in different hosts. We may discover hints of dark fusion activity as we look deeper into dark matter potential wells.

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