Double Higgs boson production at e^+e^- colliders in the two-Higgs-doublet model

Tadashi Kon,^{*} Takuto Nagura,[†] Takahiro Ueda,[‡] and Kei Yagyu[§]

*Department of Materials and Life Science, Faculty of Science and Technology,
Seikei University, 3-3-1 Kichijoji-Kitamachi, Musashino-shi, Tokyo 180-8633, Japan*

(Received 24 December 2018; published 22 May 2019)

We study the double Higgs boson production processes $e^+e^- \rightarrow hh f \bar{f}$ ($f \neq t$), with h being the 125 GeV Higgs boson in the two-Higgs-doublet model with a softly broken Z_2 symmetry. The cross section can be significantly enhanced, typically a few hundred percent, as compared to the standard model prediction due to resonant effects of heavy neutral Higgs bosons, which becomes important in the case without the alignment limit. We find a strong correlation between the enhancement factor of the cross section and the scaling factor of the $h f \bar{f}$ couplings under constraints from perturbative unitarity, vacuum stability, and current experimental data at the LHC as well as the electroweak precision data.

DOI: [10.1103/PhysRevD.99.095027](https://doi.org/10.1103/PhysRevD.99.095027)

I. INTRODUCTION

Various signatures of the discovered Higgs boson at the LHC [1,2] show that its properties such as observed production cross sections and decay branching ratios are consistent with those predicted in the standard model (SM) [3]. Although the SM assumes the minimal form of the Higgs sector composed of one isospin scalar doublet, one may ask him- or herself a natural question: whether the discovered Higgs boson comes from just one doublet or not. In fact, the discovered Higgs boson can be regarded as one of Higgs bosons arising from an extended structure of the Higgs sector, and there is no strong reason to restrict the Higgs sector to be minimal. On the other hand, extended Higgs sectors often appear as a low energy effective theory of physics beyond the SM based upon various physics motivations. The important thing is that phenomenological features in the extended Higgs sectors strongly depend on a specific scenario of the underlying theory. Therefore, as a bottom-up approach, it is important to study the structure of the Higgs sector in order to narrow down new physics models.

Among various possibilities of the extended Higgs sector, a two-Higgs-doublet model (THDM) [4] is one of the simplest but important examples, as it appears in several new physics models. For example, models proposed to solve the gauge hierarchy problem predict THDMs as their low energy effective theories such as the minimal supersymmetric SM [5] and composite Higgs models [6–8]. In addition, extra CP violating phases can arise from the multidoublet structure of the scalar potential, which are needed to realize the successful scenario for the electroweak baryogenesis [9,10]. Furthermore, the second Higgs doublet is often introduced in models beyond the SM to explain tiny neutrino masses [11–15] and dark matter in the Universe [16]. For these reasons, we consider the THDM as a reference model, in which we impose a softly broken discrete Z_2 symmetry to avoid flavor changing neutral currents at tree level [17].

There are basically two ways to test the THDM at collider experiments, namely, the direct searches for additional Higgs bosons such as a charged Higgs boson and the indirect searches finding deviations in properties of the discovered Higgs boson (h) from the SM prediction. Concerning the former way, if we discover the additional Higgs bosons, it turns out to be a direct evidence for the THDM or at least an extended Higgs sector, but no report has hitherto been provided for the discovery of such new particles at the LHC. Recent studies about the prospects of the direct searches at the high-luminosity LHC (HL-LHC) include Refs. [18,19], where possibilities to find new Higgs bosons via cascade decays of heavy scalar resonances are discussed in detail.

Recently the latter way, seeking deviations as the indirect searches, is getting much attention, since h has already been discovered and its properties will be precisely

^{*}kon@st.seikei.ac.jp

[†]dm186115@cc.seikei.ac.jp

[‡]tueda@st.seikei.ac.jp

[§]Present address: Department of Physics, Osaka University, Toyonaka, Osaka 560-0043, Japan.
yagyu@het.phys.sci.osaka-u.ac.jp

Published by the American Physical Society under the terms of the Creative Commons Attribution 4.0 International license. Further distribution of this work must maintain attribution to the author(s) and the published article's title, journal citation, and DOI. Funded by SCOAP³.

measured in the near future. For example, at the HL-LHC, the h couplings to weak gauge bosons (hVV , $V = W, Z$) and fermions (e.g., $h\tau\tau$, $hb\bar{b}$) are expected to be measured with a few percent level [20], while at the International Linear Collider (ILC), hVV and $hf\bar{f}$ ($f = \tau, c$ and b) couplings can be measured to be subpercent and one percent level, respectively [21]. In the THDM, these h couplings can deviate from the SM prediction with various patterns depending on a particular scenario as it has been clarified in Ref. [22] at tree level and in Refs. [23–27] at one-loop level. Therefore, by looking at the possible deviation in the h couplings in the future collider experiments, one can distinguish the scenarios of the THDM, which is still important even if an additional Higgs boson could be discovered via a resonant peak in the direct searches.

In this paper, we focus on the cross section of double Higgs boson production processes at future e^+e^- colliders as another important observable regarding the indirect search of the THDM. The double Higgs boson production has been discussed in the pp [28–30], e^+e^- [28,31] and $\gamma\gamma$ [28] collision in the THDM to extract the Higgs boson self-coupling constant hhh , particularly the case with the alignment limit, where all the SM-like Higgs boson couplings become the same as those in the SM prediction. In Ref. [32], the Higgs boson pair production has also been discussed at the LHC in the case with and without the alignment limit. Experimentally, it is a formidable task for the LHC to precisely determine the Higgs trilinear coupling because of the tiny cross section of the double Higgs boson production (see [33,34] for recent results). This is certainly one of the motivations of construction of new powerful colliders: the HL-LHC will be sensitive only to $\mathcal{O}(100\%)$ for the trilinear coupling while the ILC running at 500 GeV can measure it with an about 25% uncertainty at the 68% confidence level [35].

Our motivation to discuss the double Higgs production is to find the correlation between the deviation in the Higgs boson couplings and the modification of the cross section. These two variables are expected to be strongly correlated with each other, because the deviation in the h couplings appears in the case without the alignment limit at tree level, in which case the additional neutral Higgs bosons can mediate the double Higgs boson production process and can provide sizable enhancement of the cross section. For the LHC, large enhancement of the di-Higgs production cross section due to resonantly produced heavy Higgs bosons in the CP violating THDM has been studied in Ref. [29]. In the present paper, we clarify how large enhancement can be obtained at e^+e^- colliders in the case without the alignment limit. In the numerical analysis, we focus on a special case of the THDM often referred to as type I, because scenarios without the alignment limit are highly constrained by the Higgs

boson signal strengths in the other types of Yukawa interactions such as type II. Under the current theoretical and experimental constraints on the parameter space, we find a strong correlation between the enhancement of the cross section and the scaling factor of the $hf\bar{f}$ couplings. We note that Higgs boson pair production at e^+e^- colliders requires the collision energy $\sqrt{s}$ being *larger* than 250 GeV while the scaling factor can be precisely determined with $\sqrt{s} = 250$ GeV. Hence, the Higgs boson couplings will be known when experiments reach collision energy sufficient for Higgs pair boson production measurement.

This paper is organized as follows. In Sec. II, we define the model and present the relevant Higgs boson interactions. In Sec. III, we discuss general properties of the Higgs boson pair production in e^+e^- colliders. Then, we explain a mechanism that enhances the cross section in the THDM without the alignment limit and give a rough estimate of the enhancement factor. Detailed numerical analysis on the aforementioned processes at tree level is performed in Sec. IV. Finally, Sec. V is devoted to our conclusions.

II. MODEL

We consider the THDM whose Higgs sector is constructed by two isospin scalar doublet fields Φ_1 and Φ_2 . For simplicity, we consider the CP -conserving case throughout the paper. The vacuum expectation values (VEVs) of the two doublets, i.e., $\sqrt{2}\langle\Phi_{1,2}^0\rangle = v_{1,2}$, are parametrized by $v = \sqrt{v_1^2 + v_2^2} = (\sqrt{2}G_F)^{-1/2}$, with G_F being the Fermi constant and their ratio $\tan\beta = v_2/v_1$.

It is convenient to introduce the so-called Higgs basis [36,37] as follows:

$$\begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix} = \begin{pmatrix} \cos\beta & -\sin\beta \\ \sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} \Phi \\ \Phi' \end{pmatrix}, \quad (1)$$

where

$$\Phi = \begin{pmatrix} G^+ \\ \frac{h'_1 + v + iG^0}{\sqrt{2}} \end{pmatrix}, \quad \Phi' = \begin{pmatrix} H^+ \\ \frac{h'_2 + iA}{\sqrt{2}} \end{pmatrix}. \quad (2)$$

In this basis, the Nambu-Goldstone bosons $G^\pm$ and G^0 , which are absorbed into the longitudinal components of the W and Z bosons, are separated from the physical singly charged Higgs boson $H^\pm$ and the CP -odd Higgs boson A , while two CP -even Higgs states h'_1 and h'_2 are generally not mass eigenstates at this stage. By introducing another mixing angle α , the mass eigenstates are given by

$$\begin{pmatrix} h'_1 \\ h'_2 \end{pmatrix} = \begin{pmatrix} c_{\beta-\alpha} & s_{\beta-\alpha} \\ -s_{\beta-\alpha} & c_{\beta-\alpha} \end{pmatrix} \begin{pmatrix} H \\ h \end{pmatrix}, \quad (3)$$

where we have abbreviated $\sin X$ and $\cos X$ as s_X and c_X , respectively. We identify h as the discovered Higgs boson with a mass of about 125 GeV.

The kinetic terms for the Higgs doublets are given as

$$\mathcal{L}_{\text{kin}} = \sum_{i=1,2} |D_\mu \Phi_i|^2 = |D_\mu \Phi|^2 + |D_\mu \Phi'|^2, \quad (4)$$

where D_μ is the covariant derivative for the Higgs doublets. We note that from the first term on the

right-hand side, the gauge-gauge-Higgs-type interactions are obtained as

$$\mathcal{L}_{\text{kin}} \supset \left(\frac{2m_W^2}{v} W_\mu^+ W^{-\mu} + \frac{m_Z^2}{v} Z_\mu Z^\mu \right) (s_{\beta-\alpha} h + c_{\beta-\alpha} H). \quad (5)$$

From the second term in the right-hand side of Eq. (4), we obtain the Higgs-Higgs-gauge-type interactions as

$$\begin{aligned} \mathcal{L}_{\text{kin}} \supset & -i \frac{g}{2} H^+ \overleftrightarrow{\partial}^\mu (c_{\beta-\alpha} h - s_{\beta-\alpha} H - iA) W_\mu^- + \text{H.c.} - \frac{g}{2 \cos \theta_W} A \overleftrightarrow{\partial}^\mu (c_{\beta-\alpha} h - s_{\beta-\alpha} H) Z_\mu \\ & - ie H^+ \overleftrightarrow{\partial}^\mu H^- A_\mu - i \frac{g \cos 2\theta_W}{2 \cos \theta_W} H^+ \overleftrightarrow{\partial}^\mu H^- Z_\mu, \end{aligned} \quad (6)$$

with $X \overleftrightarrow{\partial}^\mu Y \equiv X(\partial^\mu Y) - (\partial^\mu X)Y$ and θ_W being the weak mixing angle.

In order to avoid Higgs boson mediating flavor changing neutral currents at tree level, we impose a discrete Z_2 symmetry [17] into the Higgs sector, where the two doublets are transformed as $\Phi_1 \rightarrow +\Phi_1$ and $\Phi_2 \rightarrow -\Phi_2$. Under the Z_2 symmetry, only one of the two Higgs doublets can couple to each up-type, down-type quark and charged lepton, by which the interaction matrices in the flavor space between neutral Higgs bosons and fermions are diagonalized in the fermion mass eigenbasis. It has been known that there are four independent types of Yukawa interactions so-called type I, II, X, and Y [38] depending on the way to assign the Z_2 charge for fermions [39,40].

The Yukawa Lagrangian for the third generation fermions is then given in the Higgs basis by

$$\begin{aligned} \mathcal{L}_Y = & -\bar{Q}_L^3 \frac{\sqrt{2}m_t}{v} (\tilde{\Phi} + \xi_t \tilde{\Phi}') t_R - \bar{Q}_L^3 \frac{\sqrt{2}m_b}{v} (\Phi + \xi_b \Phi') b_R \\ & - \bar{L}_L^3 \frac{\sqrt{2}m_\tau}{v} (\Phi + \xi_\tau \Phi') \tau_R + \text{H.c.}, \end{aligned} \quad (7)$$

where $\tilde{\Phi} = i\tau_2 \Phi^*$ and $\tilde{\Phi}' = i\tau_2 \Phi'^*$. The factors ξ_b and ξ_τ depend on the choice of the types of Yukawa interaction as

$$\begin{array}{cccc} \text{type I} & \text{type II} & \text{type X} & \text{type Y} \\ (\xi_b, \xi_\tau): & (\cot \beta, \cot \beta) & (-\tan \beta, -\tan \beta) & (\cot \beta, -\tan \beta) \quad (-\tan \beta, \cot \beta), \end{array} \quad (8)$$

while $\xi_t = \cot \beta$ for all the types of Yukawa interaction. The interaction terms of the Higgs bosons and fermions are extracted as

$$\begin{aligned} \mathcal{L}_Y \supset & - \sum_{f=t,b,\tau} \frac{m_f}{v} \bar{f} [(s_{\beta-\alpha} + \xi_f c_{\beta-\alpha}) h + (c_{\beta-\alpha} - \xi_f s_{\beta-\alpha}) H - 2i I_f \xi_f \gamma_5 A] f \\ & - \frac{\sqrt{2}}{v} \bar{t} (m_b P_R - m_t P_L) H^+ b - \frac{\sqrt{2}}{v} \bar{\nu}_\tau m_\tau P_R H^+ \tau + \text{H.c.}, \end{aligned} \quad (9)$$

where $I_t(I_{b,\tau}) = 1/2(-1/2)$ and $P_L(P_R)$ is the projection operator for the left (right) hand chirality.

The Higgs potential is generally written by eight independent real parameters when we include the soft-breaking term of the Z_2 symmetry,

$$\begin{aligned} V = & m_1^2 \Phi_1^\dagger \Phi_1 + m_2^2 \Phi_2^\dagger \Phi_2 - m_3^2 (\Phi_1^\dagger \Phi_2 + \text{H.c.}) \\ & + \frac{\lambda_1}{2} (\Phi_1^\dagger \Phi_1)^2 + \frac{\lambda_2}{2} (\Phi_2^\dagger \Phi_2)^2 + \lambda_3 (\Phi_1^\dagger \Phi_1) (\Phi_2^\dagger \Phi_2) + \lambda_4 |\Phi_1^\dagger \Phi_2|^2 + \frac{\lambda_5}{2} [(\Phi_1^\dagger \Phi_2)^2 + \text{H.c.}]. \end{aligned} \quad (10)$$

As we already mentioned in the above, we assume the CP invariance of the Higgs sector, so that m_3^2 and λ_5 parameters are taken to be real. After imposing the tadpole conditions, i.e., the requirement of vanishing the linear terms of h and H in the Higgs potential, we can eliminate the m_1^2 and m_2^2 parameters. Then, these eight parameters, i.e., six parameters in the potential and two VEVs, are expressed as follows:

$$m_{H^\pm}, m_A, m_H, m_h, M^2, \tan\beta, v, \alpha, \quad (11)$$

where $M^2 \equiv m_3^2/(s_\beta c_\beta)$. Among these eight parameters, v and m_h are fixed to about 246 and 125 GeV, respectively.

From Eq. (10), we can extract the scalar three-point couplings. In particular, the hhh and Hhh couplings will be important in the later analysis, which are expressed in terms of the parameters shown in (11) as follows:

$$\lambda_{hhh} = -\frac{m_h^2}{2v}s_{\beta-\alpha} + \frac{M^2 - m_h^2}{v}s_{\beta-\alpha}c_{\beta-\alpha}^2 + \frac{M^2 - m_h^2}{2v} \times c_{\beta-\alpha}^3(\cot\beta - \tan\beta), \quad (12)$$

$$\lambda_{Hhh} = -\frac{c_{\beta-\alpha}}{2v}\{4M^2 - 2m_h^2 - m_H^2 + (2m_h^2 + m_H^2 - 3M^2) \times [2c_{\beta-\alpha}^2 + s_{\beta-\alpha}c_{\beta-\alpha}(\tan\beta - \cot\beta)]\}, \quad (13)$$

where the above quantities are defined by the coefficient in front of the hhh and Hhh vertices in the Lagrangian.

Before closing this section, it would be worth mentioning that taking $s_{\beta-\alpha} \rightarrow 1$, all the hVV ($V = W, Z$), hff and hhh couplings become the same values as in the SM at tree level. This limit has been known as the alignment limit, where the SM-like Higgs boson h completely comes from the doublet Φ in the Higgs bases in Eq. (2).

III. DOUBLE HIGGS BOSON PRODUCTION

In this section, we discuss the general property of double Higgs boson production processes at e^+e^- colliders, and consider how much the production cross section can be different in the THDM with respect to that in the SM prediction.

The double Higgs boson production $e^+e^- \rightarrow hh + X$ is possible when the collision energy of the electron and positron $\sqrt{s}$ is larger than about 250 GeV. Figures 1 and 2 show relevant Feynman diagrams, where those in Fig. 1 are common to the SM and THDM, while those in Fig. 2 only appear in the THDM. We here ignore diagrams

FIG. 1. Feynman diagrams for the double Higgs boson production process in the e^+e^- collision in the SM with V denoting either W or Z .

FIG. 2. Additional Feynman diagrams for the double Higgs boson production process in the e^+e^- collision in the THDM with V denoting either W or Z .

induced via Yukawa couplings.¹ In the SM, there are two types of diagrams contributing to the double Higgs boson production, namely, the s -channel process $e^+e^- \rightarrow Zhh$ shown as diagrams (a)–(c) in Fig. 1 and the vector boson fusion processes $e^+e^- \rightarrow e^+e^-hh$ and $e^+e^- \rightarrow \nu_e\bar{\nu}_e hh$ shown as diagrams (d)–(f) in Fig. 1. The cross section of these processes is calculated as 0.16 (0.12), 2.6×10^{-3} (7.2×10^{-2}), and 3.6×10^{-4} (9.5×10^{-3}) fb for the $e^+e^- \rightarrow Zhh$, $\nu_e\bar{\nu}_e hh$, and $e^+e^- hh$ process at $\sqrt{s} = 500(1000)$ GeV, respectively. Therefore, the cross section of the double Higgs boson production is mainly determined by the s -channel process.

In the THDM, additional diagrams contribute to the process as seen in Fig. 2, where the extra neutral Higgs boson H or A appears in the diagrams having the same topology as those of (a), (b), (d), and (e) in Fig. 1. In order to estimate the typical size of these contributions, let us focus on the diagram (a') in Fig. 2 as an example. When the mass of H is taken to be between $250 \text{ GeV} \lesssim m_H \lesssim \sqrt{s} - m_Z$, H can be on shell. In this case, the cross section of the diagram (a') is approximately calculated by the product of the two-body cross section of $e^+e^- \rightarrow ZH$ and the branching ratio of the $H \rightarrow hh$ decay assuming $\Gamma_H \ll m_H$ with Γ_H being the total width of H .² Therefore, the size of this cross section is typically obtained by multiplying the factor of $16\pi^2 \times c_{\beta-\alpha}^2 \times \text{BR}(H \rightarrow hh)$ with respect to the cross section of the diagram (a) in the SM, where the factor $16\pi^2$ appears due to the typical ratio of the two-body and three-body phase-space factors, while $c_{\beta-\alpha}^2$ comes from the HZZ coupling normalized to the hZZ one in the SM. For example, when $s_{\beta-\alpha}$ is fixed to be 0.99 (0.995), the above factor becomes 3.14 (1.56) assuming $\text{BR}(H \rightarrow hh) = 1$. The similar enhancement can also be obtained from the diagram (b') as long as A is produced on shell. We thus can expect that the total cross section of the double Higgs boson production can be several times larger than the SM prediction.

The important thing here is that such enhancement of the cross section happens when departure of the alignment limit, i.e., $s_{\beta-\alpha} \neq 1$, is realized, because both HZZ and AZh couplings are proportional to $c_{\beta-\alpha}$. Therefore, the enhancement is strongly correlated with the deviation in the Higgs boson couplings from the SM prediction. On the other hand, the hVV ($V = W, Z$) and $hf\bar{f}$ couplings are expected to be precisely measured at future e^+e^- colliders. For example, at the ILC the hZZ , hWW , $hb\bar{b}$, $h\tau^+\tau^-$, and $hc\bar{c}$

couplings may be able to be measured with 0.38%, 1.8%, 1.8%, 1.9%, and 2.4% at 1σ level assuming 250 GeV of the collision energy and 2 ab^{-1} of the integrated luminosity [21]. Therefore, if deviations in the Higgs boson couplings are detected in future, we expect the sizable enhancement of the double Higgs boson production as well.

In the next section, we numerically evaluate how large enhancement can be obtained in the scenario without the alignment limit and show the correlation between the double Higgs boson production cross section and the deviation in the Higgs boson couplings.

IV. NUMERICAL RESULTS

In this section, we compute the cross section of the $e^+e^- \rightarrow f\bar{f}hh$ process, where f is a fermion except for the top quark. These four-body final states are obtained through the Zhh production with the decay of Z into a fermion pair and the vector boson fusion process; see Figs. 1 and 2. We note that when the Z boson from the Zhh process decays into e^+e^- ($\nu_e\bar{\nu}_e$), this process interferes with the $e^+e^- hh$ ($\nu_e\bar{\nu}_e hh$) final states from the Z (W) boson fusion process. We take into account such interference effects in the numerical analysis. We also note that the dependence of the type of Yukawa interactions slightly appears in the cross section through the decay width of H and A . In contrast, the type dependence significantly appears in the region of the parameter space allowed from experimental bounds. In particular, except for the type-I THDM, the scenario with $s_{\beta-\alpha} \neq 1$ that is considered in this section is highly constrained by the Higgs boson signal strengths; see, e.g., [41]. We thus consider the type-I THDM in what follows.³

As seen in (11), there are six free parameters in the THDM, i.e., three masses of extra Higgs bosons, M^2 , $\tan\beta$, and α . Instead of inputting α , we take $s_{\beta-\alpha}$ and the sign of $c_{\beta-\alpha}$ as inputs. To avoid large contributions to the electroweak oblique T parameter [43,44], we take the masses of $H^\pm$ and A to be the same, i.e., $m_A = m_{H^\pm}$ [45–48]. In this case, the quadratic dependence of the extra Higgs boson masses completely vanishes, and only the small logarithmic mass dependence remains, which is proportional to $c_{\beta-\alpha}^2$.

Before going to show the numerical results for the cross section, let us summarize the constraints on the parameter space what we take into account in the analysis. We impose the perturbative unitarity bound [49–52] and the vacuum stability bound [45–48] as the constraints from theoretical consistency. These bounds restrict the size of scalar quartic couplings in the potential, which can be translated into the

¹For the case of $\sqrt{s}$ larger than $2(m_t + m_h) \sim 600 \text{ GeV}$, the $t\bar{t}hh$ production is allowed and its contribution could be comparable as compared with that shown in Fig. 1. In the numerical analysis of this paper given in the next section, we focus on the collision energy to be 500 GeV, so that this process does not need to be considered.

²In our scenario, the typical size of Γ_H/m_H is 1% level or smaller.

³In the type-I THDM, constraints from flavor experiments such as those from $B \rightarrow X_s \gamma$ data on the charged Higgs boson mass are also much milder than those in the other types especially in type II. For example, $\mathcal{O}(100)$ GeV of the charged Higgs boson mass is allowed by the $B \rightarrow X_s \gamma$ data when $\tan\beta \gtrsim 2$ [42] in the type-I THDM, while $m_{H^\pm} \lesssim 600 \text{ GeV}$ is excluded with 95% confidence level in the type-II THDM.

FIG. 3. Region excluded by the theoretical and experimental bounds in the case of $m_H = m_A = m_{H^\pm} = 300$ GeV and $s_{\beta-\alpha} = 0.99$ (left) and 0.995 (right). The blue (magenta) shaded region is excluded by the perturbative unitarity, vacuum stability bounds, and/or the electroweak oblique S , T parameters (direct searches at the LEP, Tevatron, and LHC experiments and/or compatibility of the signal strength of h). The solid, dashed, and dotted curves respectively show the contours for $\text{BR}(H \rightarrow hh) = 0.25, 0.5$, and 0.75 .

bound on the Higgs boson masses and the mixing angles. As the experimental constraints, we take into account the electroweak oblique S and T parameters [53], direct searches for additional Higgs bosons at the LEP, Tevatron and LHC experiments, as well as the compatibility of the signal strengths for the discovered Higgs boson with a mass of 125 GeV. For the direct searches, we use the HiggsBounds-5.3.0beta [54]. For the signal strengths of the discovered Higgs boson, we use the combined data from ATLAS and CMS at the LHC Run-I experiments [3], and require that the prediction of the signal strengths for the $\gamma\gamma$, ZZ^* , WW^* , and $\tau\tau$ modes of the h state does not exceed the given width of the error bar. We impose these experimental bounds at the 95% confidence level.

In Fig. 3, we show the region of the parameter space excluded by the theoretical and experimental bounds explained in the above. Here, we take $m_H = m_A (= m_{H^\pm}) = 300$ GeV and $s_{\beta-\alpha} = 0.99$ (left panel) and 0.995 (right panel). The other parameters M^2 and $\tan\beta$ are scanned in these plots. Instead of showing the value of $\tan\beta$, we introduce the scaling factor κ_f for the Yukawa coupling g_{hff}^{THDM} in the THDM normalized to the SM value g_{hff}^{SM} , see Eq. (7),

$$\kappa_f \equiv \frac{g_{hff}^{\text{THDM}}}{g_{hff}^{\text{SM}}} = s_{\beta-\alpha} + c_{\beta-\alpha} \cot\beta. \quad (14)$$

We note that due to the choice of the type-I THDM, the scaling factor κ_f does not depend on the choice of a fermion f ; see Sec. II. From Eq. (14), we see that $\kappa_f < s_{\beta-\alpha}$ ($\kappa_f > s_{\beta-\alpha}$) is obtained by taking the sign of $c_{\beta-\alpha}$ to be negative (positive), and $\kappa_f = s_{\beta-\alpha}$ is given in the limit of $\tan\beta \rightarrow \infty$.

We see that the constraint from experiments (shown by the magenta shaded region) becomes important in the

region with smaller $\tan\beta$ values, i.e., larger values of $|1 - \kappa_f|$, where the direct search at the LHC, particularly for $H \rightarrow ZZ$ dominantly contributes to the exclusion. This can be understood by the fact that the production cross section $gg \rightarrow H$ is proportional to $\cot^2\beta$ in the limit of $s_{\beta-\alpha} \rightarrow 1$, so that the larger $\tan\beta$ case can avoid the constraint due to the smaller cross section. Another thing we can find in this figure is that the theory bound (shown by the blue shaded region⁴) becomes important in the case with larger $\tan\beta$ and/or larger difference between m_H^2 and M^2 . In particular, for a larger $\tan\beta$ case, quite small area with $M^2 \sim m_H^2$ is allowed, because some of the scalar quartic couplings become very sensitive to the value of $m_H^2 - M^2$. The typical behavior of the constraints does not change so much between the cases with $s_{\beta-\alpha} = 0.99$ (left panel) and 0.995 (right panel), but smaller values of $|1 - \kappa_f|$ are excluded by experimental constraints. This is simply because the value of $|1 - \kappa_f|$ given by the same value of $\tan\beta$ becomes smaller in the case of $s_{\beta-\alpha} = 0.995$ as compared to the case of $s_{\beta-\alpha} = 0.99$, as seen in Eq. (14). For reference, we also show the contours of the branching ratio of the $H \rightarrow hh$ mode, which is important to understand the behavior of the enhancement of the cross section of the double Higgs boson production.

Now we present numerical results on the cross section. For the numerical computation of the cross section, we used the public version of GRACE [55–57] with some modifications, and all the calculations were performed at tree level with $\sqrt{s} = 500$ GeV. Let us first illustrate the invariant mass distribution of the di-Higgs system m_{hh} in Fig. 4. Here, we take $m_H = 300, 350$, and 400 GeV as examples,

⁴In fact, the constraints from the S and T parameter are also taken into account in the blue shaded region, but the shape of the exclusion is almost determined by the perturbative unitarity and vacuum stability bounds.

FIG. 4. Invariant mass distribution for the di-Higgs system m_{hh} . We take $s_{\beta-\alpha} = 0.99$, $c_{\beta-\alpha} > 0$, and $\tan\beta = 2$. For the red, blue, and green histograms, we take $m_H = m_A = m_{H^\pm} = 300, 350$, and 400 GeV with $M = m_H - 20$ GeV, respectively. All these choices of the parameters are allowed by the constraints explained in this section.

FIG. 5. Correlation between κ_f and R in the case of $m_H = m_A = m_{H^\pm} = 300$ GeV. We take $s_{\beta-\alpha} = 0.99$ (red points) and 0.995 (blue points). All the points satisfy the theoretical constraints and experimental bounds explained in this section.

FIG. 6. Same as in Fig. 3 but for the case of larger masses $m_A = m_{H^\pm} = 500$ GeV while the extra CP -even neutral Higgs mass is retained, $m_H = 300$ GeV.

and the other parameters are specified as described in the caption. It is clearly seen that the sharp peaks appear at around m_H , because of the on-shell H mediation with the $H \rightarrow hh$ decay shown in the diagram (a') of Fig. 2. The height of the peak is getting lower as the mass of H increases. Because of this resonant effect, the cross section of the double Higgs boson production is sizably enhanced as we see in what follows.

Next, we consider the ratio of the di-Higgs production cross section in the type I THDM to that in the SM in order to see how large an enhancement can be obtained,

$$R \equiv \frac{\sum_f \sigma^{\text{THDM}}(e^+e^- \rightarrow f\bar{f}hh)}{\sum_f \sigma^{\text{SM}}(e^+e^- \rightarrow f\bar{f}hh)}, \quad (15)$$

where the summation for f is done over all the fermions except for $f = t$. In the following calculation, we scan $\tan\beta$ and M^2 with the ranges of $1 \leq \tan\beta \leq 30$ and $0 \leq M^2 \leq (500 \text{ GeV})^2$. The scatter plot in Fig. 5 shows the ratio R as functions of the scaling factor κ_f using sampling points passing all the constraints, namely, points in the white region of Fig. 3. Red and blue points represent for $s_{\beta-\alpha} = 0.99$ and 0.995 , respectively. As it is expected in Sec. III, a clear correlation between R and κ_f is seen from this plot, and a considerable enhancement of the cross section is observed due to the on-shell mediation of the extra neutral Higgs bosons H and A . See also the contours of the branching ratio of $H \rightarrow hh$ shown in Fig. 3 to figure out the behavior of the correlation. It is also seen that a larger value of R is obtained in the case of $s_{\beta-\alpha} = 0.99$ in comparison with the case of $s_{\beta-\alpha} = 0.995$, because the HZZ and AZh couplings are proportional to $c_{\beta-\alpha}$. The strong enhancement seen in the large $\tan\beta$ region (i.e., $\kappa_f \sim s_{\beta-\alpha} \sim 1$) can be traced back to a large value of the λ_{Hhh} coupling; see Eq. (13). This gives a considerable deviation $R > 1$ even at $\kappa_f = 1$. We find that the value of R can be maximally around 5 (3) for $s_{\beta-\alpha} = 0.99(0.995)$.

FIG. 7. Same as in Fig. 5 but for the case of larger masses $m_A = m_{H^\pm} = 500$ GeV while the extra CP -even neutral Higgs mass is retained, $m_H = 300$ GeV.

We also perform the similar calculation shown in Figs. 6 and 7, but for the case with $m_H = 300$ GeV and $m_A = m_{H^\pm} = 500$ GeV. The region allowed by the constraints is almost the same as the previous plots in Fig. 3. On the other hand, by looking at Fig. 7, we find that the shape of the scattering points is shifted to below due to the A mediation being off shell, and the maximally allowed value of R becomes around 4 and 2.5 for $s_{\beta-\alpha} = 0.99$ and 0.995, respectively.

In Fig. 8, we scan the value of $s_{\beta-\alpha}$ with $0.99 \leq s_{\beta-\alpha} \leq 1$ under the constraints described in the above. The left (right) panel shows the case with $m_A = m_{H^\pm} = 300(500)$ GeV, while m_H is fixed to be 300 GeV in both panels. The value of R is distributed in the range between 1 and 4.5 (4) for the case with $m_A = m_{H^\pm} = 300(500)$ GeV. In Fig. 9, we also show the dependence on the scaling factor of hVV ($V = W, Z$) couplings denoted as $\kappa_V \equiv g_{hVV}^{\text{THDM}}/g_{hVV}^{\text{SM}} = s_{\beta-\alpha}$. It is clarified that the value of R approaches unity

FIG. 8. Correlation between κ_f and R in the case of $m_A = m_{H^\pm} = 300$ (left) and 500 GeV (right). In the both figures, we take $m_H = 300$ GeV, while M^2 , $\tan\beta$, and $s_{\beta-\alpha}$ are scanned. All the points satisfy the theoretical constraints and experimental bounds explained in this section.

FIG. 9. Correlation between $\kappa_V (= s_{\beta-\alpha})$ and R in the case of $m_A = m_{H^\pm} = 300$ (left) and 500 GeV (right). In both figures, we take $m_H = 300$ GeV, while M^2 and $\tan\beta$ are scanned. All the points satisfy the theoretical constraints and experimental bounds explained in this section.

FIG. 10. Correlation between $m_H (= m_A = m_{H^\pm})$ and R for the case of $\tan\beta = 2$ (top), $\tan\beta = 3$ (middle), and $\tan\beta = 5$ (bottom). The sign of $c_{\beta-\alpha}$ is taken to positive (left) and negative (right). For all the plots, we take $s_{\beta-\alpha} = 0.99$ (red-circle points) and 0.995 (blue-square points).

when $\kappa_V \rightarrow 1$, because H and A do not appear in the diagram in this limit and all the couplings of h become the same value as those in the SM. However, once $\kappa_V < 1$ is taken, a significant enhancement is driven by the appearance of the H and A mediations in the diagram depending on the value of κ_f ; see also Figs. 5 and 7.

Finally, we show the mass dependence of the value of R in Fig. 10 with a fixed value of $\tan\beta$ to be 2 (top), 3 (middle), and 5 (bottom). In these plots, we take the degenerate mass of the extra Higgs bosons, i.e.,

$m_H = m_A = m_{H^\pm}$, and varying the mass range to be from 200 to 500 GeV. Again, all the points are passed all the constraints explained in the above. For the low $\tan\beta$ case such as $\tan\beta = 2$ shown in the top panels, the constraint from direct searches excludes the lower mass region, so that the points appear only in the larger mass region. For the larger $\tan\beta$ case particularly with $c_{\beta-\alpha} > 0$ (left panels), the lower mass region is allowed, and it is clearly seen that the value of R drastically increases at the mass of extra Higgs boson just above 215 and 250 GeV, because of the

on-shell $A \rightarrow Zh$ and $H \rightarrow hh$ decays open. We also find that for the larger mass region, e.g., $m_H \gtrsim 400$ GeV the value of R becomes smaller than 1, in which the H and A appearing in the Zhh production are getting off shell,⁵ so that these contributions become unimportant. The value $R < 1$ is simply explained by the fact that the contribution containing $h^* \rightarrow hh$ as the diagram (a) in Fig. 1 becomes smaller than the SM prediction because of the smaller λ_{hhh} coupling with respect to the SM value; see Eq. (12). We note that in these plots, the value of $\tan\beta$ is fixed, so that the value of κ_f is determined to be (1.06, 1.04, 1.02; 0.92, 0.94, 0.96) for $s_{\beta-\alpha} = 0.99$ and (1.04, 1.03, 1.01; 0.95, 0.96, 0.98) for $s_{\beta-\alpha} = 0.995$ where the first (second) three values are the case for $c_{\beta-\alpha} > 0$ ($c_{\beta-\alpha} < 0$) with $\tan\beta = 2, 3$, and 5.

Before closing this section, we give a brief comment on the difference of the prediction from the other models, e.g., models with an isospin singlet scalar field; see for instance [58]. In these models, both κ_V ($V = W, Z$) and κ_f are given by the same factor like $\cos\theta$, where θ is a mixing angle between two CP -even Higgs bosons. Therefore, the case ($\kappa_f \sim 1$, but $R > 1$) that we found in the above does not happen, and so our analysis would give an important result to pin down the possible models.

In addition, we comment on the possibility of the direct detection of H at future collider experiments such as the HL-LHC. In the parameter space that we considered in this paper, the branching ratio of H is approximately given by $\text{BR}(H \rightarrow hh) + \text{BR}(H \rightarrow WW) + \text{BR}(H \rightarrow ZZ) \sim 1$, where the small portion of the branching ratio is filled by the fermionic decay modes.⁶ Thus, H can be discovered via the $H \rightarrow WW/ZZ$ mode. In fact at the HL-LHC, there is a study for direct searches of a neutral scalar boson X using the $X \rightarrow ZZ$ mode, and the 95% confidence level upper limit on the cross section

⁵In fact, for the case with $m_H \sim 400$ GeV, H can still be on shell as it is produced with the Z boson at the collision energy of 500 GeV. However, we have checked that the cross section of the $e^+e^- \rightarrow ZH \rightarrow Zhh$ with $m_H = 400$ GeV is smaller than the other subdominant contributions such as the $e^+e^- \rightarrow \nu_e \bar{\nu}_e hh$ process, because of the phase-space suppression. Therefore, we cannot obtain the large enhancement of the cross section at $m_H \sim 400$ GeV or larger.

⁶For $m_H \gtrsim 350$ GeV, the branching ratio of the $H \rightarrow t\bar{t}$ mode can be 10% level for a smaller value of $\tan\beta$.

($pp \rightarrow X$) times the branching ratio ($X \rightarrow ZZ$) is taken in Ref. [59]. This bound can be translated into the constraint on the parameter space in our model, by which smaller values of $\tan\beta$ and/or m_H can be excluded. Combining such constraint into our analysis, more restricted results would be obtained.

V. CONCLUSIONS

We have discussed the correlation between the scaling factor of the Yukawa coupling for the SM like Higgs boson κ_f and the ratio of the cross section for the $e^+e^- \rightarrow hh f \bar{f}$ ($f \neq t$) process normalized to the SM prediction in the type-I THDM. We particularly concentrate on the case without the alignment limit, in which resonant effects of the extra neutral Higgs bosons H and A provide a sizable enhancement of the cross section and the value of κ_f is different from unity at tree level. Under the constraints from perturbative unitarity, vacuum stability, electroweak oblique parameters, direct searches for heavy Higgs bosons at collider experiments, and compatibility of the signal strengths of the discovered Higgs boson, we have found that the considerable enhancement of the cross section, typically a few times larger than the SM prediction, can be obtained depending on the value of κ_f and the masses of the extra Higgs bosons. The value of κ_f is expected to be precisely measured at future collider experiments such as the high-luminosity LHC and the ILC, typically with a few percent and one percent level, respectively. Therefore, if some deviations in the Higgs boson couplings are found at future colliders, we expect the sizable enhancement of the double Higgs boson production and can extract information of the mass of the extra neutral Higgs boson and/or dimensionful model parameter M^2 in the Higgs potential.

Although expected not to give so drastic a change, radiative corrections to the cross-section enhancement studied in this work may be investigated by using, for example, H-COUP [60] to incorporate one-loop electroweak-corrected vertices or GRACE-1loop [61,62] for full one-loop computation. We leave it for future works.

ACKNOWLEDGMENTS

All the authors are grateful for fruitful discussions with Masaaki Kuroda, Yoshimasa Kurihara, Kiyoshi Kato, Masato Jimbo, Tadashi Ishikawa, Junpei Fujimoto, Yusaku Kouda, and Ryotaro Nara.

- [1] G. Aad *et al.* (ATLAS Collaboration), Observation of a new particle in the search for the standard model Higgs boson with the ATLAS detector at the LHC, *Phys. Lett. B* **716**, 1 (2012).
- [2] S. Chatrchyan *et al.* (CMS Collaboration), Observation of a new boson at a mass of 125 GeV with the CMS experiment at the LHC, *Phys. Lett. B* **716**, 30 (2012).
- [3] G. Aad *et al.* (ATLAS and CMS Collaborations), Measurements of the Higgs boson production and decay rates and constraints on its couplings from a combined ATLAS and CMS analysis of the LHC pp collision data at $\sqrt{s} = 7$ and 8 TeV, *J. High Energy Phys.* **08** (2016) 045.
- [4] G. C. Branco, P. M. Ferreira, L. Lavoura, M. N. Rebelo, M. Sher, and J. P. Silva, Theory and phenomenology of two-Higgs-doublet models, *Phys. Rep.* **516**, 1 (2012).
- [5] H. E. Haber and G. L. Kane, The search for supersymmetry: Probing physics beyond the standard model, *Phys. Rep.* **117**, 75 (1985).
- [6] J. Mrazek, A. Pomarol, R. Rattazzi, M. Redi, J. Serra, and A. Wulzer, The other natural two Higgs doublet model, *Nucl. Phys. B* **853**, 1 (2011).
- [7] S. De Curtis, L. Delle Rose, S. Moretti, and K. Yagyu, Supersymmetry versus compositeness: 2HDMs tell the story, *Phys. Lett. B* **786**, 189 (2018).
- [8] S. De Curtis, L. Delle Rose, S. Moretti, and K. Yagyu, A concrete composite 2-Higgs doublet model, *J. High Energy Phys.* **12** (2018) 051.
- [9] N. Turok and J. Zadrozny, Dynamical Generation of Baryons at the Electroweak Transition, *Phys. Rev. Lett.* **65**, 2331 (1990).
- [10] A. E. Nelson, D. B. Kaplan, and A. G. Cohen, Why there is something rather than nothing: Matter from weak interactions, *Nucl. Phys. B* **373**, 453 (1992).
- [11] A. Zee, A theory of lepton number violation, neutrino majorana mass, and oscillation, *Phys. Lett.* **93B**, 389 (1980); Erratum, *Phys. Lett.* **95B**, 461(E) (1980).
- [12] A. Zee, Charged scalar field and quantum number violations, *Phys. Lett.* **161B**, 141 (1985).
- [13] E. Ma, Verifiable radiative seesaw mechanism of neutrino mass and dark matter, *Phys. Rev. D* **73**, 077301 (2006).
- [14] M. D. Campos, D. Cogollo, M. Lindner, T. Melo, F. S. Queiroz, and W. Rodejohann, Neutrino masses and absence of flavor changing interactions in the 2HDM from gauge principles, *J. High Energy Phys.* **08** (2017) 092.
- [15] D. A. Camargo, A. G. Dias, T. B. de Melo, and F. S. Queiroz, Neutrino masses in a two Higgs doublet model with a U(1) gauge symmetry, [arXiv:1811.05488](https://arxiv.org/abs/1811.05488).
- [16] R. Barbieri, L. J. Hall, and V. S. Rychkov, Improved naturalness with a heavy Higgs: An alternative road to LHC physics, *Phys. Rev. D* **74**, 015007 (2006).
- [17] S. L. Glashow and S. Weinberg, Natural conservation laws for neutral currents, *Phys. Rev. D* **15**, 1958 (1977).
- [18] F. Kling, H. Li, A. Pyarelal, H. Song, and S. Su, Exotic Higgs decays in type II 2HDMs at the LHC and future 100 TeV hadron colliders, [arXiv:1812.01633](https://arxiv.org/abs/1812.01633).
- [19] A. Adhikary, S. Banerjee, R. K. Barman, and B. Bhattacharjee, Resonant heavy Higgs searches at the HL-LHC, [arXiv:1812.05640](https://arxiv.org/abs/1812.05640).
- [20] S. Dawson *et al.* (Working Group Report: Higgs Boson), in *Proceedings, 2013 Community Summer Study on the Future of U.S. Particle Physics: Snowmass on the Mississippi (CSS2013): Minneapolis, MN, USA, 2013* (2013).
- [21] K. Fujii *et al.*, Physics case for the 250 GeV stage of the international linear collider, [arXiv:1710.07621](https://arxiv.org/abs/1710.07621).
- [22] S. Kanemura, K. Tsumura, K. Yagyu, and H. Yokoya, Fingerprinting nonminimal Higgs sectors, *Phys. Rev. D* **90**, 075001 (2014).
- [23] A. Arhrib, M. Capdequi Peyranere, W. Hollik, and S. Penaranda, Higgs decays in the two Higgs doublet model: Large quantum effects in the decoupling regime, *Phys. Lett. B* **579**, 361 (2004).
- [24] S. Kanemura, M. Kikuchi, and K. Yagyu, Radiative corrections to the Yukawa coupling constants in two Higgs doublet models, *Phys. Lett. B* **731**, 27 (2014).
- [25] S. Kanemura, M. Kikuchi, K. Sakurai, and K. Yagyu, Gauge invariant one-loop corrections to Higgs boson couplings in non-minimal Higgs models, *Phys. Rev. D* **96**, 035014 (2017).
- [26] S. Kanemura, M. Kikuchi, and K. Yagyu, Fingerprinting the extended Higgs sector using one-loop corrected Higgs boson couplings and future precision measurements, *Nucl. Phys. B* **896**, 80 (2015).
- [27] S. Kanemura, M. Kikuchi, K. Mawatari, K. Sakurai, and K. Yagyu, Loop effects on the Higgs decay widths in extended Higgs models, *Phys. Lett. B* **783**, 140 (2018).
- [28] E. Asakawa, D. Harada, S. Kanemura, Y. Okada, and K. Tsumura, Higgs boson pair production in new physics models at hadron, lepton, and photon colliders, *Phys. Rev. D* **82**, 115002 (2010).
- [29] R. Grober, M. Muhlleitner, and M. Spira, Higgs pair production at NLO QCD for CP -violating Higgs sectors, *Nucl. Phys. B* **925**, 1 (2017).
- [30] P. Basler, S. Dawson, C. Englert, and M. Muhlleitner, Showcasing HH production: Benchmarks for the (HL-) LHC, *Phys. Rev. D* **99**, 055048 (2019).
- [31] N. Sonmez, Measuring the triple Higgs self-couplings in two Higgs doublet model, *J. High Energy Phys.* **10** (2018) 083.
- [32] A. Arhrib, R. Benbrik, C.-H. Chen, R. Guedes, and R. Santos, Double neutral Higgs production in the Two-Higgs doublet model at the LHC, *J. High Energy Phys.* **08** (2009) 035.
- [33] The ATLAS Collaboration, Combination of searches for Higgs boson pairs in pp collisions at 13 TeV with the ATLAS experiment, Report No. ATLAS-CONF-2018-043.
- [34] A. M. Sirunyan *et al.* (CMS Collaboration), Combination of Searches for Higgs Boson Pair Production in Proton-Proton Collisions at $\sqrt{s} = 13$ TeV, *Phys. Rev. Lett.* **122**, 121803 (2019).
- [35] S. Di Vita, G. Durieux, C. Grojean, J. Gu, Z. Liu, G. Panico, M. Riembau, and T. Vantalon, A global view on the Higgs self-coupling at lepton colliders, *J. High Energy Phys.* **02** (2018) 178.
- [36] S. Davidson and H. E. Haber, Basis-independent methods for the two-Higgs-doublet model, *Phys. Rev. D* **72**, 035004 (2005); Erratum, *Phys. Rev. D* **72**, 099902(E) (2005).
- [37] H. Georgi and D. V. Nanopoulos, Suppression of flavor changing effects from neutral spinless meson exchange in gauge theories, *Phys. Lett.* **82B**, 95 (1979).

- [38] M. Aoki, S. Kanemura, K. Tsumura, and K. Yagyu, Models of Yukawa interaction in the two Higgs doublet model, and their collider phenomenology, *Phys. Rev. D* **80**, 015017 (2009).
- [39] V.D. Barger, J.L. Hewett, and R.J.N. Phillips, New constraints on the charged Higgs sector in two Higgs doublet models, *Phys. Rev. D* **41**, 3421 (1990).
- [40] Y. Grossman, Phenomenology of models with more than two Higgs doublets, *Nucl. Phys. B* **426**, 355 (1994).
- [41] S. Blasi, S. De Curtis, and K. Yagyu, Extracting the mass scale of a second Higgs boson from a deviation in $h(125)$ couplings, *J. High Energy Phys.* **09** (2017) 091.
- [42] M. Misiak and M. Steinhauser, Weak radiative decays of the B meson and bounds on $M_{H^\pm}$ in the Two-Higgs-Doublet Model, *Eur. Phys. J. C* **77**, 201 (2017).
- [43] M. E. Peskin and T. Takeuchi, A New Constraint on a Strongly Interacting Higgs Sector, *Phys. Rev. Lett.* **65**, 964 (1990).
- [44] M. E. Peskin and T. Takeuchi, Estimation of oblique electroweak corrections, *Phys. Rev. D* **46**, 381 (1992).
- [45] N. G. Deshpande and E. Ma, Pattern of symmetry breaking with two Higgs doublets, *Phys. Rev. D* **18**, 2574 (1978).
- [46] M. Sher, Electroweak Higgs potentials and vacuum stability, *Phys. Rep.* **179**, 273 (1989).
- [47] S. Nie and M. Sher, Vacuum stability bounds in the two Higgs doublet model, *Phys. Lett. B* **449**, 89 (1999).
- [48] S. Kanemura, T. Kasai, and Y. Okada, Mass bounds of the lightest CP even Higgs boson in the two Higgs doublet model, *Phys. Lett. B* **471**, 182 (1999).
- [49] S. Kanemura, T. Kubota, and E. Takasugi, Lee-Quigg-Thacker bounds for Higgs boson masses in a two doublet model, *Phys. Lett. B* **313**, 155 (1993).
- [50] A. G. Akeroyd, A. Arhrib, and E.-M. Naimi, Note on tree level unitarity in the general two Higgs doublet model, *Phys. Lett. B* **490**, 119 (2000).
- [51] I. F. Ginzburg and I. P. Ivanov, Tree level unitarity constraints in the 2HDM with CP violation, [arXiv:hep-ph/0312374](https://arxiv.org/abs/hep-ph/0312374).
- [52] S. Kanemura and K. Yagyu, Unitarity bound in the most general two Higgs doublet model, *Phys. Lett. B* **751**, 289 (2015).
- [53] M. Tanabashi *et al.* (Particle Data Group), Review of particle physics, *Phys. Rev. D* **98**, 030001 (2018).
- [54] P. Bechtle, O. Brein, S. Heinemeyer, O. Stål, T. Stefaniak, G. Weiglein, and K. E. Williams, HiggsBounds – 4: Improved tests of extended Higgs sectors against exclusion bounds from LEP, the tevatron and the LHC, *Eur. Phys. J. C* **74**, 2693 (2014).
- [55] F. Yuasa *et al.*, Automatic computation of cross-sections in HEP: Status of GRACE system, *Prog. Theor. Phys. Suppl.* **138**, 18 (2000).
- [56] J. Fujimoto *et al.*, GRACE/SUSY automatic generation of tree amplitudes in the minimal supersymmetric standard model, *Comput. Phys. Commun.* **153**, 106 (2003).
- [57] The Minami-Tateya Collaboration, GRACE version 2.2.1, <http://minami-home.kek.jp/>.
- [58] I. M. Lewis and M. Sullivan, Benchmarks for double Higgs production in the singlet extended standard model at the LHC, *Phys. Rev. D* **96**, 035037 (2017).
- [59] M. Cepeda *et al.* (Physics of the HL-LHC Working Group), Higgs physics at the HL-LHC and HE-LHC, [arXiv:1902.00134](https://arxiv.org/abs/1902.00134).
- [60] S. Kanemura, M. Kikuchi, K. Sakurai, and K. Yagyu, H-COUP: A program for one-loop corrected Higgs boson couplings in non-minimal Higgs sectors, *Comput. Phys. Commun.* **233**, 134 (2018).
- [61] G. Belanger, F. Boudjema, J. Fujimoto, T. Ishikawa, T. Kaneko, K. Kato, and Y. Shimizu, Automatic calculations in high energy physics and Grace at one-loop, *Phys. Rep.* **430**, 117 (2006).
- [62] J. Fujimoto, T. Ishikawa, Y. Kurihara, M. Jimbo, T. Kon, and M. Kuroda, Two-body and three-body decays of charginos in one-loop order in the MSSM, *Phys. Rev. D* **75**, 113002 (2007).