Unifying Type-II Strings by Exceptional Groups

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We construct the exceptional sigma model: a two-dimensional sigma model coupled to a supergravity background in a manifestly (formally) $E_{D(D)}$ -covariant manner. This formulation of the background is provided by exceptional field theory (EFT), which unites the metric and form fields of supergravity in $E_{D(D)}$ multiplets before compactification. The realization of the symmetries of EFT on the world sheet uniquely fixes the Weyl-invariant Lagrangian and allows us to relate our action to the usual type-IIA fundamental string action and a form of the type-IIB (m, n) action. This uniqueness “predicts” the correct form of the couplings to gauge fields in both Neveu-Schwarz and Ramond sectors, without invoking supersymmetry.

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The usual way to search for realistic lower-dimensional physics from string or M theory is to compactify. Early studies of the simplest reductions—on tori—led to the first encounter of T duality [1,2], in which string theory on a circle of radius R is equivalent to string theory on a circle of radius $1/R$. String theory on a D torus leads to an $O(D, D; \mathbb{Z})$ T duality symmetry, while M theory on a D torus has an $E_{D(D)}(\mathbb{Z})$ U duality, involving the exceptional Lie groups. These are powerful tools for understanding how these theories are unified, and point to the idea that stringy or M -theoretic probes see spacetime geometry contrary to our usual expectations.

The search for reformulations of string theory, in which the enlarged symmetries exhibited by these dualities are apparent before compactification, begins with the construction of “duality symmetric” models [3–7]. Here, the sigma model’s target space is *doubled*, and the coordinates appearing on the world sheet are the doubled pair $(Y, \tilde{Y})$. T duality then swaps $\tilde{Y}$ for Y . A chirality constraint relates the $\tilde{Y}$ back to the Y , so that the number of degrees of freedom is not increased. Similar ideas were pioneered for membranes in [8], but this approach runs into some difficulties [9].

We can also pursue this problem in the supergravity picture. Here, double field theory (DFT) [10–15] and exceptional field theory (EFT) [16–27] reformulate 10- or 11-dimensional supergravity with all bosonic fields in representations of $O(D, D)$ or $E_{D(D)}$ (the fermions appear

in representations of the maximally compact subgroups), depending on an extended set of coordinates (X^μ, Y^M) . For DFT, the Y^M are simply doubled coordinates, while for EFT, more extra coordinates are needed such that the Y^M fit into a particular representation, denoted R_1 , of $E_{D(D)}$ [so for $O(D, D)$, R_1 is the fundamental].

The extended space parametrized by the Y^M comes with a *local* $O(D, D)$ or $E_{D(D)}$ symmetry provided by “generalized diffeomorphisms” (which combine ordinary diffeomorphisms and gauge transformations). These are generated by generalized vectors Λ^M , which act on another generalized vector U^M via the generalized Lie derivative: $\delta_\Lambda U^M = \mathcal{L}_\Lambda U^M$ with

$$\mathcal{L}_\Lambda U^M = \Lambda^N \partial_N U^M - U^N \partial_N \Lambda^M + Y_{PQ}^{MN} \partial_N \Lambda^P U^Q, \quad (1)$$

where the deviation from the usual form of the Lie derivative is due to the final term involving Y_{PQ}^{MN} , which is constructed from $O(D, D)$ or $E_{D(D)}$ invariant tensors (see [19]). Now, coordinate dependence, in principle, can be on any of the Y^M , but this is, in fact, constrained, as follows from closure of algebra of generalized diffeomorphisms, for which we impose the “section condition”

$$Y_{PQ}^{MN} \partial_M \otimes \partial_N = 0. \quad (2)$$

A choice of physical Y^i coordinates satisfying this is a choice of “section.” Such a choice breaks $O(D, D)$ or $E_{D(D)}$, and it establishes the link to the usual formulation of supergravity without extended coordinates. When isometries are present, there is an ambiguity in the choice of section. This corresponds to the usual notion of duality.

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We can think of the “doubled” sigma model of [6,7] as describing strings propagating on a DFT background; upon eliminating dual coordinates the DFT background reduces to a standard supergravity one, and the doubled sigma model reduces to the conventional sigma model.

We will provide a similar world sheet picture for $E_{D(D)}$ ($D \leq 6$), which we call the exceptional sigma model. This describes a string coupling to an EFT background. This background will consist of the following EFT tensors. First, there are metriclike degrees of freedom: the “external” metric $g_{\mu\nu}(X, Y)$ [roughly, a metric on the “external space” with coordinates X^μ , which do not transform under $E_{D(D)}$] and the “generalized metric” $\mathcal{M}_{MN}(X, Y)$ (roughly, a metric on the “internal space” with coordinates Y^M). Second, we have generalized gauge fields, including a one-form A_μ in the R_1 representation of $E_{D(D)}$, and a two-form $B_{\mu\nu}$, in another representation of $E_{D(D)}$, denoted by R_2 . These are the first two fields in the “tensor hierarchy” of EFT [28,29]. The representation R_2 is always contained within the symmetric product of two R_1 representations, and so we can write the field $B_{\mu\nu} \in R_2$ as carrying a pair of symmetric R_1 indices, thus $B_{\mu\nu}^{MN}$ (symmetrization implicit).

Unsurprisingly, the representations $R_1, R_2, \dots$ that characterize these form fields are exactly the representations into which the brane ensemble of string and M theory reassembles upon toroidal reduction (see e.g. [30]); for instance, upon reducing on a T^D , $M2$ and $M5$ branes completely wrapping the torus directions appear as particles in the reduced theory, transforming in the R_1 representation. $M2$ and $M5$ branes, with one world volume direction unwrapped, appear as strings—transforming in the R_2 multiplet. And so on. The conceptual difficulty is that different kinds of branes are mapped to each other by the action of $E_{D(D)}$. A way around this is to construct $(p-1)$ -brane actions coupling to the p form in R_p , which in 10- or 11-dimensions describe only the genuine $(p-1)$ -branes that occur there, but those which can be interpreted in lower dimensions as describing the full multiplet of wrapped branes. This is the logic of the EFT particle actions ($p=1$) studied in [31]. (Alternative approaches to U duality covariant branes include [32–36]).

We now present the action. To couple the multiplet $B_{\mu\nu}^{MN}$, we introduce a set of charges q_{MN} [valued in the representation $\bar{R}_2$ of $E_{D(D)}$ inside the symmetric tensor product $\bar{R}_1 \otimes \bar{R}_1$]. We denote the world sheet coordinates by σ^α , the world sheet metric by $\gamma_{\alpha\beta}$, and the Levi-Civita symbol by $\epsilon^{\alpha\beta}$. The extended spacetime coordinates appear as world sheet scalars ($X^\mu(\sigma), Y^M(\sigma)$), and the background fields can depend on these subject to the section condition. We also need an auxiliary world sheet one-form V_α^M , which appears in the covariant world sheet differential

$$D_\alpha Y^M d\sigma^\alpha = (\partial_\alpha Y^M + A_\alpha^M + V_\alpha^M) d\sigma^\alpha, \quad (3)$$

in which the EFT one-form A_μ^M also appears (we write $A_\alpha^M \equiv \partial_\alpha X^\mu A_\mu^M$). The field V_α^M essentially serves to gauge away the dual coordinates. Consider splitting $Y^M = (Y^i, Y^A)$, such that Y^i are physical and the Y^A are dual, one obtains a shift symmetry in the Y^A (as the section condition is solved by $\partial_i \neq 0, \partial_A = 0$). In [6,7], gauging this symmetry allows one to eliminate the Y^A from the action. For this to work, we require

$$V_\alpha^M \partial_M = 0, \quad (4)$$

so that for the section $\partial_i \neq 0, \partial_A = 0$, only the components V^A are present.

The action is then given by $S = -\frac{1}{2} \int d^2 \sigma (L_{\text{kin}} + L_{\text{WZ}})$ with

$$L_{\text{kin}} = T \sqrt{-\gamma} \gamma^{\alpha\beta} \times \left(\frac{1}{2} \mathcal{M}_{MN} D_\alpha Y^M D_\beta Y^N + g_{\mu\nu} \partial_\alpha X^\mu \partial_\beta X^\nu \right), \quad (5)$$

$$L_{\text{WZ}} = q_{MN} \epsilon^{\alpha\beta} (B_{\alpha\beta}^{MN} + A_\alpha^M D_\beta Y^N + \partial_\alpha Y^M V_\beta^N), \quad (6)$$

where $B_{\alpha\beta}^{MN} = B_{\mu\nu}^{MN} \partial_\alpha X^\mu \partial_\beta X^\nu$ and T (the “tension”) is

$$T = \sqrt{\frac{1}{2(D-1)} \mathcal{M}^{MN} \mathcal{M}^{PQ} q_{MP} q_{NQ}}. \quad (7)$$

As we will explain, the construction of this action, coupling in a natural—and in particular, gauge-invariant—manner to the two-form $B_{\mu\nu}$ of the EFT tensor hierarchy, is exceptionally constrained by the requirement of invariance under the intricate $E_{D(D)}$ local symmetries of EFT, and this leads us to the *unique result* (5), (6), (7) (modulo some reasonable assumptions). Gauge invariance also restricts the choice of q_{MN} [see (10)]: for a generic 10-dimensional background (i.e., a background dependent on $D-1$ of the Y^M), one finds that q_{MN} can only select the strings known to exist in ten dimensions. Remarkably, this includes the correct couplings to the 10-dimensional two forms, otherwise fixed by supersymmetry.

The world sheet one-form V_α^M also ensures covariance under the local symmetries of EFT. As pointed out in [37,38], the natural candidate kinetic term $\mathcal{M}_{MN} D_\alpha Y^M D_\beta Y^N$, involving the generalized metric on the extended space, only transforms properly when V_α^M is present and assigned a particular transformation under the local symmetries of DFT. This generalizes naturally to EFT [31]. Consider first the $E_{D(D)}$ generalized diffeomorphisms, defined in (1). These act on the generalized metric as a tensor and as gauge transformations of A_μ^M via $\delta_\Lambda A_\mu^M = \partial_\mu \Lambda^M - \mathcal{L}_{A_\mu} \Lambda$. The world sheet action should obey a covariance requirement under generalized diffeomorphisms; namely, varying the coordinates on the world

sheet as $\bar{\delta}_\Lambda Y^M = \Lambda^M(X, Y)$ should *induce* the correct transformations δ_Λ of the background fields. This is then a symmetry of the world sheet only if Λ^M is a generalized Killing vector, i.e., $\delta_\Lambda = 0$ on all background fields. In addition, A_μ^M also transforms under one-form gauge transformations valued in R_2 , as $\delta_\lambda A_\mu^M = -Y_{PQ}^{MN} \partial_N \lambda_\mu^{PQ}$, and the world sheet action should be invariant under such gauge transformations. Imposing these requirements on the generalized metric coupling $\mathcal{M}_{MN} D_\alpha Y^M D_\beta Y^N$ fixes the transformations of the gauge field V_α^M to be

$$\begin{aligned}\bar{\delta}_\Lambda V_\alpha^M &= -Y_{PQ}^{MN} (\partial_N \Lambda^P D_\alpha Y^Q + \partial_N A_\mu^P \partial_\alpha X^\mu \Lambda^Q), \\ \delta_\lambda V_\alpha^M &= Y_{PQ}^{MN} \partial_N \lambda_\mu^{PQ} \partial_\alpha X^\mu.\end{aligned}\quad (8)$$

Note that, these preserve $V_\alpha^M \partial_M = 0$ [using the section condition (2)]. Actually, the presence of certain weight terms in the generalized Lie derivative acting on the generalized metric, in fact, forces us to introduce T as defined in (7), such that, altogether it is the combination $T \mathcal{M}_{MN} D_\alpha Y^M D_\beta Y^N$, which obeys the covariance requirement.

One can use this information to then construct the gauge invariant completion of the electric WZ coupling, beginning with the gauge transformation of $B_{\mu\nu}$, which is

$$\delta_\lambda B_{\mu\nu}^{MN} = 2\partial_{[\mu} \lambda_{\nu]}^{MN} - \mathcal{L}_{A_{[\mu}} \lambda_{\nu]}^{MN} + \frac{1}{2(D-1)} Y_{PQ}^{MN} A_{[\mu}^P \delta_\lambda A_{\nu]}^Q, \quad (9)$$

with the end result being (6).

As mentioned before, the WZ coupling is only gauge invariant if q_{MN} is constrained:

$$q_{MN} Y_{KL}^{NP} \partial_P = q_{KL} \partial_M. \quad (10)$$

[This arises from considering V and A independent terms in the gauge transformation of (6)]. The idea is to solve this constraint for the charge q_{MN} after imposing the section condition $\partial_i \neq 0$, $\partial_A = 0$. (The role of this constrained charge in simultaneously ensuring gauge invariance and selecting the allowed branes appears to be a generic feature of brane formulations in EFT, as has been proposed in [39–41].) Generically, there are no solutions for the section choice that relates EFT to 11-dimensional supergravity—unless one of the physical directions Y^i is an isometry. This reduces us to 10-dimensional type IIA, and we find that there is a single solution corresponding to the single $F1$ string of type IIA. On the type-IIB sections, one finds instead that there is a doublet of allowed solutions transforming under the unbroken $SL(2) \subset E_{D(D)}$; this corresponds to the (m, n) strings of type IIB.

So far, we have constructed the WZ coupling and the generalized metric pullback $T \mathcal{M}_{MN} D_\alpha Y^M D_\beta Y^N$ by

imposing gauge invariance under the EFT B -field gauge transformations (with parameter λ_μ^{MN}) and generalized diffeomorphisms (with parameter Λ^M). We can also write down the pullback of the external metric $T g_{\mu\nu} \partial_\alpha X^\mu \partial_\beta X^\nu$, which automatically respects both symmetries. It remains to consider the EFT “external” diffeomorphisms with parameter ξ^μ .

Remarkably, imposing an external diffeomorphism covariance requires an interplay between the kinetic and WZ pieces, thereby fixing all but one relative coefficient. (The last one is fixed later by “twisted self-duality.”) This interplay follows inescapably from the following piece in the transformation of A_μ^M ,

$$\delta_\xi A_\mu^M \supset \mathcal{M}^{MN} g_{\mu\nu} \partial_N \xi^\nu, \quad (11)$$

which must appear upon varying the WZ term. However, there is no way of generating this, as no other $\mathcal{M}$ -dependent terms appear in the variation of L_{WZ} . This suggests that we must be able to obtain it from the kinetic term. The calculation leads to the following anomalous variation, which must vanish:

$$\begin{aligned}-\frac{1}{2} \int d^2 \sigma g_{\mu\nu} \partial_K \xi^\mu \partial_\alpha X^\nu \mathcal{M}^{KM} \\ \times (T \sqrt{-\gamma} \gamma^{\alpha\beta} \mathcal{M}_{MN} D_\beta Y^K - q_{MN} \epsilon^{\alpha\beta} D_\beta Y^N).\end{aligned}\quad (12)$$

This can be compared with the variation of the action with respect to V_α^M ,

$$\begin{aligned}\delta S = -\frac{1}{2} \int d^2 \sigma \delta V_\alpha^M [T \sqrt{-\gamma} \gamma^{\alpha\beta} \mathcal{M}_{MN} D_\beta Y^N \\ - \epsilon^{\alpha\beta} q_{MN} (D_\beta Y^N - V_\beta^N)].\end{aligned}\quad (13)$$

Solving the section condition so that $\partial_i \neq 0$, $\partial_A = 0$, we know that only V_α^A appears. It turns out that the only nonzero components of the charge allowed by (10) are $q_{Ai} = q_{iA}$. Using this and the equations of motion for the V_α^A components, one can show group-by-group that (12) vanishes upon inserting the standard parametrizations for the generalized metric on the section. Alternatively, one can cancel (12) off shell by including a further transformation of V_α^M :

$$\bar{\delta}_\xi V_\alpha^M \supset -\frac{1}{T \sqrt{-\gamma}} \gamma_{\alpha\beta} \epsilon^{\beta\gamma} \mathcal{M}^{MP} q_{PQ} \mathcal{M}^{QK} \partial_K \xi^\mu g_{\mu\nu} \partial_\gamma X^\nu. \quad (14)$$

For this to work, some miraculous identities must hold involving the charge q_{MN} ; we need

$$\mathcal{M}^{MP} q_{PQ} \mathcal{M}^{QN} \partial_M \otimes \partial_N = 0, \quad (15)$$

$$\left(\delta_Q^K - \frac{1}{T^2} \mathcal{M}^{KM} q_{MN} \mathcal{M}^{NP} q_{PQ} \right) \partial_K = 0, \quad (16)$$

(where the latter can also be viewed as fixing the precise form of T). These follow from the charge constraint (10), as can be checked on a group by group basis.

The reader familiar with the doubled sigma model may find that these identities appear somewhat familiar. Let us explain. In fact, our action, (5) and (6), also describes the doubled sigma model in the formulation of [6,7,38] [the $D-1$ in the tension (7) corresponds to $O(D-1, D-1)$]. In this case, one has $q_{MN} = T_{F1} \eta_{MN}$, where T_{F1} is the tension of the fundamental string, and η_{MN} is the $O(D-1, D-1)$ structure. One has $Y_{PQ}^{MN} = \eta^{MN} \eta_{PQ}$, so the requirement (10) is identically satisfied—implying that there is always a doubled string. The identities (15) and (16) are just the statement that $S^2 = I$ for $S_N^M = \eta^{MN} \mathcal{M}_{NP}$ [saying that the generalized metric is an element of $O(D-1, D-1)$]. In that case, they hold without contractions with derivatives and imply the consistency of the “twisted self-duality” constraint $DY^M = \star S_N^M DY^N$, which in turn kills the anomalous variation (12) when V_α^M is on shell. In EFT, the identities only hold upon contractions with derivatives, but a directly analogous twisted self-duality constraint is true—that is, if we fix the last remaining relative coefficient in the exceptional sigma model Lagrangian. This is explained in [42].

We have just described the construction of the exceptional sigma model based on the local symmetries of EFT. It is curious to note how similar the procedure is to the usual method for obtaining the spacetime EFT action [43], where a collection of terms, which are separately invariant under generalized diffeomorphisms, have their relative coefficients determined by requiring invariance under the external diffeomorphisms, with intricate interplay of “gauge”- and “metric”-like terms. The details will be reported in [42], including verifying uniqueness of the resulting action. We will also discuss the inclusion of a Fradkin-Tseytlin term in which T as in (7) plays the role of a generalized dilaton.

We now outline how our action reproduces the type-II string and $D1$ -brane actions. The procedure is similar to that for the doubled sigma model [6,7,38]. On choosing a solution for the section condition as before, we impose the algebraic equation of motion for the nonzero components V_α^A . This determines $D_\alpha Y^A$ in terms of $D_\alpha Y^i$. In order to do so, we should solve (10) for the allowed nonzero components of the charge q_{MN} , finding in general that $q_{AB} = q_{ij} = 0$. Then, using the dictionary relating the EFT fields to components of the 10-dimensional supergravity fields, we find that the action reduces to that of the IIA $F1$ or the IIB (m, n) string, up to a single term involving the dual coordinates Y^A ,

$$-\frac{1}{2} \int d^2 \sigma \epsilon^{\alpha\beta} q_{Ai} \partial_\alpha Y^A \partial_\beta Y^i, \quad (17)$$

which is a total derivative. Something similar appears in the doubled sigma model, and it is cancelled by adding a

so-called topological term, which in fact ensures the quantum consistency of the model [6,7,44]. For $O(D, D)$, this involves an antisymmetric tensor Ω_{MN} , which can be interpreted as a symplectic term on the doubled space. Ordinarily this is not included in DFT, but it plays a central role in the related proposals of [45–47].

Let us briefly discuss the $E_{6(6)}$ EFT, for which dictionaries relating the EFT fields to supergravity ones are provided in [20,48]. The representation R_1 is the 27-dimensional fundamental, and the R_2 representation is its conjugate. There are two totally symmetric invariant tensors: d^{MNP} and d_{MNP} . A field $B_M \in R_2$ can be written with two upper indices as $B^{MN} = d^{MNP} B_P$, leading to the identification $q_{MN} = d_{MNP} q^P$. The Y tensor is $Y_{PQ}^{MN} = 10 d^{MNK} d_{PQK}$. In a IIB section, we split $E_{6(6)} \rightarrow \text{SL}(5) \times \text{SL}(2)$, in which $Y^M = (Y^i, Y_{ia}, Y^{[ij]}, Y_a)$, with $i, j = 1, \dots, 5$ and $a, b = 1, 2$. One then finds that the condition (10) kills all components of q^M except the $\text{SL}(2)$ doublet q_a . After some work, we find that the Lagrangian becomes that of a IIB (m, n) string,

$$T_{F1} \tau_{m,n} \sqrt{-\gamma} \gamma^{\alpha\beta} \hat{g}_{\hat{\mu}\hat{\nu}} \partial_\alpha X^{\hat{\mu}} \partial_\beta X^{\hat{\nu}} + \epsilon^{\alpha\beta} q_a \hat{C}_{\hat{\mu}\hat{\nu}}^a \partial_\alpha X^{\hat{\mu}} \partial_\beta X^{\hat{\nu}}, \quad (18)$$

where the $\text{SL}(2)$ doublet q_a is straightforwardly related to (m, n) through $q_a = \sqrt{10} T_{F1} (m, n)$, $\tau_{m,n} = \sqrt{e^{-2\Phi} n^2 + (m + C_{(0)} n)^2}$, $\hat{g}_{\hat{\mu}\hat{\nu}}$ is the 10-dimensional string frame metric, and $\hat{C}_{\hat{\mu}\hat{\nu}}^a$ is the doublet of 10-dimensional RR and NSNS two forms. The 10-dimensional coordinates are $X^{\hat{\mu}} = (X^\mu, Y^i)$. For $(m, n) = (1, 0)$, this is the $F1$ action, and for $(m, n) = (0, 1)$, we see the tension scales with g_s^{-1} as expected for the $D1$. The action for general (m, n) is related to the $F1$ action by an S duality transformation and to the usual $D1$ action by integrating out the BI vector [49]. It can also be obtained from the $\text{SL}(2)$ covariant formulation of [50,51], which can be viewed as a precursor to our exceptional sigma model.

Similarly, one can obtain the IIA fundamental string by working with a IIA solution of the section condition. If viewed as a reduction of an M theory section, for which $Y^M = (Y^i, Y_{[ij]}, Y^{\bar{i}})$, where i and $\bar{i}$ are 6-dimensional indices and $\partial_i \neq 0$, then one obtains the IIA section whenever there is no dependence on one of the M theory coordinates, say $\partial_{\bar{1}} \neq 0$. In this case, the single nonzero charge allowed by (10) is $q^{\bar{1}} \propto T_{F1}$.

Therefore, the unique exceptional sigma model action can be reduced to an action for the standard 1-branes of type-IIA and type-IIB string theory, on solving the section condition for these cases and eliminating the gauge field V_α^M . This requires the constraint (10) on the charges q_{MN} appearing in the WZ coupling of the two form. For the 10-dimensional IIA and IIB sections, the number of

solutions of this constraint are 1 and 2, respectively, leading inevitably to the IIA fundamental string and IIB $F1/D1$ bound state [52].

One could also reduce the action below 10 dimensions. The obstacles to gauge invariance or covariance all vanish when $\partial_M = 0$, in which case q_{MN} is unconstrained by (10). Here, the natural conjecture is that the exceptional sigma model describes the $E_{D(D)}$ string multiplet in $11 - D$ dimensions, obtained by toroidal reduction. For instance, the $SL(5)$ exceptional sigma model Lagrangian on a background with $\partial_M = 0$ should describe the string quintuplet in seven dimensions, consisting of the four $M2$ branes wrapped around a single compactified dimension, along with the $M5$ branes wrapping all four compactified dimensions.

The methods we used to construct the action can be systematically applied to study branes in EFT. One application of the doubled sigma model is to define and study strings in T -fold backgrounds [6], suggesting the exceptional sigma model would be relevant for U folds. A simple example of a U fold in this formulation would simply be a torus bundle over the external manifold M —in the E_6 case, we would have a 5-dimensional external manifold that is the base for a T^{27} -bundle, patched by E_6 transformations. For genuine U folds, q^M will change from patch to patch.

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