Measuring the mass, width, and couplings of semi-invisible resonances with the matrix element method

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We demonstrate the use of the matrix element method (MEM) for the measurement of masses, widths, and couplings in the case of single or pair production of semi-invisibly decaying resonances. For definiteness, we consider the two-body decay of a generic resonance to a visible particle from the Standard Model (SM) and a massive invisible particle. It is well known that the mass difference can be extracted from the endpoint of a transverse kinematic variable like the transverse mass, M_T , or the Cambridge M_{T2} variable, but measuring the overall mass scale is a very difficult problem. We show that the MEM can be used to obtain not only the absolute mass scale, but also the width of the resonance and the tensor structure of its couplings. Apart from new physics searches, our results can be readily applied to the case of SM W boson production at the CERN Large Hadron Collider (LHC), where one can repeat the measurements of the W properties in a general and model-independent framework.

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I. INTRODUCTION

The dark matter problem is the biggest mystery in particle physics today [1]. It greatly motivates the current experimental efforts to discover new physics beyond the Standard Model (BSM) at the LHC. In typical BSM models, dark matter particles are produced at the LHC as the end products of the cascade decays of heavier particles. Generally speaking, the longer the cascade, the more handles we have at our disposal to measure particle properties like masses, widths, couplings, etc. Hence, the most challenging cases are actually the simplest ones, illustrated in Fig. 1 for the case of single production [diagram (a)] and pair-production [diagram (b)]. For these processes, the classic method of kinematic endpoints [2] fails to determine the complete mass spectrum—there is only one observable endpoint,¹ and the best one can do is to

obtain a relationship² $M_W(M_\nu)$ between the mass of the parent, M_W , and the mass of the daughter particle, M_ν , leaving the overall mass scale undetermined. In particular, standard methods for measuring the W boson mass, such as via properties of the transverse mass spectra like the Jacobian peak [3,4], or by determining M_W/M_Z by comparison with Z boson events [10], are not sufficient in this scenario.

One possibility that has been suggested in the literature for determining the overall mass scale is to go beyond the leading order diagrams of Fig. 1 and consider hard initial state radiation (ISR), which provides a kick to the system in the transverse plane. In the presence of ISR, the functional dependence $M_W(M_\nu)$, derived from either M_T for the case of Fig. 1(a) [11,12] or from M_{T2} for the case of Fig. 1(b) [8,13–18], exhibits a kink at the true value, M_ν^{true} , of the daughter particle. However, the kink is not very visible unless we demand very hard ISR, which causes a significant loss in statistics.

Given the difficulty of measuring the overall mass scale, it stands to reason that one should use the MEM [19–27], which, like similar multivariate analyses (MVA) [28], has been used with success in LHC experiments. A particularly dramatic example has been the use of the MEM in the four-lepton channel for the discovery of the Higgs boson [29] and the measurement of its properties [30–32].

¹In order to preform this measurement in practice, one can use any one of several variables, e.g., the transverse mass, M_T , of the parent particle [3,4], or the transverse momentum, p_T , of the visible daughter particle for the case of Fig. 1(a), or the Cambridge M_{T2} variable [5,6], the contranverse mass M_{CT} [7], or their 1D variants [8,9] for the case of Fig. 1(b).

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²See Eq. (8) below.

FIG. 1. The event topologies considered in this paper: (a) single and (b) pair production of a W -like resonance decaying leptonically.

Such analyses used variables, such as MELA KD [33–35] or MEKD [36,37] that involve the ratio of signal and background matrix elements.³ Clearly these variables are, therefore, optimized to the appropriate signal and background hypotheses.

To our knowledge, the MEM has thus far been underutilized for the purposes of mass spectrum measurements of the sort we describe here. This is perhaps due to the practical challenges one usually encounters in the implementation of the MEM, e.g., the presence of instrumental and/or reducible physics backgrounds; the need to account for effects like the finite detector resolution, the underlying event, jet fragmentation; the challenge of integrating peaked integrand structures in phase space; incorporating higher order corrections to the matrix element, etc., [27]. Despite these practical limitations, the physical transparency of the MEM greatly motivates expanding its role in LHC data analyses. Previously, Ref. [38] applied the MEM to the case of squark pair production in SUSY [which has the same event topology as Fig. 1(b)] and demonstrated that one cannot only recover the endpoint measurement information but also place a restriction on the allowed overall mass scale. Similar conclusions were drawn in Ref. [39] for the example of smuon pair production as in Fig. 1(b). We would like to revisit these earlier studies and extend the application of the MEM to the measurement of the remaining properties of the particles involved, namely the width and the chirality of the couplings. (The discrete choices of the spin assignments of the particles in the decay chain were previously investigated in Ref. [40].)

II. FORMULATION OF THE PROBLEM

We begin by considering single production of a resonance, W^+ , which decays semi-invisibly via a two-body decay into a visible SM particle, $\bar{\ell}$, and an invisible particle, ν . For definiteness, we shall take $\bar{\ell}$ to be an antilepton (positron or antimuon) and ν to be an invisible particle,

³Strictly speaking, those analyses were using MEM-inspired optimal variables, whose distributions were used later to form the likelihood in a template method instead of as in Eq. (3).

which can be a SM neutrino or some BSM dark matter candidate. The W^+ resonance can be produced singly, as shown in Fig. 1(a), or as part of a W^+W^- pair, as in the s -channel⁴ diagram of Fig. 1(b). As suggested by our notation, this setup includes, but is not limited to, the SM production of W^+ bosons decaying leptonically. In particular, the process of Fig. 1(a) may refer to the production of a charged Higgs scalar [41], a charged slepton in supersymmetry (SUSY) models with R -parity violation [42,43], or a new W' heavy gauge boson [44,45]. Similarly, the process of Fig. 1(b) may be interpreted as the pair-production of (perhaps quarkophobic) charged Higgs bosons [46,47] or W' bosons [48], of charginos [49], Kaluza-Klein leptons [50,51], or sleptons [52,53]. However, for definiteness, in our simulations below we shall assume that the $W^\pm$ are spin-1 particles while $\ell(\bar{\ell})$ and $\nu(\bar{\nu})$ are spin 1/2, as in the SM. We shall parametrize the W couplings to leptons (quarks) as $g_R^q P_R + g_L^q P_L$ ($g_R^q P_R + g_L^q P_L$), where $P_{L,R} = (1 \mp \gamma_5)/2$ are the usual chiral projectors.

The (normalized) kinematic distributions of the leptons in the final state will depend on five model parameters:

$$\{M_W, M_\nu, \Gamma_W, \varphi_q, \varphi_\ell\}, \quad (1)$$

where M_W (M_ν) is the mass of the parent (daughter) particle, Γ_W is the width of the parent, and

$$\tan \varphi_q \equiv \frac{g_R^q}{g_L^q}, \quad \tan \varphi_\ell \equiv \frac{g_R^\ell}{g_L^\ell}, \quad (2)$$

so that the angles φ_q and φ_ℓ encode the information about the chirality of the W couplings to quarks and leptons, respectively.

Given this general setup, our main goal in this paper will be to attempt to measure *all five* of the model parameters in Eq. (1). As already discussed in the introduction, this is not a trivial task. Measuring the mass splitting is relatively straightforward (see Fig. 2 below), but fixing the remaining four parameters requires subtle measurements of the relevant kinematic distributions. We shall make use of the MEM, which is ideally suited for our purposes. Along the way we shall also study the dependence of the relevant kinematic variables on the underlying parameters (1), highlighting the cases when a certain variable depends

⁴We focus on the photon-mediated s -channel diagram for simplicity: since the W is charged, it must couple to photons, so that the diagram of Fig. 1(b) is guaranteed to exist. In principle, there can be additional t - and u -channel pair-production diagrams, but this requires that the W couples to quarks as well, in which case the single production from Fig. 1(a) should dominate. There could also be s -channel diagrams mediated by Z or other more exotic gauge bosons, but this possibility also involves additional assumptions. All of those complications can be easily incorporated in the analysis, and the MEM would still work, but our discussion would become more opaque.

FIG. 2. Unit-normalized lepton P_T distributions for single W production in the limit of $\Gamma_W = 0$. The invisible particle mass is $M_\nu = 500$ GeV; the mass of the parent particle is varied as shown.

strongly on a particular parameter. This not only provides intuitive understanding of our main results (namely, Figs. 10, 14 and 15 below) which are obtained with the MEM but also offers an alternative approach to estimate the parameters (1) by template-fitting to the corresponding sensitive kinematic variables.

III. THE MATRIX ELEMENT METHOD

In the MEM, the likelihood for a given event with a measured set of N_f^{vis} visible final state 4-momenta $\{P_j^{\text{vis}}\}$, $j = 3, \dots, N_f^{\text{vis}} + 2$, is defined as

$$\begin{aligned} \mathcal{P}(\{P_j^{\text{vis}}\}|\alpha) &= \frac{1}{\sigma_\alpha} \left[\prod_{j=1}^{N_f} \int \frac{d^3 p_j}{(2\pi)^3 2E_j} \right] W(\{P_j^{\text{vis}}\}, \{p_j^{\text{vis}}\}) \\ &\times \sum_{a,b} \frac{f_a(x_1) f_b(x_2)}{2s x_1 x_2} |\mathcal{M}_\alpha(\{p_i\}, \{p_j\})|^2 \\ &\times (2\pi)^4 \delta^4 \left(\sum_{i=1}^2 p_i - \sum_{j=3}^{N_f+2} p_j \right). \end{aligned} \quad (3)$$

Here f_a and f_b are the parton distribution functions (pdf) of the initial state partons a and b as a function of their momentum fractions x_1 and x_2 , while $\sqrt{s}$ is the center-of-mass energy of the collider. $\mathcal{M}_\alpha$ represents the theoretical matrix element for the parton-level scattering process $\{p_i\} \rightarrow \{p_j\}$ under a given hypothesis α ,⁵ while σ_α is

⁵In general $\mathcal{M}_\alpha$ includes the appropriate color factor, appropriate symmetry factors, etc., for the process under consideration, though when only one process contributes, these factors will cancel in the ratio with the total cross section.

the total cross section for the corresponding hypothesis after acceptances, efficiencies, etc., whose role is to normalize the total probability. The transfer function $W(\{P_j^{\text{vis}}\}, \{p_j^{\text{vis}}\})$ incorporates all detector effects and efficiencies by mapping the $3N_f^{\text{vis}}$ true 4-momenta $\{p_j^{\text{vis}}\}$ of the final state visible particles onto the set of measured 4-momenta $\{P_j^{\text{vis}}\}$ [54].⁶ In general, the final state will also contain a certain number, N_f^{inv} , of invisible particles, e.g., neutrinos or dark matter candidates. Since their momenta are not measured, they are simply integrated over, so that in (3) there are a total of $3N_f = 3(N_f^{\text{vis}} + N_f^{\text{inv}})$ integrations. The delta function factor on the third line of (3) ensures energy-momentum conservation.

The likelihood for a set of N events is simply the product of the individual likelihoods for each event n :

$$\mathcal{L}_\alpha = \prod_n^N \mathcal{P}(\{P_j^{\text{vis}}\}_n|\alpha); \quad (4)$$

when necessary an overall Poisson factor for the number of events can be used to construct the so-called “extended likelihood.”

IV. SINGLE W PRODUCTION

We first consider single W production [Fig. 1(a)]. The spin and color averaged squared matrix element for the process $u\bar{d} \rightarrow W^+ \rightarrow \bar{\ell}\nu$ is given by

$$\begin{aligned} \langle |\mathcal{M}|^2 \rangle &= \frac{4|V_{ud}|^2}{3[(\hat{s} - M_W^2)^2 + (\Gamma_W M_W)^2]} \\ &\times [\{(g_L^q)^2 (g_L^\ell)^2 + (g_R^q)^2 (g_R^\ell)^2\} (p_u \cdot p_{\bar{\ell}})(p_{\bar{d}} \cdot p_\nu) \\ &+ \{(g_R^q)^2 (g_L^\ell)^2 + (g_L^q)^2 (g_R^\ell)^2\} (p_{\bar{d}} \cdot p_{\bar{\ell}})(p_u \cdot p_\nu)], \end{aligned} \quad (5)$$

where the parton-level center-of-mass energy squared is $\hat{s} = (p_u + p_{\bar{d}})^2$, and we have generalized to the case of arbitrary fermion couplings to the W -like intermediate resonance. V_{ud} is the analog of the CKM matrix element and is the SM CKM matrix element if we are considering production and decay of the SM W boson.

If we take p_1 to be the momentum of the incident parton with positive z momentum, p_2 to be the momentum of the incident parton with negative z momentum, and define

$$\begin{aligned} F_1 &= f_u(x_1) f_{\bar{d}}(x_2), \\ F_2 &= f_u(x_2) f_{\bar{d}}(x_1), \\ k_1 &= (p_1 \cdot p_{\bar{\ell}})(p_2 \cdot p_\nu), \\ k_2 &= (p_1 \cdot p_\nu)(p_2 \cdot p_{\bar{\ell}}), \end{aligned} \quad (6)$$

⁶We will use P to refer to measured momentum variables and p to label truth momentum variables throughout this work.

we find that the likelihood for a particular event is proportional to

$$(F_1 + F_2)(k_1 + k_2) + \cos 2\varphi_\ell \cos 2\varphi_q (F_1 - F_2)(k_1 - k_2). \quad (7)$$

We see that only the second term depends on the helicity of the couplings. As expected, this term will not contribute in the absence of a longitudinal boost (when $F_1 = F_2$) or if the lepton is emitted perpendicular to the beam line in the rest frame of the W (when $k_1 = k_2$). We also note that k_1 and k_2 depend only on $p_{\ell z}$, not on $p_{\ell T}$, so when determining the $p_{\ell T}$ distribution we get no contribution from the second term, as the contribution from this term from each point with a given value of $(p_{\ell T}, p_{\ell z})$ is canceled by the contribution of the term with $(p_{\ell T}, -p_{\ell z})$. So it will be the $P_{\ell z}$ distribution rather than the $P_{\ell T}$ distribution that will give us sensitivity to the chirality of couplings, as we will see in more detail below.

In our subsequent analyses we generate events with MADGRAPH5_AMC@NLO [55] version 2.5.5 for the parameter point with $M_W = 1000$ GeV, $M_\nu = 500$ GeV, and $\Gamma_W = 50$ GeV at $\sqrt{s} = 13$ TeV without applying selection criteria (cuts) or detector simulation to the events. We use MADWEIGHT5 [56] for computation of the weights in MEM calculations using δ -function transfer functions, and have verified that the MADWEIGHT results can be reproduced using (5), where appropriate.

We note that there are two relevant observables: the transverse momentum $P_{\ell T}$ and the longitudinal momentum $P_{\ell z}$ of the lepton, as the only visible particle in the final state is the lepton, with fixed (zero) mass. The third momentum degree of freedom corresponds to an azimuthal angle, which cannot have a nontrivial distribution in the absence of some very unexpected physics (or detector effects) breaking the azimuthal symmetry.

V. MEASUREMENT OF THE MASS “DIFFERENCE”

One quantity, related to the difference of the squared masses of the W and the ν , can be easily measured from the endpoint of the distribution of the W transverse mass M_{WT} . In the absence of ISR, this quantity can also be measured from the kinematic endpoint of the $P_{\ell T}$ distribution, μ , as

$$\frac{M_W^2 - M_\nu^2}{2M_W} = \text{constant} \equiv \mu. \quad (8)$$

We note that μ , the maximum value of lepton p_T , is also the 3-momentum of the lepton in the center-of-mass (CM) frame (still in the absence of ISR). Equation (8) allows us to fix one of M_W and M_ν once we have measured the other. Thus, at least for the time being, we shall focus on measuring the orthogonal mass degree of freedom, i.e.,

the overall mass scale, by choosing the test masses to satisfy the relation (8).⁷ In other words, we shall vary one of the two masses, e.g., M_ν , and then compute the other mass from Eq. (8) as

$$M_W = \mu + \sqrt{\mu^2 + M_\nu^2}. \quad (9)$$

In Fig. 2, we show the lepton P_T distributions in single W production for different W masses; $M_W = 800$ GeV (green dotted), $M_W = 1000$ GeV (red solid), and $M_W = 1200$ GeV (blue dashed) in the $\Gamma_W = 0$ limit. The mass of the invisible particle is set to $M_\nu = 500$ GeV. Here we consider only left-handed fermionic couplings to the W boson. The black dashed line gives the theoretical prediction for the *true* lepton p_T distribution (in the absence of cuts) which is given by

$$\frac{1}{\sigma} \frac{d\sigma}{dp_{\ell T}} = \frac{3}{4 + 3\rho} \frac{p_{\ell T}}{\mu \sqrt{\mu^2 - p_{\ell T}^2}} \left(2 - \frac{p_{\ell T}^2}{\mu^2} + \rho \right), \quad (10)$$

where $\rho \equiv 2M_\nu^2/(M_W^2 - M_\nu^2)$. Therefore when the lepton p_T reaches its maximum value, μ , the longitudinal momentum p_z goes to zero, and we obtain the well-known Jacobian peak in the distribution (10). In principle, this equation indicates that the p_T spectrum depends on both M_W and M_ν via the quantity, ρ , and the endpoint, μ . However in practice, the dependence on ρ tends to be subtle, so in practical situations we may only be able to measure μ , and of course, a given value of μ corresponds to any M_W and M_ν satisfying Eq. (8). The main point of this paper is that in addition to measuring μ (from a kinematic endpoint), by utilizing the MEM, we can simultaneously also obtain (a) the mass scale, i.e., M_W itself, (b) the width Γ_W , and (c) the chirality of the couplings (2).

VI. MEASUREMENT OF THE MASS SCALE M_W

The mass scale is notoriously difficult to measure; even in this very simple topology it cannot be determined from kinematic endpoint measurements alone (unless we require hard ISR). Instead we have to rely on subtle effects. The two tools at our disposal are the distributions of the measured lepton $P_{\ell T}$ and $P_{\ell z}$. Interestingly, the shapes of both of these distributions encode information about the mass scale, as illustrated in Fig. 3. As seen in the top panel, the P_T distribution is rather weakly sensitive to the mass scale. However, the lower panel in Fig. 3 shows that the longitudinal momentum does contain information about the mass scale which can potentially be observed.

As a proof of principle, we perform a preliminary toy exercise to find the mass scale by simply fitting to $P_{\ell z}$

⁷We postpone the question of the simultaneous determination of both M_W and M_ν until the analysis presented in Fig. 10 below.

FIG. 3. Unit-normalized distributions of the lepton transverse momentum P_T (top) and longitudinal momentum P_z (bottom), for different values of M_W and $\Gamma_W = 0$ obtained using analytical expressions and, in the case of P_z , pdfs from LHAPDF [57]. The mass, M_ν , of the invisible particle has been fixed from the measurement (8).

templates generated for different mass spectra obeying the relation (8). The advantage of the template method is that it avoids the time consuming integrations over the invisible momenta that are needed for the MEM. The result is shown in Fig. 4, where we plot the $\chi^2/\text{d.o.f.}$ for several

FIG. 4. $\chi^2/\text{d.o.f.}$ fit to M_ν from $P_{\ell z}$ templates in W^+ production (left) and W^- production (right).

FIG. 5. The same as Fig. 2, but for fixed $M_W = 1000$ GeV, and several values of the width Γ_W (as indicated in the figure).

hypothesized values of M_ν [with M_W calculated from (9)]. The right panel of Fig. 4 shows results from the same exercise, but for the case of $d\bar{u} \rightarrow W^- \rightarrow \ell\bar{\nu}$.

The minima of the χ^2 curves in Fig. 4 are near the true value, $M_\nu = 500$ GeV; this suggests that the template method works in principle. However, the MEM will be more sensitive, as it (1) uses the correlations among $P_{\ell T}$ and $P_{\ell z}$ in the data and (2) incorporates the dependence on the remaining parameters in Eq. (1), which makes it possible to do a *simultaneous* measurement of several parameters.

VII. MEASUREMENT OF THE WIDTH Γ_W

The width effects will manifest themselves in two places. First, there will be some smearing of the $P_{\ell T}$ endpoint [58] as illustrated in Fig. 5.⁸ However, the $P_{\ell T}$ distributions resulting from different choices of M_W , M_ν , and Γ_W will be relatively similar provided M_W and M_ν give the same endpoint μ [following Eq. (8)]. Given this criterion, the distributions will tend to be more similar if the masses and the width satisfy the relation

$$\frac{\Gamma_W}{\Gamma_W^{\text{true}}} = \frac{1 + (M_\nu^{\text{true}}/M_W^{\text{true}})^2}{1 + (M_\nu/M_W)^2}, \quad (11)$$

which follows from demanding a similar distribution in the “endpoints” obtained from “off-shell” W bosons in the different scenarios. Second, the width will also affect the lepton P_z distribution, although to a much smaller extent than the mass scale. In Fig. 6, we examine how well we can

⁸The same effect would be observed in the M_T distribution [59].

FIG. 6. Results from a χ^2 fit to the one-dimensional $P_{\ell T}$ distribution for a data sample of 10 000 events. The fitted parameters are M_ν and Γ_W/M_W , with M_W computed from (8). The study point ($\times$) has $M_W = 1000$ GeV, $M_\nu = 500$ GeV, and $\Gamma_W = 50$ GeV. The dashed line marks the flat direction (11). The color bar indicates the $\chi^2/\text{d.o.f.}$ (we used 100 bins).

simultaneously measure M_ν and Γ_W by determining the χ^2 fit to the one-dimensional $P_{\ell T}$ distribution for the study point with $M_W = 1000$ GeV, $M_\nu = 500$ GeV, and $\Gamma_W = 50$ GeV, which is indicated on the plot with the $\times$ symbol. The dashed line marks the relatively flat direction in the χ^2 which is described by Eq. (11).

VIII. SIMULTANEOUS MEASUREMENT OF THE MASS SCALE AND THE WIDTH WITH THE MEM

We are now ready to measure the mass scale (M_ν) and the width (Γ_W) with the MEM. In Fig. 7 we start by measuring a single parameter, for example the mass scale, M_ν (left panel) or the width, Γ_W (right panel). In either case, one has to make an ansatz for the second parameter (the width and the mass scale, respectively). The ansatz may or may not be correct, which motivates the *simultaneous*

FIG. 7. One-dimensional scan in M_ν for a fixed width of $5\% \times M_W$ and M_W given by Eq. (9) (left) and a one-dimensional scan in Γ_W with M_W and M_ν fixed (right), for the W^+ sample, with the likelihood calculated using 10 000 events.

FIG. 8. Simultaneous measurement of the mass M_W and the width, Γ_W , of the heavy resonance with the MEM for W^+ production (left) and W^- production (right). The $\times$ (+) marks the input values (the result from the fit). The dashed line represents the relation (11). Contours represent $-2 \ln(\mathcal{L}/\mathcal{L}_{\max})$ calculated with 1000 events.

measurement of the two parameters with the MEM, as shown in Fig. 8. The input values of the parameters were $M_W = 1000$ GeV and $M_\nu = 500$ GeV, which results in $\mu = 375$ GeV. The W width was 5% of its mass, i.e., $\Gamma_W = 50$ GeV. We chose the chirality of the couplings to be left handed, i.e., $g_R^q = g_R^\ell = 0$, which was assumed to be known. This assumption is harmless, since, as we show in the next section, the chirality of the couplings to the W can also be measured analogously with the MEM.

IX. MEASUREMENT OF THE CHIRALITY OF THE COUPLINGS

Having measured the two masses and the width, the only remaining task is to measure the chirality of the couplings.⁹ In analogy to Fig. 8, Fig. 9 shows a simultaneous extraction of the chirality of the couplings to quarks and leptons, for fixed $M_W = 1000$ GeV and $\Gamma_W = 50$ GeV (the nominal values measured in Fig. 8). As one would expect from Eq. (7), the contour lines in the left plot of Fig. 9 are given by

$$\cos(2\varphi_\ell) \cos(2\varphi_q) = \text{constant} \equiv \cos(2\varphi_{\text{rel}}), \quad (12)$$

where φ_{rel} parametrizes the relative chirality of the two vertices. Figure 9 reveals that the chirality of the couplings can be measured very well [up to the degeneracy described by Eq. (12)]. This is because the $P_{\ell z}$ distribution is very sensitive to the chirality,¹⁰ as shown in the right panel of Fig. 9, where we plot the $P_{\ell z}$ distribution for the correct mass spectrum and the correct Γ_W , but for three different choices of the couplings: $\varphi_q = \varphi_\ell = 0^\circ$ (blue); $\varphi_q = \varphi_\ell = 45^\circ$ (green); and $\varphi_\ell = 0^\circ$ and $\varphi_q = 90^\circ$

⁹In practice, these measurements will be done simultaneously; see the next section.

¹⁰We note that in the case when the BSM signal of Fig. 1(a) is due to a W' gauge boson decaying to a SM neutrino, the chirality of the couplings can in principle also be determined by studying the $W' - W$ interference effects in the M_T distribution [60].

FIG. 9. Left: fit to the chirality of the quark and lepton couplings to the heavy resonance. The input study point has $M_W = 1000$ GeV, $\Gamma_W = 50$ GeV, $\varphi_\ell = 0$, and $\varphi_q = 0$. Contours represent $-2\ln(\mathcal{L}/\mathcal{L}_{\max})$ calculated with 1000 events. Right: lepton P_z distributions for different chiralities, obtained from analytic expressions and LHAPDF pdfs [57] for $M_W = 1000$ GeV and $\Gamma_W = 0$.

(orange). The lesson to be learned from all these exercises so far is that one should not attempt to do mass measurements from the shapes of the kinematic distributions unless one is sure about the chiralities of the couplings; if the chiralities are *a priori* unknown, then one should fit to the masses, widths and chiralities simultaneously [61]. This ultimate exercise is performed in the next section.

X. SIMULTANEOUS MEASUREMENT OF ALL PARAMETERS

After the preliminary exercises shown in the previous sections, we now attempt to simultaneously measure the relevant parameters (1) using the MEM.

As already shown in Eqs. (7) and (12), we cannot extract the individual chiralities φ_ℓ and φ_q , but only the relative chirality φ_{rel}

$$\cos^2 \varphi_{\text{rel}} \equiv \frac{\cos(2\varphi_\ell) \cos(2\varphi_q) + 1}{2}. \quad (13)$$

As for the remaining three parameters, M_ν , M_W , and Γ_W , they will all share a common source of uncertainty coming from the overall mass scale, causing their measured values to be highly correlated. In order to reduce the covariance between the parameters being measured, we choose to reparametrize them in terms of the daughter mass M_ν , the parent mass parameter μ from Eq. (8), and the width parameter γ ,

$$\gamma = \Gamma_W \left(1 + \frac{M_\nu^2}{M_W^2} \right), \quad (14)$$

which appears in Eq. (11). From Eqs. (8) and (14) we see that when $M_\nu = 0$, 2μ and γ are identically equal to M_W and Γ_W respectively. With this choice, we expect that 2μ and γ will be measured relatively well, while the mass scale uncertainty will only be manifested in the determination of M_ν .

FIG. 10. Simultaneous measurement of all four parameters: the daughter particle mass M_ν (blue), the parent particle mass parameter μ defined in (8) (orange), the parent particle width parameter γ defined in (14) (green), and the relative chirality φ_{rel} defined in (13) (black). We show normalized distributions of the measured values for each parameter over 100 samples of 1000 events each, using MEM. The input values for our study point were $M_\nu = 500$ GeV, $M_W = 1000$ GeV, $\Gamma_W = 50$ GeV and $\cos^2 \varphi_{\text{rel}} = 0.5$, which translates into $2\mu = 750$ GeV and $\gamma = 62.5$ GeV.

These expectations are confirmed in Fig. 10, which shows the results from our simultaneous measurement of the four parameters M_ν , 2μ , γ and $\cos^2 \varphi_{\text{rel}}$. For our purpose, we use simulated data samples of 1000 events each, and in each case we find the “measured” values of M_ν , 2μ , γ , and $\cos^2 \varphi_{\text{rel}}$ by maximizing the likelihood (3). Figure 10 shows the unit-normalized distributions of the measured values of each parameter from 100 such pseudoexperiments.

The sample mean and the standard deviation of the measured values (with the true values quoted in parentheses) are as follows:

$$M_\nu = 495 \pm 42 \text{ GeV} \quad (500 \text{ GeV}), \quad (15)$$

$$2\mu = 750 \pm 4 \text{ GeV} \quad (750 \text{ GeV}), \quad (16)$$

$$\gamma = 62.2 \pm 6.5 \text{ GeV} \quad (62.5 \text{ GeV}), \quad (17)$$

$$\cos^2 \varphi_{\text{rel}} = 0.499 \pm 0.045 \quad (0.5). \quad (18)$$

We see that the mass difference μ is very well constrained, while the measurement of the mass scale M_ν is less precise, as expected from the toy exercises performed in the lead-up to this analysis.

For completeness, we also study the correlations among the different measurements (15)–(18). They are illustrated in Fig. 11, where we show scatter plots for the individual measurements in the 100 pseudoexperiments used in Fig. 10. The four panels of Fig. 11 focus on certain

FIG. 11. Correlations among the measured values of the parameters: M_ν and M_W (top left), M_ν and 2μ (top right), M_ν and Γ_W (bottom left), and M_ν and γ (bottom right).

interesting pairs of variables: M_ν and M_W (top left), M_ν and 2μ (top right), M_ν and Γ_W (bottom left), and M_ν and γ (bottom right). In the top left corner of each panel in Fig. 11 we show the corresponding Pearson correlation coefficient¹¹ between the two variables used in the plot.

The top left panel in Fig. 11 demonstrates the significant correlation among the measured values of M_ν and M_W . This was the primary motivation for trading M_W for the parameter μ via Eq. (8). Indeed, the top right panel in Fig. 11 confirms that the correlation between M_ν and μ is much milder. Similarly, the correlation between M_ν and Γ_W (bottom left panel) gets ameliorated once we switch from Γ_W to the parameter γ defined in (14); see the bottom right panel in Fig. 11.

XI. THE CASE OF PAIR PRODUCTION

Now let us consider pair production, as in the second diagram of Fig. 1. We generate $q\bar{q} \rightarrow W^+W^- \rightarrow 2\ell + MET$ events using MADGRAPH5_AMC@NLO at $\sqrt{s} = 13$ TeV, again without cuts or detector simulation. We set $M_W = 1000$ GeV, $M_\nu = 500$ GeV, and Γ_W to 5% of the parent mass. For simplicity we keep only the s -channel diagram as the t - and u -channel diagrams are more model dependent. Also we do not include the Z -boson in the s -channel diagram because our W' -like particle may not participate in the weak interaction.

As in Fig. 2, the mass “splitting” between the W and the ν can be easily measured, this time from the endpoint of the

¹¹The Pearson correlation coefficient between variables x and y is computed as

$$r_{xy} = \frac{\sum_i (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_i (x_i - \bar{x})^2} \sqrt{\sum_i (y_i - \bar{y})^2}}. \quad (19)$$

FIG. 12. The same as Fig. 5, but for the case of pair production, where we study the M_{T2} distribution instead of P_T . In the lower panel we show the bin-by-bin ratio of the number of events for different widths, normalized to the case of $\Gamma_W = 50$ GeV.

distribution of M_{T2} instead of M_T [5]. The same combination of masses (8) will be constrained.

In analogy to Fig. 5, in Fig. 12 we show the M_{T2} distribution for several values of the width, Γ_W , illustrating the smearing of the kinematic endpoint. The figure suggests that there is sensitivity to the width, but the measurement is challenging, since the effect is concentrated in the region near the endpoint, $M_{T2} \sim 950$ – 1200 GeV. We point out that the above measurements can also be performed using the M_{CT} variable [7]—it similarly has a well-defined kinematic endpoint, which will be partially smeared by the width effects.

As in the case of single production, for the measurement of the mass scale one cannot rely on endpoint measurements alone and needs to utilize the shapes of the relevant kinematic distributions. Figure 13 depicts several variables whose distributions show sensitivity to the overall mass scale. As in Fig. 3, we only consider mass spectra which obey the relation (8) and therefore satisfy the measured M_{T2} (or M_{CT}) kinematic endpoint. More specifically, we vary the mass, M_ν , of the invisible particle as shown in each panel, then choose the parent mass, M_W , from Eq. (9). The main effect of the mass scale is to provide a different boost of the parent particles: lighter (heavier) W s will be

FIG. 13. The same as Fig. 3, but for the case of pair production. We showcase several variables whose distributions are sensitive to the overall mass scale: the dilepton invariant mass, $m_{\ell\ell}$ (upper left), the transverse lepton momentum, $P_{\ell T}$ (upper right), the larger of the two longitudinal lepton momenta (in absolute value), $\max(|P_{\ell z}|, |P_{\bar{\ell} z}|)$ (lower left), and the other longitudinal lepton momentum with its sign chosen as $\text{sgn}(P_{\ell z} P_{\bar{\ell} z}) \min(|P_{\ell z}|, |P_{\bar{\ell} z}|)$ (lower right).

produced with a higher (lower) boost. While the parent boost itself is unobservable, its effects are reflected in the kinematic distributions of the visible decay products. For example, when the W s are highly boosted in the transverse plane, we would expect the leptons to be back to back. Any time the W s are highly boosted in the CM frame (whether in the transverse plane or along the beam axis) we expect a larger invariant mass of the visible leptons, as well as higher (on average) values of their transverse and longitudinal momenta. These expectations are confirmed by Fig. 13, in which we show distributions of the dilepton invariant mass, $m_{\ell\ell}$ (upper left panel), the transverse lepton momentum, $P_{\ell T}$ (upper right panel), and the longitudinal lepton momenta (lower two panels). We note that the boost effect is seen better in the distribution of the *larger* of the two longitudinal lepton momenta (in absolute value), $\max(|P_{\ell z}|, |P_{\bar{\ell} z}|)$, which is shown in the lower left panel. The other longitudinal lepton momentum, $\min(|P_{\ell z}|, |P_{\bar{\ell} z}|)$, is then plotted in the lower right panel, with the sign chosen so that it is positive (negative) when the two longitudinal lepton momenta have equal (opposite) signs.

Figure 13 demonstrates that the kinematic distributions of the visible particles do, in principle, contain information about the mass scale, which can then be extracted from a fit to these (one-dimensional) distributions, as was done in Fig. 4. However, the MEM will have better sensitivity, as it

takes into account the *correlations* among the different variables.

XII. PARAMETER MEASUREMENTS IN THE CASE OF PAIR PRODUCTION WITH THE MEM

In analogy to single production, we now apply the MEM to measure simultaneously the mass scale, the width (Fig. 14), and the chirality of the lepton couplings (Fig. 15). As expected, the MEM is quite successful in determining all the parameters in (1). In particular, the $+$ symbol in Fig. 14 denotes the result from our fit, which is close to the input parameter values (marked with $\times$). In Fig. 15, a minimum of the negative log-likelihood distribution is always found at the true input value for the chirality (marked with a vertical dotted line). Note that in

FIG. 14. The same as the left panel in Fig. 8, but for pair production, i.e., the second diagram in Fig. 1. Contours represent $-2 \ln(\mathcal{L}/\mathcal{L}_{\max})$ calculated with 500 events. The $\times$ (+) marks the input values (the result from the fit).

FIG. 15. Fit to the chirality of the lepton couplings in the case of pair production as in the second diagram of Fig. 1 using 600 events. In the left panel, the couplings were chosen to be purely chiral, $\varphi_\ell = 0$, while in the right panel they were vectorlike: $\varphi_\ell = 45^\circ$.

general, the chirality is determined only up to a two-fold ambiguity, $\varphi_\ell \rightarrow \pi/2 - \varphi_\ell$, reflecting the symmetry of the underlying squared amplitude.

The results displayed in Figs. 14 and 15 demonstrate the power of the MEM for parameter measurements in the challenging event topology of Fig. 1(b), thus generalizing and strengthening the conclusions from the previous studies performed in Refs. [38–40].

XIII. CONCLUSIONS

We have presented methods to measure masses, widths, and couplings, and, in particular, shown that (a) all of these parameters can be simultaneously measured using the MEM and (b) that the physics which gives sensitivity to each of these parameters is more transparent than it would be with many MVAs. While we have focused on the case of W s/ W 's, the approach and conclusions can be readily

generalized to nearly any SM or BSM scenario. We therefore look forward to the utilization of such methods during the continued successful operation of the LHC.

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