Erratum: Inferring domain of interactions among particles from ensemble of trajectories [Phys. Rev. E **102**, 012404 (2020)]

Udoy S. Basak , Sulimon Sattari, Kazuki Horikawa, and Tamiki Komatsuzaki

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We found that the code used in our paper had an error, which affected some of the provided figures, but the main conclusions of the original paper remain unchanged. The sentences of the main paper that need to be altered, and the corrected Figs. 4–7, 9, and 10, are given below.

In Sec. IV, the statements below Fig. 8: “Our scheme based on ... the underlying interaction distance.” should be changed to “Our scheme based on transfer entropy infers the underlying interaction distance satisfactorily at different noise levels ranging from 0.1π to 2π whereas as noise gets lower, e.g., $\eta_0 = 0.1\pi$, cross correlation (CC) cannot estimate the underlying correct R at shorter $R(\leq 2.0)$. At low noise, the $(CC)_\lambda$ vs λ curve has a local minimum at high λ where the $\frac{d(CC)_\lambda}{d\lambda}$ drops below the local minimum near $\lambda = R$. Even whereas the AUC scores are manifestly smaller in CC than those in transfer entropy (TE), CC can also estimate the interaction radius as TE does except for short R when the timescale of the system dynamics in Eq. (4) (i.e., 1 arb. unit), and the “observation” timescale, say, τ_{obs} , are the same. In practice, the timescale of the measurement does not necessarily coincide with the timescale of the system. Consider the case where one can only sample the data once every τ_{obs} time steps. In that case, the relationship between the system dynamics at the current time t and that at the next available time step $t + \tau_{\text{obs}}$ is more nonlinear. For example, $\hat{R}$ estimated by TE and CC at $\eta_0 = 1.2\pi$ with the underlying actual interaction radius $R = 3.0$ for the observation timescale $\tau_{\text{obs}} = 1$ –4 with the total time length being 100 000 are as follows: $\hat{R}_{\text{TE}} = 3.02$ and $\hat{R}_{\text{CC}} = 3.02$ with $\tau_0 = 1$ as shown in Fig. 7, but $\hat{R}_{\text{TE}} = 3.02$ and $\hat{R}_{\text{CC}} = 3.62$ with $\tau_{\text{obs}} = 2$, and $\hat{R}_{\text{TE}} = 3.02$ and $\hat{R}_{\text{CC}} = 3.97$ with $\tau_{\text{obs}} = 4$. As the noise level gets much higher, e.g., 1.8π , it was found that both CC and TE could not assign the interaction radius due to high noise when $\tau_{\text{obs}} > 1$. This implies that TE is superior to CC in predicting the leader-follower relationship (Figs. 4 and 5) and in inferring the underlying interaction domain and when the noise level gets much higher, both TE and CC fail to capture the underlying interaction distance for the case of the observation time $\tau_{\text{obs}} > 1$.”

FIG. 4. AUC landscapes of the classifier (a) χ_{TE} and (b) χ_{CC} at different noise levels and cutoff distance λ where the interaction radius R is 2 arb. units. At very short λ there exist situations where two particles rarely enter each other’s interaction domains, and to avoid the unclassified cases $\lambda < 0.25$ arb. units are excluded. Also at very low η_0 there were cases where the symbols of a particle seldom change in time, which makes CC undefined, and, hence, $\eta_0 < 0.1\pi$ are excluded. The highest AUC values for TE and CC are found to be 0.999 (at $\eta_0 = 1.1\pi$, $\lambda = 2.0$ arb. units) and 0.859 (at $\eta_0 = 1.2\pi$, $\lambda = 2.0$ arb. units), respectively.

FIG. 5. AUC landscapes as a function of noise levels η_0 and the interaction radius R obtained by our cutoff distance scheme compared to those by the conventional scheme without-cutoff distance. (a) The classifier χ_{TE} with-cutoff distance ($\lambda = R$), (b) χ_{TE} without-cutoff distance ($\lambda = 5\sqrt{2}$), (c) χ_{CC} with the cutoff distance, and (d) χ_{CC} without the cutoff distance. Since $\lambda = 5\sqrt{2}$ corresponds to the without-cutoff distance scheme, and the AUC values with- and without-cutoff distance schemes converge to the same values as λ approaches $5\sqrt{2}$.

FIG. 6. Dependence of averaged TE and CC over the ensembles on cutoff distance λ for different interaction radii R at noise level $\eta_0 = 1.2\pi$. (a) The averaged TE $\langle TE \rangle_\lambda$, (b) the derivatives of $\langle TE \rangle_\lambda$ with respect to λ , (c) the averaged CC $\langle CC \rangle_\lambda$, and (d) the derivatives of $\langle CC \rangle_\lambda$ with respect to λ .

FIG. 7. Estimation of the interaction radius at (a) $\eta_0 = 0.1\pi$, (b) $\eta_0 = 1.2\pi$, and (c) $\eta_0 = 1.8\pi$ based on $\langle \text{TE} \rangle_\lambda$ and $\langle \text{CC} \rangle_\lambda$.

FIG. 9. The derivative of $\langle \text{TE} \rangle_\lambda$ with respect to λ . The inset: $\langle \text{TE} \rangle_\lambda$. In this figure the number of particles is $N = 50$, the interaction radius is $R = 3$, and noise is $\eta_0 = 1.2\pi$. It indicates that the minimum of the derivative approach can be used to identify the interaction radius for a larger number of particles.

FIG. 10. (a) The derivative of $\langle \text{TE} \rangle_\lambda$ with respect to λ for the different leader-follower ratios. The inset: $\langle \text{TE} \rangle_\lambda$. (b) The derivative of $\langle \text{CC} \rangle_\lambda$ with respect to λ for the different leader-follower ratios. The inset: $\langle \text{CC} \rangle_\lambda$. The actual interaction radius used for these figures is 3 and noise $\eta_0 = 1.2\pi$. It can be seen that the leader-follower ratio does not affect the average information flow (as we set $w_{LF} = 1.05$ and $w_{FL} = 1.0$ which are very close to each other), hence, the estimation of the interaction radius.