Drag on pairs of square section obstacles in free-surface flows

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(Received 21 August 2017; revised manuscript received 12 September 2018; published 18 December 2018)

The drag on pairs of square obstacles (of side D) in open channel flow is measured in experiments using a laboratory flume. The depth is uniform across the obstacles and the conditions in the flume are subcritical, with Froude number Fr < 0.59, Reynolds number Re = 4800 to 21900, and turbulence intensity $I_T \sim 8$ to 10%. The drag coefficient for an isolated square obstacle is found to be $C_D = 2.11$, in agreement with previous studies, and independent of Re (for the range covered here). The root-mean-square variation in the drag coefficient for the single obstacle decreased monotonically with Re, defined in terms of hydraulic radius, and approached 0.241 at high Re in agreement with previous research. For two obstacles, standard tandem and side-by-side arrangements are studied first, followed by a full range of relative positions covering $-5 \leq s_x/D \leq 20$ in the downstream direction and $0 \le s_y/D \le 7.0$ in the cross-stream direction. The lowest drag coefficients are observed when the downstream obstacle is shielded directly behind the upstream obstacle (tandem arrangement) when negative drag coefficients are found. The largest drag coefficients are observed for nearly side-by-side arrangements, with the peak values found to be for the slightly upstream obstacle of a pair $(s_x/D = -1)$. The blockage ratio (D/B), the relative size of the obstacle compared to the channel width) is found to be an important factor. For D/B = 12.7% the largest drag coefficient is $C_D = 3.82$, while for D/B = 6.3% the largest value is $C_D = 2.85$. For tandem obstacles, the drag on one obstacle can largely be accounted for by the change in flow speed induced by the other obstacle, except at small separations $(|s_x/D| < 3)$. The results will be useful in any applications where the force on multiple obstacles is required, such as the design of marine or riverine structures, or flood flows past buildings and vegetation.

DOI: 10.1103/PhysRevFluids.3.123802

I. INTRODUCTION

Bluff obstacles are found in a range of natural and engineered flows. For example, square cross-section obstacles are found as supports for river and coastal structures, buildings experiencing floods, and in heat exchangers [1-4]. Emergent aquatic vegetation is also often idealized as arrays of rigid obstacles, typically with a circular cross section (e.g., [5-7]), though it is argued here that square obstacles can be a more appropriate model for real vegetation under typical field conditions

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in terms of the magnitude of fluid drag and its relatively small variation with Reynolds number. We will focus here on free surface subcritical flow, with the Froude number Fr < 1, and Froude number given by

$$Fr = \frac{U}{\sqrt{gH}},\tag{1}$$

where U is the mean flow velocity, g is the acceleration due to gravity, and H is the depth of the flow. Of particular interest is the drag acting on obstacles in the flow (or equivalently the force acting on the flow), as this is important for analyzing the performance of structures in a flood, or for calculating the effect of obstacles on the flow itself. The drag force F_D is commonly expressed as

$$F_D = C_D F_P \quad \text{with} \ F_P = \frac{1}{2} \rho U^2 DH, \tag{2}$$

where C_D is the drag coefficient, ρ is the fluid density, and D is the obstacle width facing the flow direction. Similarly, the standard deviation of the drag force, $F_{D \text{ rms}}$, is a useful measure of temporally fluctuating drag forces, which can be quantified in terms of the root-mean-square drag coefficient $C_{D \text{ rms}}$:

$$F_{D\,\mathrm{rms}} = C_{D\,\mathrm{rms}} F_P,\tag{3}$$

For developed normal flow in channels the balance between drag and gravity can be expressed in terms of Manning's formula,

$$U = R_H^{2/3} S^{1/2} n^{-1}, (4)$$

where R_H is the hydraulic radius, S is the channel slope, and n is Manning's coefficient [8].

Much previous work has concentrated on flow past circular cylinders. In this case drag coefficients are found to be of order 1, and vary with the Reynolds number Re given by

$$Re = \frac{UD}{v},$$
(5)

where v is the kinematic viscosity. For a smooth cylinder, White [9] gives

$$C_D = 1 + 10 \text{Re}^{-2/3}$$
 for $1 < \text{Re} < O(10^5)$. (6)

At higher Re, delayed flow separation reduces C_D , which is sensitive to Re, surface roughness k_s , and turbulence intensity I_T . The drag coefficient is also influenced by wall constraints. If the blockage ratio D/B is large, sidewalls prevent the flow passing the cylinder from expanding laterally, causing it to accelerate and resulting in a compensating fall in pressure. Laboratory measurements on isolated cylinders are often corrected to estimate C_D in an infinitely wide channel C_{Dc} . A variety of methods exist in the literature (e.g., [11–13]) with their efficacy dependent on the Reynolds number regime [10].

Maskell [11] developed a blockage correction procedure, for both bluff and streamlined bodies, based on an approximate momentum balance outside the wake and two empirical auxiliary relations. The final form of the correction is

$$\frac{C_D}{C_{Dc}} = 1 + \varepsilon \frac{D}{B},\tag{7}$$

where ε is an experimentally determined coefficient, equal to 0.96 for flat plates. Awbi [12] proposed a modification to Maskell's theory for obstacles with a rectangular cross-section where ε is a function of the ratio of the cross-sectional dimensions of the obstacle. For square cross sections the reduction is only 3%, suggesting Maskell's original value is reasonable. Anthoine *et al.* [10] showed that Maskell's theory is valid for circular cylinders (at subcritical Reynolds numbers, i.e., $\text{Re} < 3 \times 10^5$) even for large blockages (tested for $D/B \leq 17\%$). Ranga Raju and Singh [13] proposed an alternative correction procedure for square cross-section obstacles, as well as triangular prisms and circular disks, which was validated for $D/B \leq 24\%$:

$$C_{Dc} = C_D \left(1 - \frac{D}{B} \right)^{\alpha},\tag{8}$$

where α is a function of the ratio of obstacle cross-sectional dimensions, with $\alpha \approx 1.15$ for square obstacles. These correction procedures are therefore reasonable to employ in the present study, given that they have been successfully applied to various bluff body shapes, including square obstacles at high blockage ratios.

For circular cylinder pairs, C_D is a function of s_x and s_y , which are the separation between obstacle centers in the streamwise and cross-stream directions respectively. For tandem arrangements, where one cylinder is aligned behind the other in the direction of the mean flow ($s_y = 0$), the drag on the downstream cylinder is substantially reduced due to decreased velocities and increased turbulence in the upstream cylinder wake and thus in flow approaching the downstream cylinder. This effect is known as "sheltering" [14]. Liu *et al.* [15] found that, for low $I_T(0.7\%)$, flow around smooth cylinders at Re = 80000, C_D (for the downstream cylinder) is negative at small separations and suddenly increases at a separation of $s_x = 3.5D$. This behavior disappeared as the surface was roughened or turbulence was introduced into the inflow. For side-by-side arrangements ($s_x = 0$), if the cross-stream separation is sufficiently large ($s_v > 2D$), the flow pattern is symmetrical relative to the axis passing through the middle of the gap between cylinders [16]. However, at small separations the gap flow is biased to one side, switching between the two sides at irregular intervals. One cylinder is subject to a narrow wake (low drag) and the other to a wide wake (high drag). Zdravkovich and Pridden [16] found that the sum of the high and low drag is always less than twice that on an isolated cylinder, hence interference reduces C_D at close spacing. They also found that for a given separation the minimum C_D often occurs when the obstacles are offset in both directions $(s_x \neq 0 \text{ and } s_y \neq 0)$, known as staggered arrangements, because the gap flow between cylinders shifts the stagnation point of the downstream cylinder (by up to 30°), exposing a large area on the front of the cylinder to low pressure.

In arrays of circular cylinders, C_D is dependent on Re and solid volume fraction λ (the ratio of the total solid volume to the total volume of solid and fluid). Tanino and Nepf [6] found that in random arrays C_D decreases with increasing Re (where Re is based on the mean velocity within the array, taking account of the average reduction in cross-sectional area) and increases with increasing λ for 25 < Re < 685 and 0.091 < λ < 0.35. At higher Re the opposite trend (C_D decreasing with increasing λ) has been reported, for example in the numerical model of Nepf [5] where C_D of individual cylinders is assigned based on the position of the closest upstream cylinder (8 × 10⁻⁴ < λ < 0.24, random and staggered), validated with laboratory results at Re > 1000. Cheng and Nguyen [7] introduced the concept of vegetation hydraulic radius, r_v (the ratio of the volume of water to the combined frontal area of all cylinders), to collapse laboratory data to a function of a single variable. They show that C_D decreases monotonically with Re (defined in terms of mean array velocity and r_v) and is independent of λ (with no significant difference between random and staggered arrays).

Vegetation in environmental flows is not so well represented by the idealized models or laboratory experiments. Values of I_T are generally higher, and vegetation is often not circular at all in cross section ("sedges have edges"), and certainly not perfectly circular or smooth. As a result, drag coefficients for real vegetation are typically $C_D \sim 2$ and there is less variation with Re. James *et al.* [17] found in a flume study that isolated stems of *Phragmites australis* (common reed) have higher drag coefficients than circular cylinders, with square obstacles forming a better model. They also found C_D increased further if foliage was present. In a further flume study, Wunder *et al.* [18] found that for leafy willows at high velocities C_D exceeds that of circular cylinders. Thus square obstacles, with fixed separation points and less dependence on Re, can form better models of real vegetation in terms of the magnitude of fluid drag.

Flows past square cross-section obstacles have been less intensively studied, limited mostly to single square obstacles, or special arrangements of pairs of obstacles (tandem or side by side). The value of C_D for a single obstacle has been measured by various authors with typical values of the order of 2 (see Table I). Lyn *et al.* [19] used laser-Doppler velocimetry to measure wake recovery in a closed water channel at Re = 21400 and found $C_D = 2.1$ from the integral of mean velocity profiles (and found the length of the recirculation region was about 1.4D). Yen and Yang [3] calculated C_D

Author	Re	$I_T(\%)$	D/B (%)	H/D	C _D
Norberg [20]	5000	0.06	1.1	62.5	2.21
Yen and Yang [3]	6300	0.3	4.0	25.0	1.86
Norberg [20]	13000	0.06	1.1	62.5	2.15
Yen and Liu [4]	21000	0.4	4.0	25.0	2.06
Lyn <i>et al.</i> [19]	21400	2	7.1	9.8	2.1
Yen and Yang [3]	24000	0.3	4.0	25.0	1.96
Yen and Yang [3]	36000	0.3	4.0	25.0	2.02
Lee [21]	176000	0.5	3.6	9.2	2.04
Lee [21]	176000	4.4	3.6	9.2	1.98
Lee [21]	176000	6.5	3.6	9.2	1.94
Lee [21]	176000	8	3.6	9.2	1.81
Lee [21]	176000	12.5	3.6	9.2	1.53
British Standards Institution [22]	N/A	N/A	N/A	N/A	2.1

TABLE I. Drag coefficients for square obstacles from previous studies.

from pressure transducer measurements in an open-loop wind tunnel and found C_D increased with Re from 1.86 at Re = 6300 to 2.02 at Re = 36000. Lee [21] investigated the influence of I_T on C_D in a low speed wind tunnel. The turbulence level was varied by using four different square mesh grids upstream of the obstacle and C_D was obtained from the integrated mean pressure distribution measured using 18 and 6 pressure taps on the front and rear faces respectively. A reduction in C_D was observed with an increase in I_T . This was ascribed to the fact that additional turbulence thickens shear layers which are then deflected by the obstacle's rear corners, causing the vortices to form further downstream. The measured drag coefficients are in the range obtained by other researchers at lower I_T , for I_T up to 6.5%, but then fall below this range as I_T increases to 8%, reaching the lowest value of 1.53 at the highest turbulence intensity of 12.5%. In a channel of finite width *B*, increases in blockage ratio D/B would be expected to contribute to an increase in the drag.

For square-section obstacles, an important parameter is the angle of attack θ , i.e. the angle between the mean flow direction and the upstream face of the obstacle [3,20,21]. Yen and Yang [3] investigated the influence of θ on C_D (defined in terms of the projected area which varies with θ) and flow patterns visualized using particle image velocimetry (PIV). Three flow modes were identified. In the leading-edge separation mode ($\theta < 9^\circ$) stagnation occurs at the midsection of the front face and the flow divides into two streams. Flow separates near the front vertices and two vortices form in the wake. A local maximum in C_D occurs at 0° due to the high pressure near the stagnation point. As θ increases, a vortex shed from the front corner intermittently touches the rear corner, reducing C_D . In the separation bubble mode ($9^\circ < \theta < 27^\circ$) flow separates at the leading edges, reattaches on the surface on one side, and separates again at the rear corner with the minimum C_D occurring at 12° (1.35 at Re = 6300 and 1.50 at Re = 36000). In the attached mode ($27^\circ < \theta < 45^\circ$) streamlines near the front vertices smoothly follow the obstacle shape and separation does not occur until the rear vertices. Increases in θ gradually broaden the wake, increasing C_D towards the maximum value at 45°. The present study is restricted to $\theta = 0^\circ$.

For side-by-side square obstacles Yen and Liu [4] (using pressure transducer and velocity measurements in an open-loop wind tunnel) identified three distinct modes of flow behavior. When the obstacles are sufficiently close $(1 \le s_y/D \le 1.1)$ "single mode" flow behavior is similar to that around an isolated obstacle, but the wake is wider so C_D is higher (maximum $C_D = 2.24$). As s_y/D increases ("gap-flow mode" $1.1 \le s_y/D \le 6.5$) jet flow develops between the obstacles, with no flow separation on the interfacial surfaces, so C_D is reduced (minimum $C_D = 1.68$). As s_y/D increases further ("couple vortex-shedding mode" $6.5 \le s_y/D \le 13$), interactions are minimal and C_D is almost independent of s_y/D ($C_D = 2.08$). Yen and Liu [4] also measured $C_{D \text{rms}}$ on an isolated obstacle and the average root-mean-square drag coefficient for each obstacle in a pair.

The maximum value of 0.263 occurred in the gap flow mode at $s_y/D = 1.5$, and $C_{D \text{ rms}}$ in the couple vortex-shedding mode was 0.241, close to isolated obstacle value of 0.242. For square obstacles in tandem, Yen et al. [2] identified (in experiments in a vertical water tank) three distinct modes of flow behavior depending on Re and s_x/D . When the obstacles are sufficiently close, flow patterns are similar to that of an isolated obstacle ("single mode"). As the separation increases, flow reattaches on the downstream obstacle, resulting in a substantial reduction in downstream obstacle C_D ("reattached mode"). At a separation of 3D, C_D tends to 0.5 for Re > 670. At large separations similar structures form in the wakes of the two obstacles and downstream C_D increases towards the isolated value ("binary mode"). The spacing at which the transition between the different flow modes occurs was found to decrease with an increase in Re, and the single mode was only observable for Re up to the order of 600. A number of researchers have identified a sudden change in C_D which corresponds to the transition between the reattached and binary modes (e.g., [23,24]). Kim et al. [24] used PIV to investigate the mean velocity and turbulent statistics of the flow field surrounding tandem pairs of square obstacles for $1.5 \leq s_x/D \leq 11.0$ and Re = 5300 and 16000. A discontinuous jump occurred simultaneously in C_D for both obstacles at the critical spacing of 3.5D, which was independent of Re within the range studied, and the flow patterns at $s_x/D \leq 3.0$ were drastically different from those at $s_x/D \ge 3.5$.

A number of researchers have also employed numerical models to determine drag on square obstacles (e.g., [25–27]). Rodi [25] conducted a comprehensive study comparing various Reynoldsaveraged Navier-Stokes (RANS) and large eddy simulation (LES) models for the test case with a single obstacle presented by Lyn et al. [19] ($C_D = 2.1$). For the RANS simulations, C_D was well predicted by the k-epsilon model with the Kato-Launder modification and a two-layer approach to resolving the near-wall region (2.0) and by a Reynolds-stress model with wall functions (2.15). LES models generally gave a better simulation of the details of the flow at the expense of a large increase in computation time, but there were large differences between individual calculations, particularly in terms of C_D which varied between 2.02 and 2.77, which the author noted were difficult to explain. The literature concerning simulations of the flow past obstacle pairs is far more sparse, but numerical models have been employed to investigate the flow around tandem pairs albeit at low Reynolds numbers (using 2D incompressible laminar flow models) (e.g., [26,27]). Sohankar and Etminan [26] considered obstacles separated by $s_x/D = 6$ at $1 \le \text{Re} \le 200$ and D/B = 5%, using both steady and unsteady models. The results demonstrate that the flow becomes unsteady for Re ≥ 40 , and for Re = 50 pressure drag contributes to 80% of the total drag. For Re \ge 50 upstream C_D is relatively constant, whereas the downstream obstacle C_D increases gradually due to the change of flow pattern between the obstacles. For Re \geq 100, the downstream obstacle C_D also becomes approximately constant, with C_D equal to 1.53 and 1.30 for the upstream and downstream obstacles respectively at Re = 100. Lankadasu and Vengadesan [27] studied the flow around tandem obstacles subject to both uniform and planar shear inflow, for $2 \le s_x/D \le 7$ and Re = 100. The results showed that, with an increasing shear gradient, the flow became unsteady at lower s_x/D . When $s_x/D > 4$, there was a clearly defined wake behind the upstream obstacle, but below this threshold there was no definite wake, despite the fact that the flow was unsteady for some combinations of s_x/D and the shear gradient. The downstream obstacle C_D was consistently lower than that of the upstream obstacle and was far lower than that of the single obstacle in uniform flow. Negative values were obtained at lower s_x/D for uniform flow or low shear gradients, but C_D became positive as the shear gradient increased. The root-mean-square drag coefficient was consistently higher for the downstream obstacle than the upstream one. With increasing s_x/D , $C_{D \text{ rms}}$ initially increased and then began to decrease for the upstream obstacle, whereas it increased monotonically for the downstream one. Accurate estimation of C_D for pairs of obstacles at high Reynolds numbers remains challenging.

Here we will use laboratory flume experiments to measure the drag on a pair of square obstacles in a comprehensive set of arrangements covering the range $-5 \le s_x/D \le 20$ and $0 \le s_y/D \le$ 3.5 (or 7.0 for D/B = 6.3%). Previous work only considered special cases with perpendicular alignment between the obstacles (tandem and side-by-side arrangements), whereas we extend this to the general case including other, nonperpendicular relative positions. In Sec. II we describe the



FIG. 1. Sketch of the flume used for the experiments.

experiments and give measurements of the flume flow without obstacles. Next, in Sec. III, we present the results including the main features of the velocity field with obstacles present and comprehensive results for the drag on pairs of obstacles. Finally, in Sec. IV, we discuss the results and present our main conclusions.

II. EXPERIMENTS

The experiments were conducted in a recirculating tilting flume, with main working section of length 5 m, and width and height 300 mm (Fig. 1). The cross section of the upstream pre-flume section contracts just upstream of the flume inlet, but there are no other means of turbulence suppression. Downstream of the flume test section there is an outlet with an adjustable overflow weir, which is controlled by a wheel. The maximum flow rate is approximately $Q = 0.03 \text{ m}^3 \text{s}^{-1}$.

The flow rates were adjusted after obstacles were placed in the water to give subcritical flow, with Fr < 0.59 in all cases, and the slope and weir height were adjusted to obtain uniform flow in the test section. To calibrate the slope, the flume was filled with water and both ends of the test section were sealed. From repeated measurements of the depth of still water, an accuracy of ± 0.3 mm was achievable. The uncertainty in the slope S is therefore $\pm (2 \times 0.3/5000) =$ $\pm 1.2 \times 10^{-4}$. Velocities were measured using a Nortek Vectrino ADV operating at 200 Hz. The accuracy of velocity measurements is $\pm 0.5\%$ of the measured value ± 0.001 ms⁻¹ and the Doppler uncertainty is estimated as 1% of the user-specified nominal velocity range [28]. This range was adjusted to minimize uncertainty while capturing the entire range of velocities. It was typically set to $\pm 1 \text{ ms}^{-1}$ but up to $\pm 4 \text{ ms}^{-1}$ was necessary in the wake of an obstacle. The maximum uncertainty is therefore $\pm (0.041 + 0.005U) \,\mathrm{ms^{-1}}$. The ADV probe was fully submerged when measuring velocities surrounding obstacles. For the measurement of velocities in the channel with no obstacles present, the ADV probe was fully submerged except at the maximum height where only the transmitter and bottom two (of four) receivers were submerged. This is acceptable for the measurement of the streamwise component (as well as the lateral component but not the vertical one). See Robertson [29] for further experimental details.

Mean streamwise velocity profiles, nondimensionalized by the bulk mean speed U = Q/(BH), in the absence of obstacles are shown in Fig. 2. The velocity profiles are consistent with the logarithmic law near the base of the flume. Further from the base, in the free stream, the velocity remains constant with respect to depth. This is true above $\sim H/2$ or $\sim 8 \text{ cm}$ from the base, so velocities either side of obstacles were measured above this height. The increase in free-stream velocity with downstream distance is relatively small, especially when compared to that downstream of an obstacle (see below). The velocity profile in the cross-stream direction is reasonably uniform. The velocity decreases slightly towards the center of the flume and towards the walls. The turbulence intensity varies with position but is of the order of 10%. Manning's coefficient was found to be n =



FIG. 2. Profiles of the normalized, temporally averaged, streamwise component of velocity in the flume with no obstacles present, at different downstream distances from the start of the main working section x for $Q = 0.0166 \text{ m}^3 \text{s}^{-1}$, H = 158 mm, and S = 0.0007. (a) In the vertical direction at the center of the channel and (b) in the cross-stream direction at a height of H/3 below the free surface.

0.011 m^{-1/3}s. This was computed from linear regression of U against $R_H^{2/3}S^{1/2}$ at S = 0.0013 and 0.0022 over a range of conditions where $0.0142 \le Q \le 0.0275 \text{ m}^3 \text{s}^{-1}$ and $0.093 \le H \le 0.196 \text{ m}$.

Two sizes of obstacle were used with D = 38 mm and 19 mm (D/B = 12.7% and 6.3%) always placed square on to the flow. The obstacles are made from aluminum which has a surface roughness of $ks = 1 - 2 \mu m$ [30]. Consulting a Moody chart, the obstacles can therefore be considered hydraulically smooth since the upper estimate of ks/D is of the order of 10^{-4} and the maximum Re (based on obstacle width) is of the order of 2×10^4 .

Forces on the obstacles were measured by a strain gauge recording data at 200 Hz, calibrated before and after each set of experiments by applying known loads. The difference between an applied load and that computed from calibration constants is negligible. The best fit consistently gave a correlation coefficient $R^2 > 0.999$. The uncertainty of the strain gauge measurements was estimated as $\pm (0.02 + 0.01 F_D)N$. This accounts for a potential shift in calibration constants over the course of an experiment as well as the difference between the mean drag determined over the measurement period of 120 s and over a much longer period. When the calibration was checked at the start of each experiment the difference between known and measured forces was less than 2% or the gauge was recalibrated. However, this may underestimate the uncertainty due to calibration drift at low forces as, for the check, applied loads were greater than or equal to 0.981 N, but in the experiments $F_D \ge 0.2$ N, so the uncertainty was estimated as 2% of 0.981 N. Measurements were discarded from further analysis if there was a difference of over 2% between forces computed with the calibration constants obtained between successive calibrations. In preliminary tests, the mean value of C_D was calculated over increasing periods of time, *t*. This mean value remained constant to within 1% for t > 120 s, so this period was used for calculating the mean value of C_D .

A sketch showing the arrangement with two obstacles is shown in Fig. 3. One obstacle was kept centrally positioned, mounted on the strain gauge, while a second was placed in different positions to vary the relative positions, with s_x and s_y being the position of the gauged obstacle relative to the second obstacle. Negative values of s_x indicate that drag was measured on the upstream obstacle. Keeping the obstacle attached to the strain gauge stationary minimized the uncertainty and recalibration that would have been required if this obstacle had been moved. The upstream distance from the obstacles to the start of the working section was always greater than 61*D* while



FIG. 3. Sketch showing the arrangement of obstacles (here with D/B = 12.7%). The darker obstacle is mounted on a strain gauge in the center of the channel, while the lighter obstacle is moved to different positions. A negative value of s_x indicates that the drag is measured on the upstream obstacle.

the distance downstream of the obstacles to the end of the working section was always greater than 46*D*. There is a small gap (<2 mm) between the obstacle attached to the strain gauge and the base of the flume. Other obstacles are in contact with the channel floor. For single obstacle experiments, only the gauged obstacle is present.

Some preliminary experiments were conducted with a single obstacle. This was followed by experiments with special arrangements of two obstacles ("tandem pair," $s_y = 0$, and "side-by-side pair," $s_x = 0$) and finally experiments with a range of relative positions ("staggered"). The conditions for these experiments are summarized in Table II.

A nominal flow rate of $0.0167 \text{ m}^3 \text{s}^{-1}$ was used for each of the experiments conducted under fixed flow conditions (isolated obstacle velocity, tandem pair velocity, and staggered pair drag measurements), with uniform depth throughout the test section, and C_D was computed as the ratio of F_D and F_P . When measuring the drag over a range of conditions (isolated obstacle and tandem pair drag measurements) Q, H, and S were all varied. The depth was not always uniform throughout the entire test section but there was no change in depth across the obstacle(s). For isolated obstacles $0.0064 \leq Q \leq 0.0302 \text{ m}^3 \text{s}^{-1}$ and for tandem obstacles $0.0087 \leq Q \leq 0.0239 \text{ m}^3 \text{s}^{-1}$. The mean drag coefficient was estimated from linear regression of F_D vs F_P . A minimum of five data points were analyzed and $R^2 > 0.98$ for all cases.

TABLE II. Summary of main experimental conditions for the different types of experiments.

Obstacle(s)	Measurements	Fig(s).	I_T (%)	S	H/D	Fr	Re
Isolated obstacle	Velocity	4 and 9	≈ 10	0.0022	4.9	0.22	11000
Isolated obstacle	Drag	5 and 6		0.0007-0.0022	1.4-6.0	0.21-0.59	9500-21900
Tandem pair	Velocity	7	≈ 8	0.0007	3.3	0.38	16000
Tandem pair	Drag	8		0.0007-0.0022	2.7-6.3	0.08-0.33	4800-16500
Staggered pair	Drag	8-12	≈ 8	0.0007	3.3	0.38	16000
Staggered pair ^a	Drag	8, 10–12	≈ 8	0.0007	7.7	0.33	7500

^a All have D/B = 12.7% except the final staggered pair set, which has D/B = 6.3%.



FIG. 4. Normalized, temporally averaged, streamwise component of velocity on the channel centerline at a depth of H/3 below the free surface for flow past a single obstacle with D/B = 12.7% and Re = 11000. The results from Lyn *et al.* [19] are also shown.

III. RESULTS AND ANALYSIS

A. Isolated obstacles

The mean streamwise velocities on the channel centerline at a depth of H/3 below the free surface, nondimensionalized by the velocity 10D upstream (1.0% higher than the bulk velocity), are shown in Fig. 4 for the preliminary experiments with a single obstacle. The turbulence intensity 10D upstream is 10%. Velocities are shown both upstream and downstream of the obstacle, and compared with the results from Lyn et al. [19]. In the present experiments, the velocity decreases rapidly upstream of the obstacle and is negative until 2.0D downstream. The flow recovers to 90% of its initial value by approximately 14D downstream of the obstacle. In the results of Lyn *et al.*, the wake recovery is initially more rapid with the transition to positive velocity at 1.4D. However, the wake does not appear to fully recover in the Lyn *et al.* experiments, at least within the zone measured (to 8D), and appears to plateau at $\sim 60\%$ of the free-stream value. It is possible that the wake is affected by the downstream boundary conditions, while wake recovery is also likely to be affected by the blockage ratio and turbulence intensity (in the present study D/B = 12.7% and $I_T = 10\%$, whereas for Lyn *et al.* D/B = 7.1% and $I_T = 2\%$). In the present study, the wake diffuses as it moves downstream and meets the tank boundary layer. The more complete wake recovery in the present experiments gives confidence that our results are not strongly influenced by downstream boundary conditions.

The drag force on the single obstacle with D/B = 12.7% was measured for a range of flow conditions, varying the flow depth and Reynolds number. The measured force F_D is plotted against F_P in Fig. 5, and the measurement uncertainty is also shown. The best fit line gives $C_D = 2.11 \pm 0.07$ at the 95% confidence level assuming the uncertainty of the measurements is negligible. This assumption seems reasonable as the maximum $F_D \sim 2$ N and the uncertainty in F_D is 0.02 \pm 0.01 F_D , giving a maximum value of 0.04 N (1.7%). The typical measurement uncertainty is therefore much lower than the ± 0.07 (3.3%) estimate of the maximum difference between $C_D = 2.11$ and the true mean. This result, together with the results for two comparable cases with D/B = 12.7% and 6.3%, are given in Table III. These two cases represent the flow conditions used



FIG. 5. Measured drag force on a single obstacle plotted against the expected scale for the force. The diagonal line marks the linear regression ($C_D \approx 2.1$). The vertical line in the top left corner represents the maximum measurement uncertainty in F_D .

for staggered pair drag measurements and C_D was determined using the same method, i.e., the ratio of F_D and F_P from a single measurement. The turbulence intensity in Table III is taken as the value which was measured at the same Q and H, with two obstacles present (see below), at 1/3 depth from the surface and 10D upstream of the first obstacle. This is assumed to be representative of the upstream I_T throughout the free stream.

The results from the present study for the drag coefficient on a single square obstacle are similar to those found by earlier authors (compare Tables III and I). One may anticipate that as the obstacles in this experiment are emergent the pressure drag is accompanied by a resistance due to the formation of surface waves, and that C_D would therefore be higher in this experiment than the values measured by other researchers. However, this is not generally true. The comparatively high D/B in this study would also be expected to contribute to a higher C_D . These effects may be offset by the higher I_T in this experiment. There are several other factors which may potentially account for variations in C_D including Re and H/D as well as differences in the experimental setups and measurement techniques. However, from Tables I and III, the value of drag coefficient $C_D \approx 2.1$ appears to be relatively insensitive to these differences, when compared to the behavior of C_D for circular cylinders. For circular cylinders, the design values of C_D given for structures in EUROCODE 1 [22], drop from 1.2 to 0.4 between Re = 1.81×10^5 and 3.98×10^5 , and a similar trend is given by Simiu and Scanlan [31]. Although this occurs slightly outside the range of Re in Tables I and III (up to 1.76×10^5), this is due to separation points moving further downstream, which therefore

und realized according to the methods of Masken [11] and realized realized and omen [15].							
Re	$I_T(\%)$	D/B (%)	H/D	C_D	C_{Dc} [11]	<i>C</i> _{<i>Dc</i>} [13]	
9500-21900		12.7	1.4-6.0	2.11	1.62	1.66	
16000	≈ 8	12.7	3.3	2.06	1.65	1.76	
7500	≈ 8	6.3	7.7	1.79	1.68	1.80	

TABLE III. Results for drag coefficient for a single square obstacle from the present study, with blockage corrections applied according to the methods of Maskell [11] and Ranga Raju and Singh [13].



FIG. 6. Root-mean-square drag coefficient of an isolated obstacle at the channel center plotted against Reynolds number, based on the upstream hydraulic radius. The best fit (solid line) is given by Eq. (9).

does not occur for obstacles with a square cross section. The drag coefficient of square obstacles is of the order of 2.0 for Re values higher than in Table I, e.g., between $O(2 \times 10^4)$ and $O(2 \times 10^6)$ according to Simiu and Scanlan [31]. Considering the range of values from Tables I and III, C_D drops by 31%, which is significant but much smaller than the 67% drop observed for circular cylinders over a narrow Re range. Furthermore, this large drop in C_D for square obstacles only occurs when I_T increases by two orders of magnitude.

Blockage corrections were applied to measurements from the present study and are included in Table III. The corrected values are in keeping with Lee's [21] values for high I_T (Table I). Maskell's correction [11] performs better than Ranga Raju and Singh's [13] in collapsing drag coefficients from the present study to a single value.

Figure 6 shows $C_{D \text{ rms}}$ for an isolated obstacle as a function of upstream Re, defined in terms of hydraulic radius. To extend the range of Re, additional measurements were included where the height of the weir, downstream of the test section, was not adjusted to obtain a constant flow depth across the obstacle (squares). These measurements are characterized by the upstream conditions $0.05 \leq \text{Fr} \leq 0.62$, $1.5 \leq H/D \leq 6.3$, and $0.0056 \leq Q \leq 0.0311 \text{ m}^3\text{s}^{-1}$. The upstream Reynolds number, based on *D*, is between 2900 and 26600. The root-mean-square drag coefficient decreases monotonically with Re, and data collapse reasonably well to a single curve suggesting that $C_{D \text{ rms}}$ is independent of whether the flow depth is uniform across the obstacle. An appropriate best fit function across both data series is

$$C_{D\,\rm rms} = 18.4e^{-2.13 \times 10^{-4}\rm Re} + 0.241. \tag{9}$$

The root-mean-square drag coefficient asymptotically approaches 0.241 at high Re, in close agreement with the value of 0.242 obtained by Yen and Liu [4] at higher Re (1.5×10^5) , suggesting that this is a good approximation beyond the range considered here.

B. Tandem obstacle pairs

The mean centerline streamwise velocity, nondimensionalized by the velocity 10D upstream (1.2% higher than the bulk velocity), is shown for two tandem obstacle pairs in Fig. 7, while drag



FIG. 7. Normalized, temporally averaged, streamwise component of velocity on the channel centerline for a tandem pair of obstacles with D/B = 12.7%, Re = 16000, and separations between obstacle centers of (a) 2.5D and (b) 10D.

coefficients for tandem pairs with $-5 \le s_x/D \le 20$ are shown in Fig. 8. The turbulence intensity 10D upstream of the first obstacle is 8%. The velocity upstream of the first obstacle and downstream of the second is close to the isolated obstacle case, independent of s_x . At close spacing the measured velocities between the obstacles are negative. The flow shown in Fig. 7(a) ($s_x = 2.5D$) is equivalent to the "reattached mode" identified by Yen *et al.* [2], where flow reattaches on the downstream obstacle and the downstream drag is substantially reduced. As s_x increases the flow between the obstacles recovers, reaching above 70% of the upstream value when $s_x = 10D$ [Fig. 7(b)]. This



FIG. 8. Drag coefficient for the second obstacle in a tandem pair, positioned s_x downstream of the first obstacle (so $s_x < 0$ indicates the drag is measured on the upstream obstacle).



FIG. 9. Comparison of predicted and measured drag coefficients at D/B = 12.7%. The drag was measured at Re = 16000 and the local velocity, used to compute C_D' , was measured at Re = 11000. The continuous lines represent perfect agreement. (a) The measured drag coefficient plotted against the predicted drag coefficient. (b) The ratio of the predicted and measured drag coefficient plotted against the normalized streamwise distance. Dotted lines enclose an area of agreement between the predicted and measured values to within $\pm 10\%$.

represents the "binary mode" flow where the velocity recovers to some extent behind both obstacles and the downstream drag coefficient increases towards the isolated obstacle value. Yen *et al.* found that at $s_x = 3D$, C_D tends to 0.5 for Re > 670. This is comparable to the values found in the present study (at much higher Re) although C_D is very sensitive to position for $s_x < 4D$.

Figure 9 assesses to what extent the drag on an obstacle in a tandem arrangement can be predicted by assuming it varies only because of the change in flow speed induced by the other obstacle, taking the induced flow field as being that from a single isolated obstacle. The predicted drag coefficient C_D' is then the isolated obstacle C_D scaled by the local square of the normalized mean streamwise velocity component. The prediction becomes unreliable where \bar{u} (and C_D) changes abruptly across a length comparable to D, tending to underestimate the magnitude of C_D . This suggests that at a distance far enough away from the obstacle the change in streamwise speed is the primary reason for a change in drag, but when the obstacles are closer together the nature of the flow changes, giving a very different drag. The prediction is demonstrated to be reasonably accurate provided that the obstacles are not too close together, as C_D' is within $\pm 10\%$ of C_D for $x \leq -3D$ and $x \geq 4D$.

C. Side-by-side obstacle pairs

Drag coefficients on one obstacle of a pair placed in side-by-side configuration are shown in Fig. 10. Here the drag on the obstacle in the center of the channel is shown, with the other obstacle offset nearer one of the sidewalls. The drag coefficient is consistently higher than an isolated obstacle at the same Re and D/B, in contrast to the work of Yen and Liu [4] who found that C_D is consistently less than the isolated obstacle value at Re = 21000 and D/B = 4% (except in the single mode $1 \le s_y/D \le 1.1$, which is smaller than considered here). Yen and Liu [4] showed C_D was independent of Re for Re > 17000. The value of Re in the present study (for D/B = 12.7%) is only 5.9% lower than this value, suggesting that the difference in behavior is mainly due to the blockage ratio, as well as differences in the experiment setup, as opposed to differences in Re. The blockage ratio is of higher importance in side-by-side configurations, as twice the effective fraction of the cross-section is blocked. In a narrower channel, relative to D, the flow around the obstacles is accelerated compared to the flow around the obstacles in a wide channel. Thus, obstacles in a channel with a higher blockage ratio experience a higher drag force. This also explains why



FIG. 10. Drag coefficient on one obstacle of a side-by-side pair. The drag is measured on an obstacle in the center of the channel, while the second obstacle would be against the channel sidewall at $s_y = 3.5D$ (for D/B = 12.7%) or 7.4D (for D/B = 6.3%). The position of the sidewall in each case is indicated by the dashed lines. Reynolds number is 16000 for D/B = 12.7% and 7500 for D/B = 6.3%.

 C_D increases with an increase in D/B. At 6.3% blockage, after the initially constant region, C_D decreases abruptly towards the isolated obstacle value, but, as s_y increases further, one obstacle is placed closer to the flume wall so the flow is diverted towards the test obstacle and C_D increases.

D. Staggered obstacle pairs

The drag coefficient on a second obstacle at a range of relative positions ($-5 \le s_x/D \le 20$ and $0 \le s_y/D \le 3.5$ or 7.0) is shown in Fig. 11, while the same information is presented in the form of contour plots of C_D in Fig. 12. When s_y is small, the upstream obstacle shields the downstream one from high velocity flow, tending to reduce C_D . As the obstacles are moved further apart in the *x*-direction shielding is reduced and C_D increases towards the isolated obstacle value, reaching 90% of the isolated value for $s_x = 19D$ and 17D respectively at D/B = 12.7% and 6.7%.

If s_y is large and s_x is small, twice the effective cross section is blocked so C_D is higher than the isolated value. The maximum C_D at D/B = 12.7% blockage is 3.82, compared to only 2.85 at D/B = 6.3%, and occurs with the obstacles nearly side by side, with the highest drag on the slightly upstream obstacle at $s_x/D = -1$. This is presumably because the flow acceleration in the gap between the obstacles in this configuration gives a particularly low pressure at the rear corner of the upstream obstacle, and thus a low pressure in the wake region and on the downstream face of the upstream obstacle, enhancing the total drag force. As s_x increases C_D decreases towards the isolated obstacle value, but this process is relatively slow.

A plot similar to Fig. 12 for circular cylinders at Re = 2600 was presented by Nepf [5]. Measurements were taken for $0 < s_y/D < 2$ and $0 < s_x/D < 5$, and trends further downstream $(6 < s_x/D < 20)$ were estimated from the observed decay of wake interference. As one may have anticipated, C_D is typically much higher for square obstacles. For circular cylinders $C_D = 1.17$ as s_x/D and s_y/D approach infinity, similar to the plateau in C_D of tandem square obstacles at large s_x/D . However, for circular cylinders C_D never substantially exceeds the isolated cylinder



FIG. 11. Drag coefficient on a second obstacle as a function of relative position for (a) D/B = 12.7% (Re = 16000) and (b) D/B = 6.3% (Re = 7500).

value, unlike square obstacles where interference and blockage contribute to substantially higher drag coefficients than the isolated obstacle value when s_y/D is sufficiently large. The wall position also prevents C_D from tending to this value as s_y increases. For circular cylinders at $s_x/D > 2$ the downstream cylinder drag increases as both s_x/D and s_y/D increase. Similar behavior is observed for square obstacles at 12.7% blockage. At 6.3% blockage, however, C_D increases with s_y/D towards a maximum value and then begins to decrease, increasing again slightly close to the wall. The two shapes of obstacle show similar wake recovery in terms of the size of the region where the drag coefficient is negative. For circular cylinders this occurs for $s_x/D < 2$ and $s_y/D < 0.25$. For



FIG. 12. Contours of drag coefficient on a second obstacle placed at different relative positions, with blockage ratios (a) D/B = 12.7% (Re = 16000) and (b) D/B = 6.3% (Re = 7500).

square obstacles the drag is also negative for $s_x/D < 2$ when they are in tandem, but the increments of s_y/D were not small enough to measure negative drag for $s_y/D \neq 0$. Drag coefficients recover to an appreciable fraction of the isolated obstacle value at similar downstream distances, reaching 85% at 16D for circular cylinders in tandem and at 14D and 17D respectively for square obstacles at 6.3% and 12.7% blockage.

Blockage corrections were not applied to obstacle pairs because it has been demonstrated elsewhere that applying standard procedures with an effective blockage ratio (e.g., D/B for tandem pairs and 2D/B for side-by-side cylinders) is not sufficient. Utsunomiya *et al.* [32] proposed a modification to Maskell's theory [11], for tandem obstacles with rectangular cross-sections, where ε is modified in proportion to a space factor which is a function of s_x/D . For staggered pairs one could anticipate a similarly modified ε is a function of s_x/D and s_y/D and employ various values to collapse the data at the two blockage ratios to a single function, assuming that Re and H/D effects are negligible. However, this is speculative and there is not enough data, with all other variables being equal, to verify the correction procedure is working accurately. In addition, for nearly side-by-side arrangements both the effective blockage ratio and C_D are high compared to studies with isolated obstacles. Under these conditions, agreement between Maskell [11] and Ranga Raju and Singh's [13] correction methods breaks down, suggesting that at least one of the corrections is no longer valid even for a single obstacle. An investigation of blockage correction methods for staggered obstacle pairs is recommended for further study.

IV. CONCLUSIONS

A comprehensive and careful set of experiments has been conducted to measure drag on pairs of square obstacles in subcritical open channel flow with parameters (Re, I_T) appropriate for many practical applications. The drag coefficient for a single square obstacle was found to be $C_D = 2.11$, in agreement with earlier studies, suggesting that any increase due to blockage effects or the formation of surface waves was compensated for by the comparatively high I_T . It was also confirmed that C_D is not strongly dependent on Re (unlike the drag on circular cylinders). The isolated obstacle C_D was found to provide an estimate of the drag on an obstacle in a tandem pair (for $s_x \leq -3D$ and $s_x \geq 4D$) to within 10% accuracy if U is replaced by \bar{u} in the free stream, in the absence of the obstacle under consideration, where its horizontal center would be located $(x = s_x)$. Thus, for distances far enough away from the obstacle the change in streamwise speed is the primary reason for a change in drag, but when the obstacles are closer together the nature of the flow changes, giving a very different drag. This has important implications for estimating drag where there are a number of obstacles, as it suggests that to correctly predict the drag on downstream obstacles it is important to accurately model the temporally averaged streamwise velocity. If the same scaling of drag, based on the induced speed, also applies to side-by-side pairs then blockage effects are essentially just a subset of the flow speed alterations due to the presence of the first obstacle. A more general investigation which aims to explain drag changes, in staggered pairs or arrays, in terms of induced speed changes is recommended for further study. The root-mean-square drag coefficient was found to be a monotonically decreasing function of Re, based on hydraulic radius, and approaches 0.241 at high Re in agreement with previous studies and was independent of whether the flow depth is uniform across the obstacle.

For pairs of obstacles, the minimum C_D occurred for a tandem arrangement where C_D becomes negative for the downstream obstacle. The smallest values measured were for $s_x = 1.5D$, with $C_D = -0.16 (D/B = 6.3\%)$ and $C_D = -0.40 (D/B = 12.7\%)$. The maximum C_D at D/B =12.7% is 3.82, compared to only 2.85 at D/B = 6.3%, and occurs when the obstacles are nearly side by side on the slightly upstream obstacle of a pair. The recovery of the wake and thus drag on an obstacle directly downstream is slow, with the drag on a downstream obstacle reaching 80% of the isolated value for $s_x = 14D (D/B = 12.7\%)$. Thus, laboratory and numerical models need to extend at least this far in the downstream direction to accurately capture the flow. The present study is restricted to angle of attack $\theta = 0^\circ$, but it is clear that θ is an important parameter which can greatly affect C_D . As such, the influence of θ for both obstacles in a staggered pair, particularly at 12° and 45° where the minimum and maximum C_D occur respectively for isolated obstacles, is recommended for further study, as this is important for practical applications such as wind induced airflow past nearby buildings.

The drag values found here can be used in a number of practical applications where it is necessary to estimate the forces on downstream structures or other bluff obstacles with similar shapes. The results can also be used to validate numerical models, where getting accurate results for the drag on a downstream obstacle (even for a square obstacle with well-defined separation points) is found to be challenging. In numerous applications (such as vegetation in streams or on flood plains and marine or riverine structures) obstacles are present in regular or random arrays, and the drag on arrays of square obstacles is the subject of our further study.

ACKNOWLEDGMENT

Francis H. Robertson was supported by a studentship from EPSRC (Grant Reference No. 1376075).

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