Erratum: Analytic Solutions to Coherent Control of the Dirac Equation [Phys. Rev. Lett. 119, 173203 (2017)]

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The following typos are present in the published Letter: The phase factors $e^{-ie\sigma_3}$ are missing on the right-hand side of Eqs. (6) and (9) in the Letter. The corrected Eq. (6) reads

$$\Psi = e^{-(eB_0/4\hbar)[(x-r_0 \cos \omega t)^2 + (y-r_0 \sin \omega t)^2]} B(\mathbf{u}(t)) e^{-imc^2 t' \sigma_3 / \hbar}, \quad (1)$$

with $u^0(t) = 1/\sqrt{1 - [\dot{\mathbf{r}}(t)/c]^2}$, $\mathbf{u}(t) = u^0(t)\dot{\mathbf{r}}(t)/c$, $ct' = u^0(t)ct - \mathbf{u}(t) \cdot \mathbf{r} = u^0(t)ct - u^1(t)x - u^2(t)y$; whereas, the corrected Eq. (9) is

$$\Psi = \frac{1}{\sqrt{u^0(t)}} e^{-\{eB_0[x^2 + (y-Y(t))^2]/4\hbar\}} B(\mathbf{u}(t)) e^{-imc^2 t' \sigma_3 / \hbar}. \quad (2)$$

The proper velocity $u(t) = (u^0(t), \mathbf{u}(t))$ and the trajectory $\mathbf{r}(t)$ are parametrized by the time t , while $\mathbf{r} = (x, y, 0)$.

We also would like to provide an additional verification of the reported solutions by a direct substitution into the Dirac equation. The expanded Eq. (6) of the Letter, defining the spinor for a dispersionless wave packet following a circular trajectory, reads

$$\Psi = \sqrt{\rho} \begin{pmatrix} \frac{\sqrt{u^0+1}}{\sqrt{2}} & \frac{r\omega[-i\cos(t\omega) - \sin(t\omega)]u^0}{\sqrt{2}c\sqrt{u^0+1}} \\ \frac{ir\omega[\cos(t\omega) + i\sin(t\omega)]u^0}{\sqrt{2}c\sqrt{u^0+1}} & \frac{\sqrt{u^0+1}}{\sqrt{2}} \end{pmatrix} e^{-ie\sigma_3}, \quad (3)$$

$$\sqrt{\rho} = \exp\left(-\frac{eB_0[-2r_0x\cos(t\omega) - 2r_0y\sin(t\omega) + r_0^2 + x^2 + y^2]}{4\hbar}\right), \quad (4)$$

$$\epsilon = \frac{mu^0[c^2t + r_0x\omega\sin(t\omega) - r_0y\omega\cos(t\omega)]}{\hbar}, \quad (5)$$

where r_0 is the radius of the electron's circular orbit. This state is equivalent to the following column spinor in the Dirac representation:

$$\psi = \sqrt{\rho} \begin{pmatrix} \frac{\sqrt{u^0+1}}{\sqrt{2}} \\ 0 \\ 0 \\ \frac{ir\omega u^0[\cos(t\omega) + i\sin(t\omega)]}{\sqrt{2}c\sqrt{u^0+1}} \end{pmatrix} e^{-i\epsilon}. \quad (6)$$

In this case, the inverted vector potential [Eq. (2) of the Letter] is

$$\begin{aligned}
eA^0 &= \frac{\omega u^0 \{r_0(eB_0 + 2m\omega)[x \cos(t\omega) + y \sin(t\omega)] - eB_0 r_0^2 + \hbar\}}{2c} - \frac{\omega \hbar}{2c}, \\
eA^1 &= \frac{u^0}{4c^2} [-eB_0 r_0^2 x \omega^2 \sin(2t\omega) + 2r_0 \sin(t\omega)(c^2 eB_0 - \omega^2 \hbar) + eB_0 r_0^2 y \omega^2 \cos(2t\omega) + eB_0 y(r_0^2 \omega^2 - 2c^2)], \\
eA^2 &= \frac{u^0}{4c^2} [eB_0 r_0^2 x \omega^2 \cos(2t\omega) + 2r_0 \cos(t\omega)(\omega^2 \hbar - c^2 eB_0) + eB_0 r_0^2 y \omega^2 \sin(2t\omega) + eB_0 x(2c^2 - r_0^2 \omega^2)], \\
eA^3 &= 0.
\end{aligned}$$

It is now straightforward to show that the spinor Eq. (6) satisfies the Dirac equation

$$i\hbar\gamma^0 \frac{\partial\psi}{\partial t} + i\hbar c\gamma^1 \frac{\partial\psi}{\partial x} + i\hbar c\gamma^2 \frac{\partial\psi}{\partial y} + i\hbar c\gamma^3 \frac{\partial\psi}{\partial z} - c(eA^0\gamma_0 + eA^1\gamma_1 + eA^2\gamma_2 + eA^3\gamma_3)\psi - mc^2\psi = 0. \quad (7)$$

This derivation and verification can be found in the *Mathematica* notebook provided in the Supplemental Material of the Letter.

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