Erratum: Superstatistics and quantum entanglement in the isotropic spin-1/2 XX dimer from a nonadditive thermodynamics perspective [Phys. Rev. E 104, 024139 (2021)]

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The computation of Eqs. (16) is wrong. The mistake is derived from Eq. (14) where it was stated that $Z_1(\beta) = \mathcal{Z}(\beta^*)$. The latter equation is not correct [i.e., $Z_1(\beta) \neq \mathcal{Z}(\beta^*)$]. To eliminate nonphysical temperatures, it must be corrected.

From the identity $\hat{\rho}(\beta) = \hat{\rho}(\beta^*)$ and from Eqs. (7) and (12), Eq. (16b) is given by

$$U = \frac{\text{Tr}[\hat{\rho}^q(\beta^*)\hat{H}]}{\text{Tr}[\hat{\rho}^q(\beta^*)]} = \frac{\mathcal{U}}{\text{Tr}[\hat{\rho}^q(\beta^*)]} = -\frac{\partial_{\beta^*} \ln_q \mathcal{Z}}{\text{Tr}[\hat{\rho}^q(\beta^*)]}. \quad (16b)$$

FIG. 3. (a) Entropy as a function of the internal energy, for the non-extensive formalism of Eqs. (6) and (7) (green squares), compared with the formalism related to $\hat{q}$ (blue line) and the GB case $q \rightarrow 1$ (red line) for $B/J = 1$ and $q = 0.2$. The black stars are non-physical (positive) temperatures provided by parametrization of Eq. (15), where the thermal observables are not functions of T . (b) Identification of the non-physical values of T by using panel (a). In panel (c) S vs. U is computed from $\hat{\rho}$ for $B/J = 4$ and $q = 2$. The black (pink) stars are the non-physical positive (negative) parametrized temperatures which are identified in panel (d).

On the other hand, Eq. (16a) is not necessary given that S is already defined in Eq. (6).

The above analysis modifies Fig. 3, but the prescription to eliminate nonphysical temperatures is the same: We discard points where $S(U)$ and $\partial S/\partial U = 1/T$ are not well-behaved or real single-valued functions. With this in mind, the modifications of Figs. 3(a) and 3(c) [Figs. 3(b) and 3(d) remain unaltered] are given in the present document. The black (pink) stars are nonphysical positive (negative) temperatures so that in Fig. 3(a) they correspond to points where U and S are complex functions, and in Fig. 3(c) S is not a single-valued function of U .

The named corrections do not affect the main results of the paper, namely, the behavior of the concurrence. Hence, our conclusions remain valid.