Erratum: Critical exponents in mean-field classical spin systems [Phys. Rev. E **100**, 032131 (2019)]

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In Appendix C of the paper, we have written: “The single-spin Hamiltonian H_0 has a separatrix which consists of the stable and unstable manifolds of the fixed point $(S_x, S_y, S_z) = (-1, 0, 0)$ and encloses the point $O = (1, 0, 0)$ on the phase space of the unit sphere.” Unfortunately, this sentence is not generally correct.

The single-spin Hamiltonian H_0 is

$$H_0 = Dp^{2n} - m_0\sqrt{1-p^2}\cos\phi, \quad (1)$$

where $m_0 > 0$ is the spontaneous magnetization assumed to point in the x direction without loss of generality. Being interested in the neighborhood of the critical point, we take m_0 to be sufficiently small. Then, analyzing the zero points of the gradient ∇H_0 , we have for $n = 1$ one center type fixed point at $(\phi, p) = (0, 0)$ (the point O), one saddle at $(\phi, p) = (\pi, 0)$, and two additional center type fixed points on the line $\phi = \pi$. For $n \geq 2$, both $(0, 0)$ and $(\pi, 0)$ become center type fixed points, and one has, in addition, two other center type fixed points and two saddle points all lying on the line $\phi = \pi$. All fixed points lying on the line $\phi = \pi$ give negative contribution to the magnetization. It is, therefore, reasonable to assume that for obtaining a normal linear response, the main contribution comes from a neighborhood of the fixed point O , as we have performed in Appendix C.

We provide here some additional information on the approximation of H_0 around the point O . The Taylor expansions of $\sqrt{1-p^2}$ and $\cos\phi$ give

$$H_0(\phi, p) \simeq Dp^{2n} - m_0\left(1 - \frac{p^2}{2} - \frac{\phi^2}{2}\right) + \dots, \quad (2)$$

and we have two choices of Dp^{2n} and $m_0p^2/2$ as the main kinetic energy term. It is true that m_0 is sufficiently small, but p is also small inside the separatrix enclosing the point O . However, the reasonable choice is Dp^{2n} because the width of separatrix in the p direction may be estimated to be unreasonable, of $O(m_0^0)$, if we choose the $m_0p^2/2$ term.

We regret the inconvenience caused by the error pointed out in the foregoing. Nevertheless, we emphasize that the estimation of Appendix C still holds even after the correction reported herein because the estimation correctly focuses on the contribution to L_2 from the neighborhood of the fixed point O .