Simple interpretations of lepton anomalies in the lepton-specific inert two-Higgs-doublet model

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There exist about 3.7 σ positive and 2.4 σ negative deviations in the muon and electron anomalous magnetic moments (g - 2). Also, some ratios for lepton universality in τ decays have almost 2 σ deviations from the Standard Model. In this paper, we propose a lepton-specific inert two-Higgs-doublet model. After imposing all the relevant theoretical and experimental constraints, we show that these lepton anomalies can be explained simultaneously in many parameter spaces with $m_H > 200$ GeV and $m_A(m_{H^{\pm}}) > 500$ GeV for appropriate Yukawa couplings between leptons and inert Higgs. The key point is that these Yukawa couplings for μ and τ/e have opposite sign.

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I. INTRODUCTION

The Standard Model (SM) describes the elementary particles, as well as the fundamental interactions between them. In particular, such description is sensitive to the quantum corrections. For example, since Schwinger's seminar calculation of the electron anomalous magnetic moment $a_e = \alpha/2\pi$ [1], the charged lepton anomalous magnetic moments have become the powerful precision tests of quantum electrodynamics (QED), and subsequently the full SM. The muon anomalous magnetic moment g - 2 has been a long-standing puzzle since the announcement by the E821 experiment in 2001 [2]. The experimental value has an approximate 3.7σ discrepancy from the SM prediction [3]

$$\Delta a_{\mu} = a_{\mu}^{\exp} - a_{\mu}^{\rm SM} = (274 \pm 73) \times 10^{-11}.$$
 (1)

Very recently, an improvement in the measured mass of atomic cesium used in conjunction with other known mass ratios and the Rydberg constant leads to the most precise value of the fine structure constant [4]. As a result, the experimental value of the electron g - 2 has a 2.4σ deviation from the SM prediction [5–7]

$$\Delta a_e = a_e^{\exp} - a_e^{\rm SM} = (-87 \pm 36) \times 10^{-14}, \qquad (2)$$

which is opposite in sign from the muon g - 2.

The lepton flavor universality (LFU) in the τ decays is an excellent way to probe new physics. The HFAG collaboration reported three ratios from pure leptonic processes, and two ratios from semihadronic processes, $\tau \rightarrow \pi/K\nu$ and $\pi/K \rightarrow \mu\nu$ [8]:

$$\begin{pmatrix} \frac{g_{\tau}}{g_{\mu}} \end{pmatrix} = 1.0011 \pm 0.0015, \qquad \begin{pmatrix} \frac{g_{\tau}}{g_{e}} \end{pmatrix} = 1.0029 \pm 0.0015, \\ \begin{pmatrix} \frac{g_{\mu}}{g_{e}} \end{pmatrix} = 1.0018 \pm 0.0014, \qquad \begin{pmatrix} \frac{g_{\tau}}{g_{\mu}} \end{pmatrix}_{\pi} = 0.9963 \pm 0.0027, \\ \begin{pmatrix} \frac{g_{\tau}}{g_{\mu}} \end{pmatrix}_{K} = 0.9858 \pm 0.0071,$$
(3)

where the ratios of g_{τ}/g_e and $\left(\frac{g_{\tau}}{g_{\mu}}\right)_K$ have almost 2σ deviations from the SM.

Muon g-2 anomaly can be simply explained in the lepton-specific two-Higgs-doublet model (2HDM) and aligned 2HDM. However, the tree-level diagram mediated by the charged Higgs gives a negative contribution to the decay $\tau \rightarrow \mu\nu\bar{\nu}$ [9–11], which will raise the discrepancy in the LFU in τ decays. In addition, these two types of 2HDM do not explain the muon and electron g-2 simultaneously since there is an opposite sign between them. Therefore, we shall propose a lepton-specific inert 2HDM to explain all three anomalies of muon and electron g-2 as well as

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LFU in τ decay simultaneously. In our model, for the extra Higgses (*H*, *A*, *H*[±]), the Yukawa couplings for μ and τ/e have opposite sign. In 2012, Giudice *et al.* used the approach of the effective operator to discuss the contributions of light scalar to the muon and electron g-2. The contributions of two-loop Barr-Zee–type diagrams can be positive or negative depending on the relative sign of the Yukawa couplings for muon, electron, and tau [12]. Although the muon and electron g-2 have been addressed simultaneously in a few recent papers [13–17], it seems to us that our model is simpler from the renormalized theory point of view.

II. LEPTON-SPECIFIC INERT 2HDM

We introduce an inert Higgs doublet Φ_2 in the SM as well as a discrete Z_2 symmetry under which Φ_2 is odd while all the SM particles are even. The scalar potential for the SM Higgs field Φ_1 and inert doublet Φ_2 is

$$V = Y_{1}(\Phi_{1}^{\dagger}\Phi_{1}) + Y_{2}(\Phi_{2}^{\dagger}\Phi_{2}) + \frac{\lambda_{1}}{2}(\Phi_{1}^{\dagger}\Phi_{1})^{2} + \frac{\lambda_{2}}{2}(\Phi_{2}^{\dagger}\Phi_{2})^{2} + \lambda_{3}(\Phi_{1}^{\dagger}\Phi_{1})(\Phi_{2}^{\dagger}\Phi_{2}) + \lambda_{4}(\Phi_{1}^{\dagger}\Phi_{2})(\Phi_{2}^{\dagger}\Phi_{1}) + \left[\frac{\lambda_{5}}{2}(\Phi_{1}^{\dagger}\Phi_{2})^{2} + \text{H.c.}\right].$$
(4)

We focus on the *CP*-conserving case where all λ_i are real. The two complex scalar doublets can be written as

$$\Phi_1 = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}(v+h+iG_0) \end{pmatrix}, \qquad \Phi_2 = \begin{pmatrix} H^+ \\ \frac{1}{\sqrt{2}}(H+iA) \end{pmatrix}.$$
(5)

The Φ_1 field has the vacuum expectation value (VEV) v = 246 GeV, and the VEV of Φ_2 field is zero. Y_1 is fixed by the scalar potential minimization condition. The H^+ and A are the mass eigenstates of the charged Higgs boson and CP-odd Higgs boson. Their masses are given as

$$m_{H^{\pm}}^2 = Y_2 + \frac{\lambda_3}{2}v^2, \qquad m_A^2 = m_{H^{\pm}}^2 + \frac{1}{2}(\lambda_4 - \lambda_5)v^2.$$
 (6)

The h and H have no mixing, and they are two mass eigenstates of the *CP*-even Higgses. In this paper, we take the light *CP*-even Higgs h as the SM-like Higgs. Their masses are given as

$$m_h^2 = \lambda_1 v^2 \equiv (125 \text{ GeV})^2, \qquad m_H^2 = m_A^2 + \lambda_5 v^2.$$
 (7)

The fermions obtain the mass terms from the Yukawa interactions with Φ_1 :

$$-\mathcal{L} = y_u \bar{Q}_L \tilde{\Phi}_1 u_R + y_d \bar{Q}_L \Phi_1 d_R + y_l \bar{L}_L \Phi_1 e_R + \text{H.c.}, \quad (8)$$

where $Q_L^T = (u_L, d_L)$, $L_L^T = (\nu_L, l_L)$, $\tilde{\Phi}_1 = i\tau_2 \Phi_1^*$, and y_u , y_d and y_{ℓ} are 3×3 matrices in family space. In addition, only in the lepton sector we introduce the Z_2 symmetry-breaking Yukawa interactions of Φ_2 ,

$$-\mathcal{L} = \sqrt{2}\kappa_e \bar{L}_{1L} \Phi_2 e_R + \sqrt{2}\kappa_\mu \bar{L}_{2L} \Phi_2 \mu_R + \sqrt{2}\kappa_\tau \bar{L}_{3L} \Phi_2 \tau_R + \text{H.c.}$$
(9)

Such the Z_2 symmetry-breaking effect only for the lepton sector can be realized in the high-dimensional brane world scenario, which will be studied elsewhere. From Eq. (9), we can obtain the lepton Yukawa couplings of extra Higgses (*H*, *A*, and H^{\pm}). The neutral Higgses *A* and *H* have no couplings to *ZZ*, *WW*.

III. NUMERICAL RESULTS

According to Eqs. (6) and (7), the values of λ_1 , λ_5 and λ_4 can be determined by m_h (= 125 GeV), m_H , m_A and $m_{H^{\pm}}$. λ_2 controls the quartic couplings of extra Higgses, but does not affect the physics observables. So we simply take $\lambda_2 = \lambda_1$. Because the precision electroweak data favor small mass splitting between m_A and $m_{H^{\pm}}$, we simply choose $m_A = m_{H^{\pm}}$. We employ the 2HDMC [18] to implement the theoretical constraints from vacuum stability, unitarity and perturbativity, as well as the constraints of the oblique parameters (*S*, *T*, *U*). We scan over several key parameters in the following ranges

$$0.5 < \kappa_{\tau} < 1, \quad -0.25 < \kappa_{\mu} < 0, \quad 0 < \kappa_{e} < 0.01,$$

200 GeV $< m_H < 350$ GeV, 500 GeV $< m_A = m_{H^{\pm}} < 700$ GeV. (10)

In such ranges of κ_{τ} , κ_{μ} and κ_{e} , the corresponding Yukawa couplings do not become nonperturbative. At the tree-level, the SM-like Higgs has the same couplings to the SM particles as the SM, and no exotic decay mode. The masses of extra Higgses are beyond the exclusion range of the searches for the neutral and charged Higgs at the LEP. Since the extra Higgses have no couplings to quarks due to Z_2 symmetry, we can safely neglect the limits from the observables of meson. The extra Higgs bosons are dominantly produced at the LHC via electroweak processes. We generate the Monte Carlo events using MG5_AMC-2.4.3 [19] with PYTHIA6 [20], and adopt the constraints from all the analysis for the 13 TeV LHC in CHECKMATE 2.0.7 [21]. The latest multilepton searches for electroweakino [22-25] are further applied because of the dominated multilepton final states in our model.

In the model, the extra one-loop contributions to muon g-2 are given as [26]

$$\Delta a_{\mu}(1\text{loop}) = \frac{1}{2\pi^2} \sum_{i} \kappa_{\mu}^2 r_{\mu}^i F_i(r_{\mu}^i), \qquad (11)$$



FIG. 1. The samples within 2σ ranges of Δa_{μ} (left panel), Δa_{e} (middle panel), and both Δa_{μ} and Δa_{e} (right panel). All the samples satisfy the constraints of the theory and oblique parameters.

where i = H, A, H^{\pm} , $r_{\mu}^{i} = m_{\mu}^{2}/M_{i}^{2}$. For $r_{\mu}^{i} \ll 1$ we have

$$F_H(r) \simeq -\ln r - 7/6, \qquad F_A(r) \simeq \ln r + 11/6,$$

 $F_{H^{\pm}}(r) \simeq -1/6.$ (12)

The contributions of the two-loop diagrams with a closed fermion loop are given by

$$\Delta a_{\mu}(2\text{loop}) = \frac{m_{\mu}}{8\pi^2} \frac{\alpha_{\text{em}}}{\pi} \sum_{i,\ell} Q_{\ell}^2 \kappa_{\mu} \frac{\kappa_{\ell}}{m_{\ell}} r_{\ell}^i G_i(r_{\ell}^i), \quad (13)$$

where i = H, A, $\ell = \tau$, and m_{ℓ} and Q_{ℓ} are the mass and electric charge of the lepton ℓ in the loop. The functions $G_i(r)$ are given in Refs. [27,28],

$$G_H(r) = \int_0^1 dx \frac{2x(1-x)-1}{x(1-x)-r} \ln \frac{x(1-x)}{r}, \quad (14)$$

$$G_A(r) = \int_0^1 dx \frac{1}{x(1-x) - r} \ln \frac{x(1-x)}{r}.$$
 (15)

We also consider the contributions of the two-loop diagrams with a closed charged Higgs loop, and find that their contributions are much smaller than the fermion loop. The calculations of Δa_e are similar to Δa_{μ} , but for the contributions of the two-loop diagrams, we include both μ loop and τ loop.

The HFAG collaboration reported three ratios from pure leptonic processes, and two ratios from semihadronic processes, $\tau \rightarrow \pi/K\nu$ and $\pi/K \rightarrow \mu\nu$ [8]. In the model, we have the ratios

$$\left(\frac{g_{\tau}}{g_{\mu}}\right)^{2} \equiv \bar{\Gamma}(\tau \to e\nu\bar{\nu})/\bar{\Gamma}(\mu \to e\nu\bar{\nu}) \approx \frac{1+2\delta_{\text{loop}}^{\tau}}{1+2\delta_{\text{loop}}^{\mu}},$$

$$\left(\frac{g_{\tau}}{g_{e}}\right)^{2} \equiv \bar{\Gamma}(\tau \to \mu\nu\bar{\nu})/\bar{\Gamma}(\mu \to e\nu\bar{\nu}) \approx \frac{1+2\delta_{\text{tree}}+2\delta_{\text{loop}}^{\tau}}{1+2\delta_{\text{loop}}^{\mu}},$$

$$\left(\frac{g_{\mu}}{g_{e}}\right)^{2} \equiv \bar{\Gamma}(\tau \to \mu\nu\bar{\nu})/\bar{\Gamma}(\tau \to e\nu\bar{\nu}) \approx 1+2\delta_{\text{tree}},$$

$$\left(\frac{g_{\tau}}{g_{\mu}}\right)^{2}_{\pi} = \left(\frac{g_{\tau}}{g_{\mu}}\right)^{2}_{K} = \left(\frac{g_{\tau}}{g_{\mu}}\right)^{2},$$

$$(16)$$

where $\overline{\Gamma}$ denotes the partial width normalized to its SM value. δ_{tree} and $\delta_{\text{loop}}^{\tau,\mu}$ obtain corrections from the tree-level and one-loop diagrams mediated by the charged Higgs, respectively. They are given as [9,11]

$$\delta_{\text{tree}} = \frac{v^4 \kappa_\tau^2 \kappa_\mu^2}{8m_{H^\pm}^4} - \frac{v^2 m_\mu}{m_{H^\pm}^2 m_\tau} \kappa_\tau \kappa_\mu \frac{g(m_\mu^2/m_\tau^2)}{f(m_\mu^2/m_\tau^2)}, \qquad (17)$$

$$\delta_{\text{loop}}^{\tau,\mu} = \frac{1}{16\pi^2} \kappa_{\tau,\mu}^2 \left[1 + \frac{1}{4} \left(H(x_A) + H(x_H) \right) \right], \quad (18)$$

where $f(x) \equiv 1-8x+8x^3-x^4-12x^2\ln(x)$, $g(x) \equiv 1+9x-9x^2-x^3+6x(1+x)\ln(x)$, and $H(x_{\phi}) \equiv \ln(x_{\phi})(1+x_{\phi})/(1-x_{\phi})$ with $x_{\phi} = m_{\phi}^2/m_{H^{\pm}}^2$.

The correlation matrix for the above five observables is

$$\begin{pmatrix} 1 & +0.53 & -0.49 & +0.24 & +0.12 \\ +0.53 & 1 & +0.48 & +0.26 & +0.10 \\ -0.49 & +0.48 & 1 & +0.02 & -0.02 \\ +0.24 & +0.26 & +0.02 & 1 & +0.05 \\ +0.12 & +0.10 & -0.02 & +0.05 & 1 \end{pmatrix}.$$
(19)

We perform χ_{τ}^2 calculations for these five observables. The covariance matrix constructed from the data of Eqs. (3) and (19) has a vanishing eigenvalue, and the corresponding



FIG. 2. The surviving samples fit the data of LFU in τ decay within the 2σ range. All the samples satisfy the constraints of the theory and oblique parameters.

degree of freedom is removed in our calculation. In our discussions we require $\chi_{\tau}^2 < 9.72$, which corresponds to be within the 2σ range for four observables, and is smaller than the SM value, $\chi_{\tau}^2(SM) = 12.25$.

The measured values of the ratios of the leptonic Z decay branching fractions are given as [29]

$$\frac{\Gamma_{Z \to \mu^+ \mu^-}}{\Gamma_{Z \to e^+ e^-}} = 1.0009 \pm 0.0028,$$

$$\frac{\Gamma_{Z \to \tau^+ \tau^-}}{\Gamma_{Z \to e^+ e^-}} = 1.0019 \pm 0.0032,$$
(20)

with a correlation of +0.63. In the model, the width of $Z \rightarrow \tau^+ \tau^-$ can have sizable deviation from the SM value

due to the loop contributions of the extra Higgs bosons, because they strongly interact with charged leptons. The calculations of quantities in Eq. (20) are similar to Ref. [30].

After imposing the constraints of the theory and the oblique parameters, in Fig. 1 we show the surviving samples which are consistent with Δa_{μ} and Δa_{e} at the 2σ level. Both one-loop and two-loop diagrams give positive contributions to Δa_{μ} . For Δa_{e} , the contributions of one-loop are positive and those of two-loop are negative. Only the contributions of two-loop can make Δa_{e} to be within the 2σ range. Δa_{μ} and Δa_{e} respectively favor negative κ_{μ} and positive κ_{e} for increasing m_{H} , and m_{H} is required to be smaller than 320 GeV from Δa_{e} . A large mass splitting between m_{A} and m_{H} can lead to sizable corrections to Δa_{μ} and Δa_{e} . Therefore, the right panel of Fig. 1 shows that m_{A} is favored for increasing m_{H} , especially for a large m_{H} .

After imposing the constraints of the theory and the oblique parameters, we show the surviving samples with $\chi_{\tau}^2 < 9.72$ in Fig. 2. Such samples fit the data of LFU in τ decay within the 2σ range. Because κ_{μ} is opposite in sign from κ_{τ} , the second term of δ_{tree} in Eq. (17) is positive, which gives a well fit to g_{τ}/g_e . Figure 2 shows that χ_{τ}^2 can be as low as 7.4, which is much smaller than the SM value (12.25). The value of χ_{τ}^2 decreases with an increase of $-\kappa_{\mu}\kappa_{\tau}$ and increases with $m_{H^{\pm}}$.

In Fig. 3 we show the surviving samples after imposing the constraints of theory, the oblique parameters, Δa_{μ} , Δa_{e} , the data of LFU in τ decay and Z decay, and the direct searches at LHC. The model can give sizable corrections to $Z \rightarrow \tau^+ \tau^-$ for large κ_{τ} and mass splitting between m_A and m_H . Therefore, the region of the small m_H and large κ_{τ} is excluded by the data of LFU in Z decay, as shown in the middle panel of Fig. 3. The left panel of Fig. 3 shows that the exclusion limits from the direct searches at LHC favor large m_H , m_A , and $m_{H^{\pm}}$. After imposing the theoretical constraint and relevant experimental constraints, the model can explain the anomalies of Δa_{μ} , Δa_e and LFU in the τ



FIG. 3. The allowed samples (dots with gray edge) and excluded samples (crosses) by the direct search limits from the LHC at 95% confidence level. The colors indicate κ_{τ} , m_A and m_H in left, middle, and right panels, respectively. All the samples satisfy the constraints of theory, the oblique parameters, Δa_{μ} , Δa_{e} , the data of LFU in τ decays, and Z decay.

decay in many parameter spaces of 200 GeV $< m_H <$ 320 GeV, 500 GeV $< m_A = m_{H^{\pm}} < 680$ GeV, 0.0066 $< \kappa_e <$ 0.01, $-0.25 < \kappa_{\mu} < -0.147$, and 0.53 $< \kappa_{\tau} < 1.0$. By normalizing event yields in the signal regions of Ref. [23] to higher luminosities, we find that these parameter spaces can be fully detected at 95% confidence level with about 80 fb⁻¹ 13 TeV LHC data.

Note the Z_2 breaking term $\mu(\Phi_1^{\dagger}\Phi_2 + \text{H.c.})$ is inevitable when we consider the renormalization of one-loop divergent integral. Although it can be set to be zero at some energy scale, radiative corrections will regenerate it at different scales. We can denote it as the μ problem in our model. The vanishing of the μ term does not induce an enhanced symmetry so that nothing prevents it to be large via quantum corrections. However, we have to figure out that the two-Higgs-doublet model is not UV consistent theory since the Higgs mass hierarchy problem is not solved. These two hierarchy problems, i.e., μ problem and Higgs mass problem, motivate us to consider new physics around TeV such as supersymmetry, which will be studied in the future paper. Therefore, the intrinsic cutoff for quantum correction is around TeV. The mixing mass term like "hH" can be generated at one-loop by the exchange of SM leptons in the loop, but is sizably suppressed by the loop factor of $\frac{1}{16\pi^2}$ and $h\tau\bar{\tau}$ coupling of $\frac{m_{\tau}}{v}$. For the cutoff of TeV, we can obtain a small value of μ through the cancellation between the bare term and the quadratic loop correction. The price we paid is that we have to accept fine-tuning. As a result, we can still obtain a Z_2 symmetric model with Z_2 breaking terms being very small.

IV. CONCLUSION

We have proposed a lepton-specific inert 2HDM, where an inert Higgs doublet field with a discrete Z_2 symmetry is introduced to the SM. Considering all the current theoretical and experimental constraints, we showed that our model can provide a simple explanation for the anomalies of muon g-2, electron g-2, and LFU of the τ decays in many viable parameter spaces.

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