Safe trinification

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We provide a UV safe trinification theory in which the Standard Model is embedded. Using recently developed large number-of-flavor techniques, safety is achieved by adding to the theory gauged vectorlike fermions. We find that all gauge, scalar quartic, and Yukawa couplings achieve an interacting ultraviolet fixed point below the Planck scale. We find renormalization group flow solutions matching the Standard Model in the IR, indicating a truly UV completion of the Standard Model. Imposing constraints that realistic top, Higgs, bottom, and tau masses are recovered, we find the set of allowed solutions to be quite restrictive. Furthermore, we find there exists a lower bound of about 10 TeV for the symmetry breaking scale, making this model vulnerable to experiment.

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I. INTRODUCTION

According to Wilson [1,2], a theory is fundamental if it features an ultraviolet (UV) fixed point that is either non-interacting (asymptotically free) [3–12] or interacting (asymptotically safe) [13–15]. The first indisputable and precisely calculable example of a four-dimensional, non-supersymmetric, complete asymptotically safe quantum field theory without gravity was discovered in [13]. Recently, starting with the conjecture of a *safe* rather than *free* QCD [16], the first implementation of a large N_F (number of flavors) technique [18–22] to the whole Standard Model (with summation only in the gauge couplings) was studied in [23], opening the way to the various safe extensions of the Standard Model [23–28]. Later on, extension of the large N_F summation to both the gauge and Yukawa couplings was studied in [29], a gaugeless study with only Yukawa summation appeared in [30], Yukawa summation in Abelian gauge theory in [31] and for the first time to all the couplings in [25,32], including the semisimple gauge group in [32]. These studies have led, e.g., to enrich the original conformal window [33,34], reviewed in [35,36], with a novel asymptotically safe region (“conformal window 2.0”) in [37].

At the grand unified theory (GUT) frontier, the first asymptotically safe Pati-Salam model by using the large number of flavor dynamics is studied in [28], providing a

realistic possibility that the Standard Model can be UV complete. The gauge, the Yukawa, and scalar quartic couplings are unified by a dynamical rather than a symmetry principle. In this work, by using the acquired knowledge of the large N_F technique, we construct a novel “safe trinification” extension by adding vectorlike fermions and showing that all couplings acquire a UV fixed point at energies that are far below the Planck scale. The trinification model was first proposed in [38–40], and for more recent studies see, e.g., [6,41–47]. The separation of scales between the UV fixed point and the Planck scale allows us to study the physics around the UV fixed point while ignoring the gravitational corrections. The interplay with gravity has been investigated in several recent investigations [48–54] and it will not be considered here. Differently from the usual grand unified scenarios [55] in which only the gauge couplings unify because of their embedding into a larger group structure and then they eventually become free, in the present scenario we have that Yukawa and scalar self-couplings are intimately linked because of the safe dynamics with their high energy behavior tamed by the presence of an interacting fixed point. It should also be noted that there is actually a need for UV safety in minimal trinification models. It has been shown in [6] that to make all the gauge, Yukawa, and quartic couplings asymptotically free is highly nontrivial and the first exploration of the asymptotically free trinification model failed for more than two Higgs multiplets. Later on, in [41], it was shown that trinification models could be rendered asymptotically free by going beyond the minimal construction, with the infrared matching of the Standard Model still an unanswered question. In this paper we focus on the minimal trinification model for which UV safety is the only choice.

Our paper is organized as follows. In Sec. II we review and introduce the trinification [56] extension of the SM.

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In Sec. III, we construct the minimal vectorlike extension able to support a safe scenario. We analyze and classify the UV fixed point structure of the model and also develop the large N_F improved renormalization group (RG) equations and determine the couplings' evolution. In Sec. IV, we provide a detailed analysis to show the possibility of the safe trinification model to match the SM at IR. We offer our conclusions in Sec. V. In the Appendix we summarize the one-loop RG equations for the trinification model investigated here.

II. TRINIFICATION EXTENSION OF THE STANDARD MODEL

Consider the time-honored trinification gauge symmetry group G_{TR} [38–40]:

$$G_{\text{TR}} = SU(3)_C \otimes SU(3)_L \otimes SU(3)_R, \quad (2.1)$$

with gauge couplings g_3 , g_L , and g_R , respectively. Here $SU(3)_C$ denotes the SM color gauge group. The gauge couplings in the trinification model (g_c, g_L, g_R) and the ones in the Standard Model (g_3, g_2, g_Y) are related by

$$g_c = g_3, \quad g_L = g_2, \quad g_R = \frac{2g_2g_Y}{\sqrt{3g_2^2 - g_Y^2}}. \quad (2.2)$$

Compared with Pati-Salam theory, the SM quark and lepton fields are *not* unified into the G_{TR} irreducible representations. However, this disadvantage becomes an advantage in the sense that it is easier to realize the quark/lepton mass splitting. The colored matter content of the minimal trinification model is given by

$$\begin{aligned} \psi_{Q_L} &= \begin{pmatrix} u_L^1 & u_L^2 & u_L^3 \\ \mathcal{D}_L^1 & \mathcal{D}_L^2 & \mathcal{D}_L^3 \\ \mathcal{D}_L^1 & \mathcal{D}_L^2 & \mathcal{D}_L^3 \end{pmatrix} \sim (3, \bar{3}, 1), \\ \psi_{Q_R} &= \begin{pmatrix} u_R^1 & u_R^2 & u_R^3 \\ \mathcal{D}_R^1 & \mathcal{D}_R^2 & \mathcal{D}_R^3 \\ \mathcal{D}_R^1 & \mathcal{D}_R^2 & \mathcal{D}_R^3 \end{pmatrix} \sim (3, 1, \bar{3}), \end{aligned} \quad (2.3)$$

where $i = 1, 2, 3$ is a color index; $\mathcal{D}'$ denotes both a new color triplet and an $SU(2)$ singlet quark with the same quantum number as the Standard Model down quark d . Thus, $\mathcal{D}'$ and $\mathcal{D}$ will mix and the actual SM down quark d (and the actual mass eigenstate of the new heavy quark d') will be a linear combination of $\mathcal{D}'$ and $\mathcal{D}$. On the other hand, the lepton content in this minimal trinification model is given by

$$\psi_E = \begin{pmatrix} \bar{\nu}_L' & e_L' & e_L \\ \bar{e}_L' & \nu_L' & \nu_L \\ \bar{e}_R & \bar{\nu}_R & \nu' \end{pmatrix} \sim (1, 3, \bar{3}), \quad (2.4)$$

where $\mathcal{L} = (e_L, \nu_L)$ is the usual lepton doublet, while $\mathcal{E} = (e_L', \nu_L')$ denotes the heavier lepton doublet with the same hypercharge. Similar to the quark case, the two lepton doublets $\mathcal{L}$ and $\mathcal{E}$ will also mix and the actual mass eigenstates after mixing are denoted as L and E (shown in more detail later on).

In order to induce the breaking of G_{TR} to the SM gauge group, we introduce two scalar triplet fields Φ_1, Φ_2 which transform under the G_{TR} as $(1, 3, \bar{3})$:

$$\Phi_a = \begin{pmatrix} \phi_1^a & \phi_2^a & \phi_3^a \\ S_1^a & S_2^a & S_3^a \end{pmatrix}, \quad (a = 1, 2), \quad (2.5)$$

where ϕ_i^a ($i = 1, 2, 3$) denotes the Higgs doublets, while S_i^a ($i = 1, 2, 3$) denotes the singlets. Note that it is argued in [41] that in order to match correctly three generations of the Standard Model matter content, three scalar triplets are required. However, in this work, for simplicity, we only focus on the case with two scalar triplets which are sufficient to address the correct flavor structure for the third generation. The vacuum configuration of the scalar triplet is given as

$$\langle \Phi_1 \rangle = \begin{pmatrix} u_1 & 0 & 0 \\ 0 & u_2 & 0 \\ 0 & 0 & v_1 \end{pmatrix}, \quad \langle \Phi_2 \rangle = \begin{pmatrix} n_1 & 0 & n_3 \\ 0 & n_2 & 0 \\ v_2 & 0 & v_3 \end{pmatrix}, \quad (2.6)$$

where normally v_3 and v_1 play the role to break G_{TR} to left-right model, i.e., $SU(3)_C \times SU(3)_L \times SU(3)_R \rightarrow SU(3)_C \times SU(2)_R \times SU(2)_L \times U(1)_{B-L}$, while v_2 as an intermediate scale (between the trinification symmetry breaking scale and the electroweak scale) will further break the left-right symmetry to the Standard Model gauge group. On the other hand u_1, u_2 and n_1, n_2, n_3 are at the electroweak scale to trigger the electroweak symmetry breaking. In our work, for simplicity, we set $v_3 = 0$ (v_1 itself is sufficient to trigger the trinification symmetry breaking) and assume v_2 is at the same scale as v_1 . Thus, the trinification model breaks directly to the SM without the left-right model as an intermediate step.

A. The Yukawa sector

The Yukawa terms for the quark sector are given by

$$\mathcal{L}_{\text{Yuk}}^Q = \bar{\psi}_{Q_R} \psi_{Q_L} (y_{\psi_{Q1}} \Phi_1 + y_{\psi_{Q2}} \Phi_2) + \text{H.c.} \quad (2.7)$$

In terms of the $SU(2)_L$ doublet, Eq. (2.7) reads

$$\begin{aligned} \mathcal{L}_{\text{Yuk}}^Q &= m_{d'} \bar{d}_R' d_L' + \sum_{a=1}^2 y_{\psi_{Qa}} [(-s_a \bar{d}_R + c_a \bar{d}_R') Q \phi_1^a + \bar{u}_R Q \phi_2^a \\ &\quad + (c_a \bar{d}_R + s_a \bar{d}_R') Q \phi_3^a] + \text{H.c.}, \end{aligned} \quad (2.8)$$

where Q denotes the SM quark $SU(2)_L$ doublet; $s_\alpha \equiv \sin \alpha$, $c_\alpha \equiv \cos \alpha$ are the mixing angles between $\mathcal{D}$, $\mathcal{D}'$ defined below:

$$\begin{pmatrix} d \\ d' \end{pmatrix} = \begin{pmatrix} -s_\alpha & c_\alpha \\ c_\alpha & s_\alpha \end{pmatrix} \begin{pmatrix} \mathcal{D} \\ \mathcal{D}' \end{pmatrix}, \quad \tan \alpha = \frac{y_{\psi_{Q1}} v_1}{y_{\psi_{Q2}} v_2}; \quad (2.9)$$

and the mass eigenvalue $m_{d'}$ is given by

$$m_{d'} = \sqrt{y_{\psi_{Q1}} v_1^2 + y_{\psi_{Q2}} v_2^2}. \quad (2.10)$$

After electroweak symmetry breaking, ϕ_1^1, ϕ_2^1 obtain the vacuum expectation values (VEVs) u_1 and u_2 , respectively, and the Standard Model quarks acquire masses (for simplicity, we have set $n_1 = n_2 = n_3 = 0$):

$$m_t = y_{\psi_{Q1}} u_2, \quad m_b = y_{\psi_{Q1}} u_1 s_\alpha, \quad (2.11)$$

where m_t and m_b denote the top and bottom quarks, respectively. Since $(u_1^2 + u_2^2)^{1/2} = 246$ GeV as the vacuum expectation value of the Higgs field at the electroweak scale, we thus have

$$u_1 = \frac{246}{\sqrt{1 + (\frac{m_t}{m_b} \sin \alpha)^2}}, \quad u_2 = \frac{246}{\sqrt{1 + (\frac{m_b}{m_t} \sin \alpha)^2}}. \quad (2.12)$$

Similarly, the Yukawa terms for the Lepton sector are given by

$$\mathcal{L}_{\text{Yuk}}^E = \frac{1}{2} \psi_E \psi_E (y_{\psi_{E1}} \Phi_1 + y_{\psi_{E2}} \Phi_2) + \text{H.c.}, \quad (2.13)$$

where the contraction is realized by the ϵ symbol. In terms of the $SU(2)_L$ doublet, Eq. (2.13) can be written out explicitly as

$$\begin{aligned} \mathcal{L}_{\text{Yuk}}^E &= m_E \bar{E} E \\ &+ \sum_{a=1}^2 y_{\psi_{Ea}} \{ -[\bar{E}(-c_\beta \bar{\nu}_R - s_\beta \nu') - (c_\beta E + s_\beta L) \bar{e}_R] \phi_1^a \\ &+ (E \nu' - L \bar{\nu}_R) \phi_2^a \\ &+ [\bar{E}(s_\beta \bar{\nu}_R - c_\beta \nu') - (-s_\beta E + c_\beta L) \bar{e}_R] \phi_3^a \} + \text{H.c.}, \end{aligned} \quad (2.14)$$

where L denotes the SM quark $SU(2)_L$ doublet; $s_\beta \equiv \sin \beta$, $c_\beta \equiv \cos \beta$ are the mixing angles between $\mathcal{L}$, $\mathcal{E}$ defined below:

$$\begin{pmatrix} E \\ L \end{pmatrix} = \begin{pmatrix} -s_\beta & c_\beta \\ c_\beta & s_\beta \end{pmatrix} \begin{pmatrix} \mathcal{E} \\ \mathcal{L} \end{pmatrix}, \quad \tan \beta = \frac{y_{\psi_{E1}} v_1}{y_{\psi_{E2}} v_2}; \quad (2.15)$$

and the mass eigenvalue m_E is given by

$$m_E = \sqrt{y_{\psi_{E1}} v_1^2 + y_{\psi_{E2}} v_2^2}. \quad (2.16)$$

After electroweak symmetry breaking, the SM lepton masses are given by

$$m_e = y_{\psi_{E1}} u_1 s_\beta, \quad m_{\nu_L, \nu_R} = y_{\psi_{E1}} u_2, \quad m_{\nu'} \sim \frac{y_{\psi_{E1}}^2 u_1 u_2 s_\beta}{m_E}. \quad (2.17)$$

It is clear that the above tree level neutrino mass Eq. (2.17) has the problem that the left- and right-handed neutrino masses are degenerate. Fortunately, the radiative loop corrections can cure this problem and provide a reasonable neutrino mass spectrum.

B. Radiative neutrino mass

From Fig. 1, the neutrino masses receive large radiative loop contributions. The total contribution to the two-point function is proportional to

$$F_E = \frac{m_E}{(4\pi)^2} \frac{1}{2} \left(\frac{m_{H1}^2}{m_E^2 - m_{H1}^2} \log \frac{m_{H1}^2}{m_E^2} - \frac{m_{H2}^2}{m_E^2 - m_{H2}^2} \log \frac{m_{H2}^2}{m_E^2} \right), \quad (2.18)$$

where m_{H1} and m_{H2} are the masses of the two Higgs doublet fields.

The one-loop neutrino mass matrix is thus written as

$$M_\nu^{\text{1loop}} = \begin{bmatrix} 0 & -y_{\psi_{E1}} u_2 & 0 \\ -y_{\psi_{E1}} u_2 & s_{\alpha-\beta} c_\beta y_{\psi_{E1}}^2 F_E & (s_{2\beta} s_\alpha - c_\alpha) y_{\psi_{E1}}^2 F_E \\ 0 & (s_{2\beta} s_\alpha - c_\alpha) y_{\psi_{E1}}^2 F_E & c_{\alpha-\beta} s_\beta y_{\psi_{E1}}^2 F_E \end{bmatrix}. \quad (2.19)$$

FIG. 1. This figure shows the one-loop radiative correction to the neutrino mass.

To obtain a phenomenologically viable neutrino mass, the elements of the matrix Eq. (2.19) should have the following form:

$$M_\nu^{\text{loop}} = \begin{bmatrix} 0 & 10 \text{ GeV} & 0 \\ 10 \text{ GeV} & 0\text{--}1 \text{ TeV} & 0.33\text{--}1 \text{ TeV} \\ 0 & 0.33\text{--}1 \text{ TeV} & 1 \text{ KeV} \end{bmatrix}, \quad (2.20)$$

where the three mass eigenvalues will correspond to the physical mass of the two sterile neutrinos (ν_R, ν') and the SM neutrino ν_L . Comparing Eqs. (2.19) and (2.20), we find from the (3,3) element of the matrix that $c_{\alpha-\beta} s_\beta \sim 10^{-8}\text{--}10^{-9}$. There are only two solutions to satisfy the above constraint, either $\beta \ll 1$ or $\alpha - \beta - \frac{\pi}{2} \sim \pm 10^{-9}$. The case $\beta \ll 1$, from Eq. (2.17), leads to an extremely large Yukawa $y_{\psi_{E1}}$, which is not acceptable. Thus, we choose

$$\alpha - \beta - \frac{\pi}{2} \sim \pm 10^{-9}, \quad (2.21)$$

in the analysis later on.

C. Color scalar field and proton decay issue

Sometimes, it occurs in the literature that color scalars are introduced to make the Yukawa sector satisfy a cyclic permutation symmetry. The introduction of the color scalar fields has the advantage of incorporating much larger radiative corrections to the neutrinos. In this case, in Eq. (2.18), the m_{H1} should be replaced by the new colored scalar field $m_{B_{H1}}$ and the lepton doublet m_E should be replaced by the heavy quark m_B . The radiative corrections to the neutrino mass can be much larger because color scalar fields are not constrained to be around the electroweak scale and thus $m_{B_{H1}} \gg m_{H1}$, resulting in much larger radiative loop corrections.

However, colored scalar fields have a problem generating tree level proton decay and thus in this work we only introduce the scalar fields that are color singlets. The leptonic scalar in this work does not introduce proton decay because it carries the same baryon number in both the quark and leptonic Yukawa interactions.

D. The scalar sector

In this minimal model, we have only introduced two Higgs triplets Φ_1 and Φ_2 . The most general scalar potentials can be written as

$$V = V_{1111} + V_{2222} + V_{1122} + V_{1222} + V_{1112}, \quad (2.22)$$

where in the following we could further simplify the potential by introducing a Z_2 discrete symmetry between

Φ_1 and Φ_2 and thus only the terms V_{1111} , V_{2222} , and V_{1122} remain. We have

$$\begin{aligned} V_{1111} &= \lambda_{1a} \text{Tr}(\Phi_1^\dagger \Phi_1)^2 + \lambda_{1b} \text{Tr}(\Phi_1^\dagger \Phi_1 \Phi_1^\dagger \Phi_1) \\ V_{2222} &= \lambda_{2a} \text{Tr}(\Phi_2^\dagger \Phi_2)^2 + \lambda_{2b} \text{Tr}(\Phi_2^\dagger \Phi_2 \Phi_2^\dagger \Phi_2) \\ V_{1122} &= \lambda_a \text{Tr}(\Phi_1^\dagger \Phi_1) \text{Tr}(\Phi_2^\dagger \Phi_2) + \lambda_b |\text{Tr}(\Phi_1^\dagger \Phi_2)|^2 \\ &\quad + \lambda_c \text{Tr}(\Phi_1^\dagger \Phi_1 \Phi_2^\dagger \Phi_2) + \lambda_d \text{Tr}(\Phi_1 \Phi_1^\dagger \Phi_2 \Phi_2^\dagger) \\ &\quad + \text{Re} \lambda_e \text{Tr}(\Phi_1^\dagger \Phi_2)^2 + \text{Re} \lambda_f \text{Tr}(\Phi_1^\dagger \Phi_2 \Phi_1^\dagger \Phi_2). \end{aligned} \quad (2.23)$$

III. RENORMALIZATION GROUP ANALYSIS

In this section, we perform an RG analysis of the minimal trinification model. After introducing the large- N beta functions of the gauge, Yukawa, and quartic couplings, we will demonstrate the existence of UV fixed point solutions. By using the RG equation as a bridge to connect UV and IR, we will then study the IR phenomenological implications. All gauge, Yukawa, and scalar couplings are listed in Table I, while their corresponding one-loop RG beta functions are listed in the Appendix.

A. Large- N beta function

To proceed, we work in the large N_F limit, employing the $1/N_F$ expansion [18–20,57], which was recently first applied to the whole SM [23] and to the minimal Pati-Salam model [28]. We introduce $N_F \gg 1$ vectorlike fermions, which transform nontrivially under G_{TR} . In this framework, the leading order RG contributions in the $1/N_F$ expansion of the relevant Feynman diagrams can be resummed, as shown in Fig. 2 (only gauge coupling cases are shown), and a closed form of the resummation is provided. This nonperturbative effect induces an interacting fixed point for both the Abelian and non-Abelian gauge interactions of the SM [23], whose existence is guaranteed due to the pole structure in the closed form expression [19,20].

We introduce three sets of vectorlike fermions charged under G_{TR} with the following charge assignment:

$$N_{F_C}(3, 1, 1) \oplus N_{F_L}(1, 3, 1) \oplus N_{F_R}(1, 1, 3), \quad (3.1)$$

TABLE I. Gauge, Yukawa, and scalar quartic couplings of the trinification model.

Gauge couplings	Yukawa couplings	Scalar couplings
$SU(3)_C : g_c$	$\psi_{Q_{L/R}} : y_{\psi_{Q1}}, y_{\psi_{Q2}}$	$V_{1111} : \lambda_{1a}, \lambda_{1b}$
$SU(3)_L : g_L$	$\psi_E : y_{\psi_{E1}}, y_{\psi_{E2}}$	$V_{2222} : \lambda_{2a}, \lambda_{2b}$
$SU(3)_R : g_R$		$V_{1122} : \lambda_a, \lambda_b, \lambda_c, \lambda_d, \lambda_e, \lambda_f$

FIG. 2. Feynman diagrams for gauge field renormalization at order $1/N_F$. Diagrams (a) and (b) are present in both the Abelian and non-Abelian two-point functions, while (c) and (d) only exist in the non-Abelian theory.

where we have chosen each set of vectorlike fermions to have nontrivial charges only under one simple gauge group to avoid extra contributions in the summation of the semisimple group.

B. Large- N gauge beta function

To leading order in $1/N_F$, the higher order (ho) contributions of the bubble diagrams (i.e., those in Fig. 2) to the gauge beta functions were first calculated in [18] for only an Abelian group and later generalized to the non-Abelian simple group case [20]. Here we summarize the results.

The leading $1/N_F$ order bubble diagrams' contributions are given by

$$\beta_i^{\text{ho}} = \frac{2A_i\alpha_i}{3} \frac{H_{1_i}(A_i)}{N_{F_i}},$$

$$\alpha_i \equiv \frac{g_i^2}{(4\pi)^2} \quad (i = L, R, C), \quad (3.2)$$

with the functions H_{1_i} and the 't Hooft couplings A_i given by

$$A_i = 4\alpha_i T_R N_{F_i}$$

$$H_{1_i} = -\frac{11}{2} N_{c_i} + \int_0^{A_i/3} I_1(x) I_2(x) dx \quad (N_{c_i} = 3)$$

$$I_1(x) = \frac{(1+x)(2x-1)^2(2x-3)^2 \sin(\pi x)^3}{(x-2)\pi^3}$$

$$\times (\Gamma(x-1)^2 \Gamma(-2x))$$

$$I_2(x) = \frac{N_{c_i}^2 - 1}{N_{c_i}} + \frac{(20 - 43x + 32x^2 - 14x^3 + 4x^4)}{2(2x-1)(2x-3)(1-x^2)} N_{c_i}. \quad (3.3)$$

The Dynkin indices are $T_R = 1/2(N_{c_i})$ for the fundamental (adjoint) representation. The RG functions of the gauge couplings (see the Appendix), including the contributions from the resummed bubble diagrams, are

$$\beta_{\alpha_L}^{\text{tot}} = \beta_{\alpha_L}^{\text{loop}} + \beta_{\alpha_L}^{\text{ho}} = -(10 + n_H) \alpha_L^2 + \frac{2A_L \alpha_L}{3} \frac{H_{1_L}(A_L)}{N_{F_L}}$$

$$\beta_{\alpha_R}^{\text{tot}} = \beta_{\alpha_R}^{\text{loop}} + \beta_{\alpha_R}^{\text{ho}} = -(10 + n_H) \alpha_R^2 + \frac{2A_R \alpha_R}{3} \frac{H_{1_R}(A_R)}{N_{F_R}}$$

$$\beta_{\alpha_C}^{\text{tot}} = \beta_{\alpha_C}^{\text{loop}} + \beta_{\alpha_C}^{\text{ho}} = -10 \alpha_C^2 + \frac{2A_C \alpha_C}{3} \frac{H_{1_C}(A_C)}{N_{F_C}}, \quad (3.4)$$

where n_H denotes the number of scalar triplets ($n_H = 2$ in our case). The $\beta_{a_L}^{\text{1loop}}, \beta_{a_R}^{\text{1loop}}, \beta_{a_c}^{\text{1loop}}$ denote the original one-loop RG beta functions of the three gauge couplings without bubble diagram contributions, whereas $\beta_{a_L}^{\text{tot}}, \beta_{a_R}^{\text{tot}}, \beta_{a_c}^{\text{tot}}$ are the total RG beta functions including the higher order bubble diagram contributions up to $1/N_F$ order. There are two reasons why only one-loop RG beta functions of the gauge couplings are used here. First, the current large N_F improved beta functions of Yukawa and quartic couplings are built upon their corresponding one-loop beta functions only (see Sec. III C for more details). Second, since gauge beta functions at the one-loop level are decoupled from the other couplings it is easier to perform an RG analysis along the gauge coupling directions (see Sec. III F for more details).

The $H(A)$ function in Eq. (3.3) has the pole structure $H(A) \sim \log(1 - \frac{A}{3})$, which will provide a sufficiently large negative contribution slightly before reaching the pole. Hence the UV fixed point for the gauge coupling subsystem (g_c, g_L, g_R) is guaranteed by the pole structure in the bubble diagram summation. For all the non-Abelian gauge groups, the poles in the functions H_i ($i = L, R, C$) always occur at $A_i = 3$, which determines the UV fixed point of the non-Abelian gauge couplings when the set of N_{F_i} is chosen. The IR initial conditions of g_L, g_R , and g_c are obtained by using the matching conditions of Eq. (2.2) and the SM couplings are running from the EW (electroweak) scale to the trinification symmetry breaking scale. Also, for simplicity, we have assumed all the vectorlike fermions were introduced at the trinification symmetry breaking scale v_T . This is in contrast to the Pati-Salam model whose symmetry breaking scale is severely constrained by the kaon decay process $K_L \rightarrow \mu^\pm e^\mp$ (see, e.g., [58,59]), which implies the symmetry breaking scale must be larger than 2000 TeV. Trinification symmetry breaking is only constrained by the masses of the extra gauge bosons such as the Z' and W_R , which have lower bounds of a few TeV. Trinification therefore has the advantage of being within experimental reach of an upgraded LHC and future colliders.

C. Large- N Yukawa and quartic beta function

In the previous section, we exhibited the bubble diagram contributions in only the gauge coupling subsystem and presented the large N gauge beta functions. We now consider bubble diagram contributions to the Yukawa and quartic beta functions [29,32]. In the following we briefly summarize the procedure.

If the beta functions of quartic and Yukawa couplings are already known to one-loop order, the corresponding large N_F beta functions (at leading $1/N_F$ order) can be obtained by simply employing the following recipe. The large- N_F Yukawa beta function can be written in the following compact form:

$$\beta_y = c_1 y^3 + y \sum_{\alpha} c_{\alpha} g_{\alpha}^2 I_y(A_{\alpha}), \quad \text{with}$$

$$I_y(A_{\alpha}) = H_{\phi} \left(0, \frac{2}{3} A_{\alpha} \right) \left(1 + A_{\alpha} \frac{C_2(R_{\phi}^{\alpha})}{6(C_2(R_{\chi}^{\alpha}) + C_2(R_{\xi}^{\alpha}))} \right)$$

$$H_{\phi}(x) = H_0(x) = \frac{(1 - \frac{x}{3})\Gamma(4-x)}{3\Gamma^2(2 - \frac{x}{2})\Gamma(3 - \frac{x}{2})\Gamma(1 + \frac{x}{2})}, \quad (3.5)$$

where the information of the resummed fermion bubbles is already encoded and c_1, c_{α} are the standard one-loop coefficients for the Yukawa beta function, while $C_2(R_{\phi}^{\alpha}), C_2(R_{\chi}^{\alpha}), C_2(R_{\xi}^{\alpha})$ are the Casimir operators of the corresponding scalar and fermion fields. Thus, when c_1 and c_{α} are known, the full Yukawa beta function including the bubble diagram contributions can be obtained. Similarly, for the quartic coupling we write

$$\beta_{\lambda} = c_1 \lambda^2 + \lambda \sum_{\alpha} c_{\alpha} g_{\alpha}^2 I_{\lambda g^2}(A_{\alpha}) + \sum_{\alpha} c'_{\alpha} g_{\alpha}^4 I_{g^4}(A_{\alpha})$$

$$+ \sum_{\alpha < \beta} c_{\alpha\beta} g_{\alpha}^2 g_{\beta}^2 I_{g_1^2 g_2^2}^{\text{tot}}(A_{\alpha}, A_{\beta}), \quad (3.6)$$

with $c_1, c_{\alpha}, c'_{\alpha}, c_{\alpha\beta}$ the known one-loop coefficients for the quartic beta function and

$$I_{\lambda g^2}(A_{\alpha}) = H_{\phi} \left(0, \frac{2}{3} A_{\alpha} \right)$$

$$I_{g^4}(A_{\alpha}) = H_{\lambda} \left(1, \frac{2}{3} A_{\alpha} \right) + A_{\alpha} \frac{dH_{\lambda}(1, \frac{2}{3} A_{\alpha})}{dA_{\alpha}}$$

$$I_{g_1^2 g_2^2}^{\text{tot}}(A_{\alpha}, A_{\beta}) = \frac{1}{3} [I_{g_1^2 g_2^2}(A_{\alpha}, 0) + I_{g_1^2 g_2^2}(0, A_{\beta}) + I_{g_1^2 g_2^2}(A_{\alpha}, A_{\beta})]$$

$$I_{g_1^2 g_2^2}(A_{\alpha}, A_{\beta}) = \frac{1}{A_{\alpha} - A_{\beta}} \left[A_{\alpha} H_{\lambda} \left(1, \frac{2}{3} A_{\alpha} \right) - A_{\beta} H_{\lambda} \left(1, \frac{2}{3} A_{\beta} \right) \right],$$

where

$$H_{\lambda}(1, x) = \left(1 - \frac{x}{4} \right) H_0(x)$$

$$= \frac{(1 - \frac{x}{4})(1 - \frac{x}{3})\Gamma(4-x)}{3\Gamma^2(2 - \frac{x}{2})\Gamma(3 - \frac{x}{2})\Gamma(1 + \frac{x}{2})} \quad (3.7)$$

are from the resummed fermion bubbles. Thus we have now the full quartic beta function including the bubble diagram contributions when $c_1, c_{\alpha}, c'_{\alpha}, c_{\alpha\beta}$ are known. Following the above recipe, the bubble diagram improved Yukawa beta function $\beta_{y_{Q1}}$, e.g., can be written as

$$(4\pi)^2 \beta_{y_{Q1}} = (-4g_L^2 I_y(A_L) - 4g_R^2 I_y(A_R) - 8g_c^2 I_y(A_3)$$

$$+ 6y_{\psi_{Q1}}^2 + 6y_{\psi_{Q2}}^2 + 2y_{\psi_{E1}}^2) y_{\psi_{Q1}}$$

$$+ 12y_{\psi_{E1}} y_{\psi_{E2}} y_{\psi_{Q2}}. \quad (3.8)$$

The bubble diagram improved quartic beta function $\beta_{\lambda_{1a}}$ reads,

$$\begin{aligned} (4\pi)^2 \beta_{\lambda_{1a}} = & 52\lambda_{1a}^2 + 12\lambda_{1b}^2 + 2\lambda_a\lambda_b + 6\lambda_a\lambda_c + 6\lambda_a\lambda_d + 2\lambda_c\lambda_d + 9\lambda_a^2 + \lambda_b^2 + 4\lambda_e^2 + 4\lambda_f^2 \\ & + \lambda_{1a}(-16g_L^2 I_{\lambda g^2}(A_L) - 16g_R^2 I_{\lambda g^2}(A_R) + 48\lambda_{1b} + 8y_{\psi_{E1}}^2 + 12y_{\psi_{Q1}}^2) \\ & + \frac{10}{3}g_L^2 g_R^2 \times \frac{1}{3}(I_{g_1^2 g_2^2}(A_L, A_R) + I_{g_1^2 g_2^2}(0, A_R) + I_{g_1^2 g_2^2}(A_L, 0)) \\ & + \frac{11}{12}g_L^4 I_{g^4}(A_L) + \frac{11}{12}g_R^4 I_{g^4}(A_R) - 2y_{\psi_{E1}}^4. \end{aligned} \quad (3.9)$$

D. Symmetric and asymmetric cases

From the Yukawa beta functions in the Appendix, we notice that there exists a symmetry between $y_{\psi_{Q1}}$ and $y_{\psi_{Q2}}$ as well as $y_{\psi_{E1}}$ and $y_{\psi_{E2}}$. Consequently there is not a unique UV fixed point solution for the Yukawa couplings when the N_F is fixed (actually there could be infinitely many solutions).

It is convenient to divide the fixed point solutions into two cases: symmetric and asymmetric. By “symmetric” we mean that $y_{\psi_{Q1}} = y_{\psi_{Q2}}$ and $y_{\psi_{E1}} = y_{\psi_{E2}}$ at the UV fixed point. In the symmetric case, all the coupling values at the UV fixed point can be determined when N_F is fixed. In addition, from Eqs. (2.9) and (2.15), we know $|\tan \alpha| = |\tan \beta| = \frac{v_1}{v_2}$. Furthermore, a phenomenologically viable neutrino mass requires $|\alpha - \beta| \sim \frac{\pi}{2}$, providing $|\tan \alpha| = |\tan \beta|^{-1}$. Hence

$$|\tan \alpha| = |\tan \beta| = \frac{v_1}{v_2} = 1. \quad (3.10)$$

By “asymmetric” we mean that these Yukawa couplings are no longer constrained to be equal at the UV fixed point; the dynamical constraints from requiring a UV interacting fixed point in this case are not sufficient to provide a unique set of UV fixed point solutions. We therefore add one more phenomenological constraint:

$$\frac{y_b}{y_\tau} = \frac{y_{\psi_{Q1}}}{y_{\psi_{E1}}} \frac{\sin \alpha}{\sin \beta} \sim 2, \quad (3.11)$$

which is that of requiring the ratio of the bottom quark mass and the tau lepton mass to be around 2. We can further get rid of the v_1^2/v_2^2 dependence in (3.11) and write it as a function of $(y_{\psi_{Q1}}, y_{\psi_{Q2}}, y_{\psi_{E1}}, y_{\psi_{E2}})$ only. By using $|\tan \alpha| = |\tan \beta|^{-1}$ we obtain

$$\frac{v_1^2}{v_2^2} = \frac{y_{\psi_{Q2}} y_{\psi_{E2}}}{y_{\psi_{Q1}} y_{\psi_{E1}}}, \quad (3.12)$$

yielding

$$\frac{y_b}{y_\tau} = \frac{y_{\psi_{Q1}}^{3/2} (y_{\psi_{Q2}} y_{\psi_{E2}} y_{\psi_{E1}} + y_{\psi_{E2}}^2 y_{\psi_{Q1}})^{1/2}}{y_{\psi_{E1}}^{3/2} (y_{\psi_{Q1}} y_{\psi_{Q2}} y_{\psi_{E2}} + y_{\psi_{Q2}}^2 y_{\psi_{E1}})^{1/2}} \sim 2, \quad (3.13)$$

and using (3.13), we are able to find UV fixed point solutions.

E. UV fixed point solutions in the gauge-Yukawa-quartic system

In this section we will prove the existence of UV fixed point solutions for the whole gauge-Yukawa-quartic system given in Table I. Note that the gauge couplings at the UV fixed point can be treated as background values (i.e., as constants in the RG functions of other couplings) since their values at the UV fixed point are fixed when N_F is chosen. By using the one-loop RG functions in the Appendix and following the recipes from (3.5) and (3.6), we obtain the total large- N beta functions of all the Yukawa and scalar couplings. To find the UV fixed point solutions, we set $\{\beta_i = 0\}$, where i denotes all the Yukawa and scalar couplings in Table I.

Our analysis yields quite a few UV candidate fixed points for different choices of N_F . For example, in the symmetric case, for $N_{F_C} = 43$, $N_{F_L} = 93$, $N_{F_R} = 182$, we find 56 sets of candidate UV fixed point solutions. Requiring the vacuum stability conditions [60]

$$\begin{aligned} \lambda_{1a} + \lambda_{1b} &> 0 \quad (\text{for } \lambda_{1b} < 0); \\ \lambda_{1a} + \frac{1}{3}\lambda_{1b} &> 0 \quad (\text{for } \lambda_{1b} > 0), \end{aligned} \quad (3.14)$$

which guarantee that the scalar potential is bounded from below, reduces the number of candidate UV fixed point solutions to 18. One of these solutions in the symmetric case is shown in Table II, where we have also classified the fixed point solutions according to relevant (Rev) and irrelevant (Irev) characteristics. By choosing the RG flow direction from UV to IR, relevant and irrelevant correspond, respectively, to the RG flows running away from the UV fixed point or towards the UV fixed point. The relevant or irrelevant characteristics can be easily determined by plotting the beta functions as functions of the corresponding couplings and reading the direction of the RG flows. The number of relevant couplings define the dimension of

TABLE II. This table summarizes the sample UV fixed point solution in the symmetric case with sample values ($N_{F_C} = 43$, $N_{F_L} = 93$, $N_{F_R} = 182$) involving the bubble diagram contributions in the Yukawa and quartic RG beta functions. Classifications of the UV fixed point solutions of the couplings with relevant (Rev) and irrelevant (Irev) characteristics are listed.

λ_{1a}	λ_{1b}	λ_a	λ_b	λ_c	λ_d	λ_e	λ_f	$y_{\psi_{Q1}}$	$y_{\psi_{Q2}}$	$y_{\psi_{E1}}$	$y_{\psi_{E2}}$
0.10	-0.03	-0.82	0.43	1.22	-0.27	-0.30	0.50	0.80	0.80	0.24	0.24
Irev	Irev	Irev	Irev	Irev	Rev	Irev	Irev	Irev	Irev	Irev	Irev

the critical surface and only the RG flows in the critical surface come out of the UV fixed point. Hence the smaller the dimension of the critical surface, the stronger are the constraints on the RG flows, which are connected to the UV fixed point, and so the predictive power is likewise stronger. From Table II, it is clear that most of the couplings are UV irrelevant; only λ_d is relevant, implying the system is highly predictive. An alternative candidate solution in the asymmetric case is given in Tables III and IV, where we see that the symmetry between $(y_{\psi_{Q1}}, y_{\psi_{E1}})$ and $(y_{\psi_{Q2}}, y_{\psi_{E2}})$ is broken.

F. RG flow

To determine the RG flow of the system we can either consider a flow from the IR to the UV or vice versa. We shall primarily focus on the UV to IR approach, only briefly commenting on the IR to UV approach.

For the UV to IR approach one simply starts from the UV fixed point and makes the RG flows run toward the IR.

TABLE III. This table summarizes the sample UV fixed point solution (for Yukawa couplings) in the asymmetric case with sample values ($N_{F_C} = 95$, $N_{F_L} = 165$, $N_{F_R} = 62$) involving the bubble diagram contributions in the Yukawa and quartic RG beta functions. Classifications of the UV fixed point solutions of the couplings with relevant (Rev) and irrelevant (Irev) characteristics are listed.

$y_{\psi_{Q1}}$	$y_{\psi_{Q2}}$	$y_{\psi_{E1}}$	$y_{\psi_{E2}}$
0.78	0.56	0.42	0.30
Irev	Irev	Irev	Irev

TABLE IV. This table summarizes the sample UV fixed point solution (for quartic couplings) in the asymmetric case with sample values ($N_{F_C} = 95$, $N_{F_L} = 165$, $N_{F_R} = 62$) involving the bubble diagram contributions in the Yukawa and quartic RG beta functions. Classifications of the UV fixed point solutions of the couplings with relevant (Rev) and irrelevant (Irev) characteristics are listed.

λ_{1a}	λ_{1b}	λ_{2a}	λ_{2b}	λ_a	λ_b	λ_c	λ_d	λ_e	λ_f
-0.08	0.26	0.02	0.02	-0.90	0.14	0.12	1.11	-0.36	0.48
Irev	Irev	Rev	Rev	Rev	Irev	Rev	Irev	Irev	Irev

Here we use the fact that at one-loop order, the beta functions of the three gauge couplings are completely decoupled from the other couplings. In addition, since the gauge couplings are UV relevant, we can freely choose the (IR) initial conditions as the gauge coupling matching conditions at a certain trinification symmetry breaking scale. We can therefore first solve the gauge coupling RG trajectories separately and then run the remaining couplings only along the determined RG trajectories of the gauge couplings. In principle, we could also run the RG flows along the alternative relevant coupling (e.g., λ_d in the symmetric case of Table II). However, since λ_d is not decoupled from the other couplings this would be more difficult to handle.

In the IR to UV approach, the RG flow of the irrelevant couplings is constrained on the separatrices which are defined to divide the RG flow into distinct physical regions. We can therefore solve the set of equations $\beta_i = 0$ (i corresponding to all the irrelevant couplings) for all the irrelevant couplings as a function of the relevant couplings. We are thus free to choose the IR initial conditions of these relevant couplings to be compatible with the known phenomenological constraints while preserving safety at the UV scale. The disadvantage of this approach for a complicated system like trinification is that it is very hard to disentangle relevant and irrelevant couplings and analytically solve all the irrelevant couplings as a function of the relevant couplings.

We report our results in Figs. 3–6, where we show the running of the gauge, Yukawa, and scalar couplings by using the UV to IR approach for ($N_{F_C} = 95$, $N_{F_L} = 165$, $N_{F_R} = 62$). The corresponding UV fixed point solution is the one shown in Tables III and IV. The RG flows of the gauge couplings are determined once the IR conditions are given as previously noted. The IR initial conditions for g_L , g_R , and g_c are obtained by using the matching conditions (2.2) and the SM couplings run from the EW scale to the trinification symmetry breaking scale. For simplicity, the vectorlike fermions' masses are taken to be the trinification symmetry breaking scale at 10 TeV. From Figs. 3–6, it is clear that all couplings (i.e., gauge, Yukawa, and scalar quartic) achieve a safe UV fixed point. The transition scale, above which the UV fixed point is reached, is about 10^7 GeV for all the couplings. Note that this transition scale is dependent on the number of vectorlike fermions N_F . By increasing or decreasing N_F , the transition scales will correspondingly decrease or increase.

FIG. 3. RG running of the gauge couplings by using the UV to IR approach. We have chosen $N_{FC} = 95$, $N_{FL} = 165$, $N_{FR} = 62$. We set the initial conditions for g_L , g_R , g_c at the IR using the matching conditions there [see Eq. (2.2)]. For simplification, we have assumed that the vectorlike fermions under different gauge symmetry groups are exactly introduced at the trinification breaking scale, $t_{SB} = 10$ TeV, marked by a vertical dashed line.

IV. MATCHING THE STANDARD MODEL

Before we proceed, we summarize all the assumptions and simplifications we have implemented so far. First, only the third generation of the SM fermions is considered.

FIG. 4. RG running of the Yukawa couplings by using the UV to IR approach. Sub-Figs. (a), (b), (c), (d) correspond respectively to the Yukawa couplings of $y_{\psi_{Q1}}$, $y_{\psi_{E1}}$, $y_{\psi_{Q2}}$, $y_{\psi_{E2}}$. We have chosen $N_{FC} = 95$, $N_{FL} = 165$, $N_{FR} = 62$. For simplification, we have assumed that the vectorlike fermions under different gauge symmetry groups are exactly introduced at the trinification breaking scale, $t_{SB} = 10$ TeV, marked by a vertical dashed line.

Second, only two Higgs multiplets are introduced. Third, we choose to introduce all the new vectorlike fermions at the lower bound of the trinification symmetry breaking scale, which is 10 TeV. Fourth, we choose the particular vacuum configuration in Eq. (2.6) to make the trinification theory directly break to the SM without going through the intermediate step of a left-right model. Fifth, the four free input parameters in this system are N_{FC} , N_{FL} , N_{FR} , and the trinification symmetry breaking scale, which is chosen to be the value of its lower bound at 10 TeV. Finally, we will further assume there are only two light Higgs doublets when matching the scalar mass spectrum (see below).

A. Scalar sector

The low energy effective scalar field sector of the trinification model contains four Higgs doublets. For simplification we shall assume the two Higgs doublets coming from the second scalar triplet play a less important role (decoupled) at the electroweak scale [this corresponds to a special case where $n_1 = n_2 = 0$ in Eq. (2.6)] and focus only on two of the four Higgs doublets. We note that the two Higgs doublets from the second scalar multiplet are required to be sufficiently heavy—otherwise they will

FIG. 5. RG running of the scalar quartic couplings using the UV to IR approach for $N_{FC} = 95$, $N_{FL} = 165$, $N_{FR} = 62$. Sub-Figs. (a), (b), (c), (d) correspond respectively to the scalar couplings of λ_{1a} , λ_{1b} , λ_{2a} , λ_{2b} . All the vectorlike fermions appear (dashed line) at the symmetry breaking scale of the trinification group, which is around 10 TeV.

contribute to the light scalar mass spectrum even at tree level and spoil our analysis below.

Thus, after trinification symmetry breaking these scalar triplets should match the conventional two Higgs doublet model, which is defined as

$$\begin{aligned}
 V_H = & m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - (m_{12}^2 \Phi_1^\dagger \Phi_2 + \text{H.c.}) \\
 & + \frac{1}{2} \bar{\lambda}_1 (\Phi_1^\dagger \Phi_1)^2 + \frac{1}{2} \bar{\lambda}_2 (\Phi_2^\dagger \Phi_2)^2 + \bar{\lambda}_3 (\Phi_1^\dagger \Phi_1) (\Phi_2^\dagger \Phi_2) \\
 & + \bar{\lambda}_4 (\Phi_1^\dagger \Phi_2) (\Phi_2^\dagger \Phi_1) \\
 & + \left[\frac{1}{2} \bar{\lambda}_5 (\Phi_1^\dagger \Phi_2)^2 + \bar{\lambda}_6 (\Phi_1^\dagger \Phi_1) (\Phi_1^\dagger \Phi_2) \right. \\
 & \left. + \bar{\lambda}_7 (\Phi_2^\dagger \Phi_2) (\Phi_1^\dagger \Phi_2) + \text{H.c.} \right]. \quad (4.1)
 \end{aligned}$$

Comparing with (4.1) with (2.23), we find

$$\begin{aligned}
 \bar{\lambda}_1 &= 2(\lambda_{1a} + \lambda_{1b}), & \bar{\lambda}_2 &= 2(\lambda_{1a} + \lambda_{1b}), \\
 \bar{\lambda}_3 &= 2(\lambda_{1a} + \lambda_{1b}), & \bar{\lambda}_4 &= -2\lambda_{1b} \\
 \bar{\lambda}_5 &= 0, & \bar{\lambda}_6 &= 0, & \bar{\lambda}_7 &= 0, \quad (4.2)
 \end{aligned}$$

where the two Higgs doublet in the second triplet has been eliminated since it is much heavier and decoupled in our

special scenario. The electroweak two Higgs doublet mainly comes from the first scalar triplet.

Given a set of values for (N_{FC}, N_{FL}, N_{FR}) and a trinification symmetry breaking scale, we can obtain all the coupling values at the trinification symmetry breaking scale in both the symmetric and asymmetric cases via RG running from the UV fixed point. Note that we will have multiple sets of possible values of the couplings at the symmetry breaking scale, corresponding to different UV fixed point solutions since each set of choices for (N_{FC}, N_{FL}, N_{FR}) provides multiple UV fixed point solutions. We can then use the coupling values obtained at the trinification symmetry breaking scale as new initial conditions [also implementing the matching conditions (4.2)]. Eventually, by using the two Higgs doublet RG beta functions [61], we can obtain all the coupling values at the electroweak scale.

In addition, the mass matrix (neutral scalar fields) of the two Higgs doublet model (setting $m_{12}^2 \rightarrow 0$ for simplicity) is given by

$$M_{\text{neutral}}^2 = \begin{bmatrix} 2\bar{\lambda}_1 v_1^2 & (\bar{\lambda}_3 + \bar{\lambda}_4 + \bar{\lambda}_5) v_1 v_2 \\ (\bar{\lambda}_3 + \bar{\lambda}_4 + \bar{\lambda}_5) v_1 v_2 & 2\bar{\lambda}_2 v_2^2 \end{bmatrix}. \quad (4.3)$$

Note that this mass matrix is scale dependent and is defined at the electroweak scale. By using the coupling

FIG. 6. RG running of the scalar quartic couplings using the UV to IR approach for $N_{FC} = 95$, $N_{FL} = 165$, $N_{FR} = 62$. Sub-Figs. (a), (b), (c), (d), (e), (f) correspond respectively to the scalar couplings of λ_a , λ_b , λ_c , λ_d , λ_e , λ_f . All the vectorlike fermions appear at the symmetry breaking scale of the trinification group, which is around 10 TeV (dashed line).

values obtained previously at the electroweak scale, we can calculate the mass eigenvalues. We expect both eigenvalues of the mass matrix should be positive and the smaller of the two eigenvalues to be close to the 125 GeV Higgs mass. It can be shown that in the asymmetric case, by choosing $N_{FC} = 95$, $N_{FL} = 165$, $N_{FR} = 62$, we obtain

$$\begin{aligned} \bar{\lambda}_1 &= 0.234, & \bar{\lambda}_2 &= 0.331, & \bar{\lambda}_3 &= 0.213, & \bar{\lambda}_4 &= -0.499, \\ \bar{\lambda}_5 &= 0, & y_{\psi_{Q1}} &= 0.806, & y_{\psi_{E1}} &= 0.435, \end{aligned} \quad (4.4)$$

which leads to two neutral scalars with masses 125.4 GeV for the lighter Higgs and 765.4 GeV for the heavier one. The quartic couplings are at around the electroweak scale, while the Yukawa couplings are at the trinification symmetry breaking scale ~ 10 TeV. Note that, considering the approximations we have made in our analysis, we are not necessarily matching the Higgs mass at exactly 125 GeV. Rather we just present the RG flow solution that matches best. In Sec. IV C, we will scan the parameter space that matches the SM light Higgs mass in the region of (110 GeV, 140 GeV).

FIG. 7. In this figure, we present two mass eigenvalues of the CP -even neutral Higgs mass matrix as a function of m_{12} . M_{H1} denotes the lighter Higgs, while M_{H2} denotes the heavier one.

It is interesting to note that the above conclusion is dependent on the choice of the m_{12} mass parameter in (4.1). When setting $m_{12} = 0$, the light Higgs will be massless, while the heavy Higgs will be around 125 GeV. However, for a slight increase in the m_{12} parameter, the light Higgs will increase correspondingly (see Fig. 7) until $m_{12} \sim 25$ GeV. After that the light Higgs mass is frozen at around 125 GeV, whereas the heavy Higgs mass keeps increasing with increasing m_{12} . More interestingly, the trinification symmetry breaking scale cannot be too much smaller than 10 TeV or the prediction of the light Higgs mass will be a little bit too small, providing a dynamical explanation of both the symmetry breaking scale and why new physics has not yet been found. Note that this observation of the 10 TeV lower bound is obtained after we have explored a large parameter space of N_{FC} , N_{FL} , N_{FR} .

Comparison of the trinification results with the Pati-Salam case is also warranted. In the Pati-Salam case, in order to match the correct Higgs mass at the electroweak scale a constraint is placed on the Pati-Salam symmetry breaking scale that it not be below 10000 TeV [28]. This is much higher than the trinification scale obtained here and so asymptotically safe trinification theory is considerably more amenable to experimental tests.

The reason that a lower viable symmetry breaking in the trinification model is obtained could be as follows. A larger Higgs mass prediction requires a large scalar quartic coupling at the electroweak scale. However, there are only two ways that can yield a large scalar quartic coupling at the electroweak scale. One is that of increasing the symmetry breaking scale, which will provide a longer scale running distance for the quartic coupling to be increased by the top Yukawa coupling. The other is to increase the value of the quartic coupling at the fixed point. This quantity is further determined by the quartic beta function, which is balanced by the contributions of the gauge couplings. Hence the larger the gauge coupling contributions are at the quartic beta functions, the larger the Higgs mass at the electroweak

scale. Fortunately the gauge couplings in the trinification model have larger contributions relative to the Pati-Salam model simply due to the group structure.

It can be shown that in the symmetric case, we are able to obtain coupling solutions with features similar to the asymmetric case. However, to obtain a reasonable (sufficiently large) Higgs mass, the trinification symmetry breaking scale in the symmetric case is required to be much larger (say around 100 TeV), making the symmetric case somewhat less experimentally accessible. By choosing $N_{FC} = 92$, $N_{FL} = 182$, $N_{FR} = 38$, we obtain

$$\begin{aligned} \bar{\lambda}_1 &= 0.067, & \bar{\lambda}_2 &= 0.336, & \bar{\lambda}_3 &= 0.056, \\ \bar{\lambda}_4 &= 0.089, & \bar{\lambda}_5 &= 0, \end{aligned} \quad (4.5)$$

which yields two neutral scalars with masses 126.6 GeV for the lighter Higgs and 541.3 GeV for the heavier one, choosing $m_{12} \sim 100$ GeV.

B. Yukawa sector

The low energy effective field theory of the trinification model is the type II two Higgs doublet model in which one scalar field couples only to the up-type quarks and the other couples to the down-type quarks and leptons. The Yukawa sector of the two Higgs doublet model can be written as [68]

$$\begin{aligned} \mathcal{L}_{yuk} = & -\frac{M_u}{v} \left(\frac{\cos \tilde{\alpha}}{\sin \tilde{\beta}} \right) \bar{u} u h - \frac{M_d}{v} \left(\frac{\sin \tilde{\alpha}}{\cos \tilde{\beta}} \right) \bar{d} d h \\ & - \frac{M_e}{v} \left(\frac{\sin \tilde{\alpha}}{\cos \tilde{\beta}} \right) \bar{e} e h, \end{aligned} \quad (4.6)$$

where $\tilde{\alpha}$, $\tilde{\beta}$ denote, respectively, the mixing angles of the two neutral CP -even and CP -odd Higgs states, with $\tan \tilde{\beta} = \frac{u_2}{u_1}$ and $|\tilde{\alpha} - \tilde{\beta}| \sim \pi/2$. For simplicity, in Eq. (4.6), we have only written down explicitly the terms with only the 125 GeV light scalar state h . Comparing Eqs. (2.11) and (2.17) with Eq. (4.6), we obtain the following relationships between the Standard Model Yukawa couplings and the Yukawa couplings of the trinification model:

$$\begin{aligned} y_{top}^{SM} &= y_{\psi_{Q1}} \cos \tilde{\alpha}, & y_{bottom}^{SM} &= y_{\psi_{Q1}} \sin \alpha \sin \tilde{\alpha}, \\ y_{tau}^{SM} &= y_{\psi_{E1}} \sin \beta \sin \tilde{\alpha}, \end{aligned} \quad (4.7)$$

where y_{top}^{SM} , y_{bottom}^{SM} , y_{tau}^{SM} denote, respectively, the SM Yukawa couplings of the top quark, bottom quark, and tau lepton. We find in the asymmetric case, by choosing $N_{FC} = 95$, $N_{FL} = 165$, $N_{FR} = 62$, we obtain

$$\begin{aligned} \sin \alpha &\sim \sin \beta \sim \sqrt{2}/2, & u_1 &= 8.4 \text{ GeV}, \\ u_2 &= 245.86 \text{ GeV}, \end{aligned} \quad (4.8)$$

where we have used Eq. (2.21) to provide a reasonable neutrino mass. Using Eqs. (4.4), (4.7), and (4.8), we obtain

$$\begin{aligned} y_{\text{top}}^{\text{Pre}} &= 0.806, & y_{\text{bottom}}^{\text{Pre}} &= 0.019, & y_{\text{tau}}^{\text{Pre}} &= 0.011 \\ y_{\text{top}}^{\text{SM}} &= 0.780, & y_{\text{bottom}}^{\text{SM}} &= 0.019, & y_{\text{tau}}^{\text{SM}} &= 0.008, \end{aligned} \quad (4.9)$$

where the first line denotes the Yukawa coupling predictions from the safe trinification model at the symmetry breaking scale 10 TeV, while the second line denotes the SM Yukawa coupling at the same scale for comparison. It is clear that both top and bottom Yukawa couplings match extremely well. The tau lepton has a 27% difference, which might be addressed by a more careful RG running procedure such as including the threshold contributions.

C. Overview of the parameter space

In this section, we try to provide an intuitive picture of the viable parameter space. In view of the assumptions and approximations we have made, it is important to consider solutions not matching the exact values of the SM Higgs mass and couplings but some values nearby. In the following analysis, we put the viable Higgs mass cut in the range of (110 GeV, 140 GeV) and the cut of the top quark Yukawa at the trinification scale (10 TeV) in the range of (0.7, 0.85). A scan of the whole parameter space, where N_{FC} , N_{FL} , N_{FR} all range from 50 to 200, indicates that the only viable region (for which the solutions can match the SM) is where $N_{FC} \in (88, 105)$; $N_{FL} \in (160, 170)$; $N_{FR} \in (58, 68)$.

In Fig. 8, we show the 3D parameter space constructed by the values of $N_{FC} \in (70, 105)$; $N_{FL} \in (160, 170)$; $N_{FR} \in (50, 68)$, where the orange dots correspond to UV fixed point solutions that do not match the SM at the IR. The black dots correspond to viable UV fixed point solutions. It is clear that of the many UV fixed point solutions only very few can roughly match the SM in the IR. We also note that when N_{FC} gets smaller, the white

FIG. 8. In this figure, we show the 3D parameter space constructed by the values of $N_{FC} \in (70, 105)$; $N_{FL} \in (160, 170)$; $N_{FR} \in (50, 68)$ in the asymmetric case. The orange dot corresponds to a UV fixed point solution which does not match the SM at IR while the black dots correspond to the viable UV fixed point solutions.

region, which corresponds to the parameter space without any fixed point solutions, gets much larger.

V. CONCLUSIONS

A truly fundamental theory requires the presence of scale invariance at short distances [1, 2, 62–67]. Fundamentality and naturality are complementary concepts. Short distance scale invariance implies fundamentality, while (near) long distance conformality and/or controllably broken symmetries (e.g., the Coleman-Weinberg mechanism) help with naturality [62–66].

We have here constructed a realistic asymptotically safe trinification model in which the SM is embedded. Employing large N_F techniques, we demonstrated that all the couplings (i.e., the gauge, scalar quartic, and Yukawa couplings) achieve a UV interacting fixed point far below the Planck scale. Different from the conventional GUT scenario, the unification of *all* types of couplings occurs due not only to a symmetry principle but also to a dynamical principle, namely, the presence of a UV fixed point. We emphasize that we have shown that starting from a UV fixed point, a few RG flows can nicely match the SM Higgs mass, the Yukawa couplings of the top and bottom quarks, and the reasonable neutrino masses, implying a truly UV completion of the Standard Model. The very few viable solutions in a 3D scan of the whole parameter space are indicative of the highly predictive power of asymptotically safe theories, which significantly narrow the parameter space of the theory at IR.

It is also very intriguing that the trinification symmetry breaking scale cannot be too much smaller than 10 TeV in order to procure a viable 125 GeV light Higgs mass. In comparison with the 10000 TeV symmetry breaking scale in the asymptotically safe Pati-Salam model, the trinification model is much more vulnerable to experiment.

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APPENDIX: ONE-LOOP RG EQUATIONS OF THE TRINIFICATION MODEL

1. Gauge couplings

$$\begin{aligned} (4\pi)^2 \frac{dg_L}{d \ln \mu} &= - \left(5 + \frac{n_H}{2} \right) g_L^2 \\ (4\pi)^2 \frac{dg_R}{d \ln \mu} &= - \left(5 + \frac{n_H}{2} \right) g_R^2 \\ (4\pi)^2 \frac{dg_c}{d \ln \mu} &= -5g_c^2 \end{aligned} \quad (A1)$$

2. Yukawa couplings

$$\begin{aligned}
(4\pi)^2 \frac{dy_{\psi_{Q1}}}{d \ln \mu} &= y_{\psi_{Q1}} (-4g_L^2 - 4g_R^2 - 8g_c^2 + 2y_{\psi_{E1}}^2 + 6y_{\psi_{Q2}}^2) \\
&\quad + 2y_{\psi_{E1}} y_{\psi_{E2}} y_{\psi_{Q2}} + 6y_{\psi_{Q1}}^3 \\
(4\pi)^2 \frac{dy_{\psi_{Q2}}}{d \ln \mu} &= y_{\psi_{Q2}} (-4g_L^2 - 4g_R^2 - 8g_c^2 + 2y_{\psi_{E2}}^2 + 6y_{\psi_{Q1}}^2) \\
&\quad + 2y_{\psi_{E1}} y_{\psi_{E2}} y_{\psi_{Q1}} + 6y_{\psi_{Q2}}^3 \\
(4\pi)^2 \frac{dy_{\psi_{E1}}}{d \ln \mu} &= y_{\psi_{E1}} (-8g_L^2 - 8g_R^2 + 6y_{\psi_{E2}}^2 + 3y_{\psi_{Q1}}^2) \\
&\quad + 6y_{\psi_{E1}}^3 + 3y_{\psi_{E2}} y_{\psi_{Q1}} y_{\psi_{Q2}} \\
(4\pi)^2 \frac{dy_{\psi_{E2}}}{d \ln \mu} &= y_{\psi_{E2}} (-8g_L^2 - 8g_R^2 + 6y_{\psi_{E1}}^2 + 3y_{\psi_{Q2}}^2) \\
&\quad + 6y_{\psi_{E2}}^3 + 3y_{\psi_{E1}} y_{\psi_{Q1}} y_{\psi_{Q2}} \quad (A2)
\end{aligned}$$

3. Quartic couplings

For the single Higgs multiplet, $n_H = 1$ and $y_{\psi_{E1}} := y_{\psi_E}$, $y_{\psi_{Q1}} := y_{\psi_Q}$, $y_{\psi_{E2}} = y_{\psi_{Q2}} = 0$. The RGEs (renormalization group equations) for the quartic couplings are

$$\begin{aligned}
(4\pi)^2 \frac{d\lambda_a}{d \ln \mu} &= \lambda_a (-16g_L^2 - 16g_R^2 + 48\lambda_b + 12y_{\psi_Q}^2 + 8y_{\psi_E}^2) \\
&\quad + \frac{10}{3} g_L^2 g_R^2 + \frac{11g_L^4}{12} + \frac{11g_R^4}{12} + 52\lambda_a^2 \\
&\quad + 12\lambda_b^2 - 2y_{\psi_E}^4 \quad (A3)
\end{aligned}$$

$$\begin{aligned}
(4\pi)^2 \frac{d\lambda_b}{d \ln \mu} &= \lambda_b (-16g_L^2 - 16g_R^2 + 24\lambda_a + 12y_{\psi_Q}^2 + 8y_{\psi_E}^2) \\
&\quad - 2g_L^2 g_R^2 + \frac{5g_L^4}{4} + \frac{5g_R^4}{4} + 24\lambda_a^2 - 6y_{\psi_Q}^4 - 2y_{\psi_E}^4. \quad (A4)
\end{aligned}$$

For the two Higgs multiplet, $n_H = 2$. The RGEs for the quartic couplings are

$$\begin{aligned}
(4\pi)^2 \frac{d\lambda_{1a}}{d \ln \mu} &= \lambda_{1a} (48\lambda_{1b} - 16g_L^2 - 16g_R^2 + 8y_{\psi_{E1}}^2 + 12y_{\psi_{Q1}}^2) \\
&\quad + 52\lambda_{1a}^2 + 12\lambda_{1b}^2 + 2\lambda_a \lambda_b + 6\lambda_a \lambda_c + 6\lambda_a \lambda_d \\
&\quad + 9\lambda_a^2 + \lambda_b^2 + 2\lambda_c \lambda_d + 4\lambda_e^2 + 4\lambda_f^2 + \frac{10}{3} g_L^2 g_R^2 \\
&\quad + \frac{11}{12} g_L^4 + \frac{11}{12} g_R^4 - 2y_{\psi_{E1}}^4 \quad (A5)
\end{aligned}$$

$$\begin{aligned}
(4\pi)^2 \frac{d\lambda_{1b}}{d \ln \mu} &= \lambda_{1b} (24\lambda_{1a} - 16g_L^2 - 16g_R^2 + 8y_{\psi_{E1}}^2 + 12y_{\psi_{Q1}}^2) \\
&\quad + 24\lambda_{1b}^2 + 2\lambda_b \lambda_c + 2\lambda_b \lambda_d + 3\lambda_c^2 + 3\lambda_d^2 + 8\lambda_e \lambda_f \\
&\quad - 2g_L^2 g_R^2 + \frac{5}{4} g_L^4 + \frac{5}{4} g_R^4 - 2y_{\psi_{E1}}^4 - 6y_{\psi_{Q1}}^4 \quad (A6)
\end{aligned}$$

$$\begin{aligned}
(4\pi)^2 \frac{d\lambda_{2a}}{d \ln \mu} &= \lambda_{2a} (48\lambda_{2b} - 16g_L^2 - 16g_R^2 + 8y_{\psi_{E2}}^2 + 12y_{\psi_{Q2}}^2) \\
&\quad + 52\lambda_{2a}^2 + 18\lambda_{2b}^2 + 2\lambda_a \lambda_b + 6\lambda_a \lambda_c + 6\lambda_a \lambda_d \\
&\quad + 9\lambda_a^2 + \lambda_b^2 + 2\lambda_c \lambda_d + 4\lambda_e^2 + 4\lambda_f^2 \\
&\quad + \frac{10}{3} g_L^2 g_R^2 + \frac{11}{12} g_L^4 + \frac{11}{12} g_R^4 - 2y_{\psi_{E2}}^4 \quad (A7)
\end{aligned}$$

$$\begin{aligned}
(4\pi)^2 \frac{d\lambda_{2b}}{d \ln \mu} &= \lambda_{2b} (24\lambda_{2a} - 16g_L^2 - 16g_R^2 + 8y_{\psi_{E2}}^2 + 12y_{\psi_{Q2}}^2) \\
&\quad + 36\lambda_{2b}^2 + 2\lambda_b \lambda_c + 2\lambda_b \lambda_d + 3\lambda_c^2 \\
&\quad + 3\lambda_d^2 + 8\lambda_e \lambda_f - 2g_L^2 g_R^2 + \frac{5}{4} g_L^4 + \frac{5}{4} g_R^4 \\
&\quad - 2y_{\psi_{E2}}^4 - 6y_{\psi_{Q2}}^4. \quad (A8)
\end{aligned}$$

For the scalar mixing couplings,

$$\begin{aligned}
(4\pi)^2 \frac{d\lambda_a}{d \ln \mu} &= \lambda_a (40\lambda_{1a} + 24\lambda_{1b} + 40\lambda_{2a} + 24\lambda_{2b} - 16g_L^2 \\
&\quad - 16g_R^2 + 4y_{\psi_{E1}}^2 + 4y_{\psi_{E2}}^2 + 6y_{\psi_{Q1}}^2 + 6y_{\psi_{Q2}}^2) \\
&\quad + 4\lambda_{1a} \lambda_b + 12\lambda_{1a} \lambda_c + 12\lambda_{1a} \lambda_d + 4\lambda_{1b} \lambda_c \\
&\quad + 4\lambda_{1b} \lambda_d + 4\lambda_{2a} \lambda_b + 12\lambda_{2a} \lambda_c + 12\lambda_{2a} \lambda_d \\
&\quad + 4\lambda_{2b} \lambda_c + 4\lambda_{2b} \lambda_d + 2\lambda_{2b}^2 + 4\lambda_a^2 + 2\lambda_b^2 \\
&\quad + 2\lambda_c^2 + 2\lambda_d^2 + 8\lambda_e^2 + \frac{2}{3} g_L^2 g_R^2 + \frac{11}{6} g_L^4 \\
&\quad + \frac{11}{6} g_R^4 - 4y_{\psi_{E1}}^2 y_{\psi_{E2}}^2 \quad (A9)
\end{aligned}$$

$$\begin{aligned}
(4\pi)^2 \frac{d\lambda_b}{d \ln \mu} &= 2\lambda_b (2\lambda_{1a} + 2\lambda_{2a} + 9\lambda_b - 8g_L^2 - 8g_R^2 + 2y_{\psi_{E1}}^2 \\
&\quad + 2y_{\psi_{E2}}^2 + 3y_{\psi_{Q1}}^2 + 3y_{\psi_{Q2}}^2 + 4\lambda_a + 6\lambda_c + 6\lambda_d) \\
&\quad + 6\lambda_{2b}^2 + 8\lambda_c \lambda_d + 96\lambda_e \lambda_f + 88\lambda_e^2 \\
&\quad + 16\lambda_f^2 + 6g_L^2 g_R^2 - 4y_{\psi_{E1}}^2 y_{\psi_{E2}}^2 \quad (A10)
\end{aligned}$$

$$\begin{aligned}
(4\pi)^2 \frac{d\lambda_c}{d\ln\mu} = & \lambda_c(4\lambda_{1a} + 12\lambda_{1b} + 4\lambda_{2a} + 12\lambda_{2b} + 8\lambda_a - 16g_L^2 \\
& - 16g_R^2 + 4y_{\psi_{E1}}^2 + 4y_{\psi_{E2}}^2 + 6y_{\psi_{Q1}}^2 + 6y_{\psi_{Q2}}^2) \\
& + 4\lambda_{1b}\lambda_b + 4\lambda_{2b}\lambda_b + 4\lambda_b\lambda_d + 16\lambda_e\lambda_f \\
& + 6\lambda_{2b}^2 + 6\lambda_c^2 + 24\lambda_f^2 - 2g_L^2g_R^2 + \frac{5}{2}g_R^4 \\
& - 4y_{\psi_{E1}}^2y_{\psi_{E2}}^2 - 12y_{\psi_{Q1}}^2y_{\psi_{Q2}}^2
\end{aligned} \quad (A11)$$

$$\begin{aligned}
(4\pi)^2 \frac{d\lambda_d}{d\ln\mu} = & \lambda_d(4\lambda_{1a} + 12\lambda_{1b} + 4\lambda_{2a} + 12\lambda_{2b} + 8\lambda_a - 16g_L^2 \\
& - 16g_R^2 + 4y_{\psi_{E1}}^2 + 4y_{\psi_{E2}}^2 + 6y_{\psi_{Q1}}^2 + 6y_{\psi_{Q2}}^2) \\
& + 4\lambda_{1b}\lambda_b + 4\lambda_{2b}\lambda_b + 4\lambda_b\lambda_c + 16\lambda_e\lambda_f + 6\lambda_{2b}^2 \\
& + 6\lambda_d^2 + 24\lambda_f^2 - 2g_L^2g_R^2 + \frac{5}{2}g_L^4 - 4y_{\psi_{E1}}^2y_{\psi_{E2}}^2 \\
& - 12y_{\psi_{Q1}}^2y_{\psi_{Q2}}^2
\end{aligned} \quad (A12)$$

$$\begin{aligned}
(4\pi)^2 \frac{d\lambda_e}{d\ln\mu} = & \lambda_e(4\lambda_{1a} + 4\lambda_{2a} + 8\lambda_a + 40\lambda_b + 12\lambda_c + 12\lambda_d \\
& - 16g_L^2 - 16g_R^2 + 4y_{\psi_{E1}}^2 + 4y_{\psi_{E2}}^2 + 6y_{\psi_{Q1}}^2 + 6y_{\psi_{Q2}}^2) \\
& + 4\lambda_{1b}\lambda_f + 4\lambda_{2b}\lambda_f + 2\lambda_{2b}^2 + 24\lambda_b\lambda_f + 4\lambda_c\lambda_f \\
& + 4\lambda_d\lambda_f - 2y_{\psi_{E1}}^2y_{\psi_{E2}}^2
\end{aligned} \quad (A13)$$

$$\begin{aligned}
(4\pi)^2 \frac{d\lambda_f}{d\ln\mu} = & \lambda_f(4\lambda_{1a} + 4\lambda_{2a} + 8\lambda_a + 12\lambda_c + 12\lambda_d - 16g_L^2 \\
& - 16g_R^2 + 4y_{\psi_{E1}}^2 + 4y_{\psi_{E2}}^2 + 6y_{\psi_{Q1}}^2 + 6y_{\psi_{Q2}}^2) \\
& + 4\lambda_{1b}\lambda_e + 4\lambda_{2b}\lambda_e + 6\lambda_{2b}^2 + 4\lambda_c\lambda_e + 4\lambda_d\lambda_e \\
& - 2y_{\psi_{E1}}^2y_{\psi_{E2}}^2 - 6y_{\psi_{Q1}}^2y_{\psi_{Q2}}^2.
\end{aligned} \quad (A14)$$

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