Multiquark matrix elements in the proton and three gluon exchange for exclusive η_c production in photon-proton diffractive scattering

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Exclusive production of a η_c pseudoscalar meson in $\gamma^{(*)} + p \rightarrow \eta_c + p$ scattering at high energies involves a C -odd exchange in the t -channel. We formulate the description of this process within the high-energy framework of eikonal dipole scattering. We obtain expressions for the light-cone wave function of the η_c required in this framework as well as for the C -odd amplitudes due to exchange of a single photon, of a photon plus two gluons, and of three gluons. We relate these amplitudes to correlators of the plus component of the quark current in the light-cone wave function of the proton. For high transverse momenta, these correlators correspond to (leading twist) generalized parton distributions given by diagrams where all exchanged gauge bosons attach to a single quark in the proton. Diagrams involving multiquark matrix elements potentially sensitive to correlations screen infrared singularities. Moreover, they are numerically important for configurations where the exchanged bosons nearly share the total momentum transfer. Using two simple models for the three-quark Fock state of the proton at $x \simeq 0.1$, we find that single photon exchange dominates for $|t| < 1.5 \text{ GeV}^2$. Here, the quark generalized parton distribution could be measured cleanly in $\gamma^{(*)} + p \rightarrow \eta_c + p$ via single photon exchange. For higher momentum transfer, three-gluon (“Odderon”) exchange is dominant.

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I. INTRODUCTION

The existence of a color singlet three-gluon exchange with negative C -parity in QCD at high energies was established long ago [1]. Such an exchange could provide some understanding of the difference of particle and antiparticle cross sections and of the violation of the Pomanchuk theorem in high-energy scattering. An “Odderon” exchange was proposed nearly 50 years ago within the framework of Regge theory to explain the different cross sections in C -conjugate channels [2]. For a review of the theory and experimental searches for the Odderon up until the year 2003, we refer to Ref. [3].

The TOTEM Collaboration at the CERN LHC has recently measured the differential cross section for pp elastic scattering at $\sqrt{s} = 2.76 \text{ GeV}$ [4]. They observe a significant difference from the data by the D0 Collaboration for $p\bar{p}$ scattering at $\sqrt{s} = 1.96 \text{ GeV}$ [5]. Assuming that the difference in energy is negligible, they conclude that these results provide evidence for a color singlet three-gluon exchange. Even though these measurements are very exciting, the data do not quite correspond to a kinematic regime where perturbative QCD may be reliable.

Exclusive production of pseudoscalar η_c mesons in (virtual) photon-proton scattering has been highlighted [6–8] as the cleanest channel for discovery of C -odd three-gluon (Odderon) exchange.¹ Here, the large mass of the c quark ensures that (at high energy) the process

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¹Also, see Ref. [9] for a calculation of $\gamma p \rightarrow \eta_c p$ using Regge theory and effective Odderon-proton and Odderon- η_c vertices. Moreover, one could search for Odderon exchange also in exclusive $\pi^+ \pi^-$ pion pair electroproduction [10] or through exclusive vector meson production in proton-proton scattering [11].

corresponds to scattering of a small dipole of transverse extent much less than the QCD color neutralization scale, from the proton. The focus in these papers was on η_c production at rather high energies and small parton momentum fractions x , at Hadron-Electron Ring Accelerator (HERA). However, the searches at HERA did not observe exclusive η_c production. The cross section for this process is small; in fact, our estimates below are substantially lower yet than old predictions from the literature [7,8]. Thus, such searches for $\mathcal{C}$ -odd three-gluon exchange at a future high-luminosity Electron Ion Collider would be more promising.² A search for η_c at Jefferson Laboratory (JLab) may also be possible, although our high-energy scattering approximations would not provide accurate predictions for near threshold energies. JLab capabilities for measuring J/Ψ production have been outlined in Ref. [16].

Exclusive measurements in photon-proton scattering offer the opportunity to extract fundamental nonperturbative QCD physics contained in the light-cone wave function of the proton [17,18] and its generalized parton distributions (GPDs) [19–25]. Specifically, these processes in fact involve correlators of multiple “+” currents evaluated as matrix elements between multiparton states [26]. Here, we illustrate explicitly how these correlators probe multiparton correlations in the proton. We recover the description in terms of a leading twist GPD when the transverse momenta of the exchanged gluons or photon are large and generic.

We consider the following kinematic window in this paper. The virtuality Q of the photon is taken to be less than the mass m_c of the charm quark, where the cross sections for J/Ψ or η_c production depend only weakly on Q . For higher photon virtuality, the η_c cross section decreases further with increasing Q . We then consider collision energies $W \sim 7\text{--}10$ GeV such that the process probes quark momentum fractions in the proton of about $x \simeq 0.1$; cf. Eq. (52) below. At such x , it should be a reasonable first approximation to describe the $\gamma^{(*)} \rightarrow J/\Psi, \eta_c$ diffraction as eikonal dipole scattering. Indeed, the longitudinal momentum transfer is much smaller than the transverse momentum transfer considered here, cf. Eq. (53) below. At the same time, the proton state should still be dominated by its valence quark Fock state. In the eikonal dipole scattering picture, the dominant dipole size in η_c production is about $r \sim 1$ GeV⁻¹ (slightly less for J/Ψ production). At the same time, we shall find that momentum transfers $|t| \simeq 1.5$ GeV² or greater will be required to detect $\mathcal{C}$ -odd three-gluon exchange in η_c production. For such r and t , an interpretation in terms of gluon exchanges (between the quarks of the proton and the $c\bar{c}$ dipole) appears reasonable.

²On the other hand, successful fits of exclusive J/Ψ production at HERA energies (e.g., Ref. [12]) can be interpreted to provide evidence for two-gluon Pomeron exchange [13,14] supplemented by QCD high-energy evolution [15].

II. SETUP

The light-cone state of an unpolarized on-shell proton with four-momentum $P^\mu = (P^+, P^-, \vec{P}_\perp)$ is written as [17,18]

$$|P\rangle = \frac{1}{\sqrt{6}} \int \frac{dx_1 dx_2 dx_3}{\sqrt{x_1 x_2 x_3}} \delta(1 - x_1 - x_2 - x_3) \times \int \frac{d^2 k_1 d^2 k_2 d^2 k_3}{(16\pi^3)^3} 16\pi^3 \delta(\vec{k}_1 + \vec{k}_2 + \vec{k}_3) \times \psi_3(x_1, \vec{k}_1; x_2, \vec{k}_2; x_3, \vec{k}_3) \times \sum_{i_1, i_2, i_3} \epsilon_{i_1 i_2 i_3} |p_1, i_1, f_1; p_2, i_2, f_2; p_3, i_3, f_3\rangle. \quad (1)$$

The n -parton Fock space amplitudes ψ_n are universal and process independent. They encode the nonperturbative structure of hadrons. Here, we have restricted ourselves to the valence quark Fock state, assuming that the process probes parton momentum fractions of order $x \sim 0.1$ or greater. The three on-shell quark momenta are specified by their light-cone momenta $p_i^+ = x_i P^+$ and their transverse momenta $\vec{p}_i = x_i \vec{P}_\perp + \vec{k}_i$. Colors and flavors of the quarks are denoted by $i_{1,2,3}$ and $f_{1,2,3}$, respectively. In Eq. (1), we have assumed that the wave function ψ_3 is flavor blind, and we omit helicity quantum numbers as they play no role in our analysis. ψ_3 is symmetric under exchange of any two of the quarks and is normalized according to

$$\int dx_1 dx_2 dx_3 \delta(1 - x_1 - x_2 - x_3) \times \int \frac{d^2 k_1 d^2 k_2 d^2 k_3}{(16\pi^3)^3} (16\pi^3) \delta(\vec{k}_1 + \vec{k}_2 + \vec{k}_3) |\psi_3|^2 = 1. \quad (2)$$

This corresponds to the proton state normalization

$$\langle K | P \rangle = 16\pi^3 P^+ \delta(P^+ - K^+) \delta(\vec{P}_\perp - \vec{K}_\perp). \quad (3)$$

Below, we neglect plus momentum transfer so that $\xi = (K^+ - P^+)/P^+ \rightarrow 0$. This approximation is valid at high energies and $|t| \gg |t_{\min}|$. Accordingly, the light-cone momentum of the produced meson is close to that of the incoming photon.

For numerical estimates, we shall employ the “harmonic oscillator” and “power law” model wave functions of Brodsky and Schlumpf [27],

$$\psi_{\text{H.O.}}(x_1, \vec{k}_1; x_2, \vec{k}_2; x_3, \vec{k}_3) = N_{\text{H.O.}} \exp(-\mathcal{M}^2/2\beta^2), \\ \psi_{\text{Power}}(x_1, \vec{k}_1; x_2, \vec{k}_2; x_3, \vec{k}_3) = N_{\text{Power}} (1 + \mathcal{M}^2/\beta^2)^{-p}. \quad (4)$$

The invariant mass $\mathcal{M}$ of the configuration is given by

$$\mathcal{M}^2 = \sum_{i=1}^3 \frac{\vec{k}_{\perp i}^2 + m^2}{x_i}. \quad (5)$$

β determines the color neutralization scale and the typical transverse momentum of quarks in the proton. The parameters β and m^2 were obtained in Ref. [28] by fitting to electroweak parameters of the baryon octet: $m = 0.26$ GeV, $\beta = 0.55$ GeV for $\psi_{\text{H.O.}}$ and $m = 0.263$ GeV, $\beta = 0.607$ GeV, $p = 3.5$ for ψ_{Power} . The normalization constants $N_{\text{H.O.}}$ and N_{Power} are obtained from the normalization condition (2). Other models and parameter sets can be found in Refs. [29–33].

Following Ref. [26], we introduce the charge density operators corresponding to the light-cone plus component of the quark currents

$$\rho(x_k, \vec{k}) = \sum_{f,i} \int \frac{dx_q}{\sqrt{x_q(x_q + x_k)}} \int \frac{d^2q}{16\pi^3} b_{q,i,f}^\dagger b_{k+q,i,f} \\ \xrightarrow{x_k \ll 1} \sum_{f,i} \int \frac{dx_q}{x_q} \int \frac{d^2q}{16\pi^3} b_{x_q, \vec{q}, i, f}^\dagger b_{x_q, \vec{k} + \vec{q}, i, f} \quad (6)$$

$$\rho^a(x_k, \vec{k}) = \sum_{f,i,j} \int \frac{dx_q}{\sqrt{x_q(x_q + x_k)}} \int \frac{d^2q}{16\pi^3} b_{q,i,f}^\dagger b_{k+q,j,f}(t^a)_{ij} \\ \xrightarrow{x_k \ll 1} \sum_{f,i,j} \int \frac{dx_q}{x_q} \int \frac{d^2q}{16\pi^3} b_{x_q, \vec{q}, i, f}^\dagger b_{x_q, \vec{k} + \vec{q}, j, f}(t^a)_{ij}. \quad (7)$$

These equations define the densities of electric and color charge, respectively; factors of e and g will be attached in Eq. (8) below. $b_{q,i,f}^\dagger$ and $b_{q,i,f}$ denote creation and annihilation operators for quarks with plus momentum $q^+ = x_q P^+$, transverse momentum $\vec{q}$, color i , and flavor f .

In what follows, we shall neglect longitudinal momentum transfer to the quarks and use the kinematic approximation where $x_k \sim 0.1 \ll 1$. This allows us to simplify the color charge operators as indicated above. Moreover, we will assume that the scattering of a energetic $c\bar{c}$ dipole from the valence charges in the proton is eikonal, to first approximation; see Eq. (19) below. Kinematic finite- x corrections are suppressed by powers of the light-cone momentum P^+ . Of course, for quantitative comparisons to future experiments, it will be important to quantify these corrections.

The charge densities are the sources for the static electromagnetic and color fields in covariant gauge,

$$\int dx^- A^+(x^-, \vec{k}) = \frac{e}{k^2} \rho(x_k = 0, \vec{k}), \\ \int dx^- A^a(x^-, \vec{k}) = \frac{g}{k^2} \rho^a(x_k = 0, \vec{k}). \quad (8)$$

The physical picture of representing the quarks with large light-cone momenta as static (x^+ independent) color charge densities sourcing soft gluon fields was introduced

by McLerran and Venugopalan [34]. In their effective theory, however, $\rho^a(x_k = 0, \vec{k})$ corresponds to a *classical* color charge vector describing a large ensemble of quarks in a high-dimensional representation of color- $SU(3)$. Here, instead, the operator $\rho^a(x_k = 0, \vec{k})$ acts on single quarks, and color charge correlators will be evaluated over the light-cone wave function of the proton.

III. CORRELATORS OF CHARGE DENSITY OPERATORS IN THE PROTON

In this section, we provide expressions for correlators of various (electric and color) charge density operators in the proton. Some of these have been considered long before, especially for forward $K_T \rightarrow 0$ scattering (see, e.g., Ref. [35]). Here, we are interested in nonforward matrix elements for single-photon, two-gluon, photon-plus-two-gluon, and three-gluon exchanges. In particular, we follow the approach of Ref. [26] to relate these matrix elements explicitly to the light-cone wave function of the proton and to analyze their GPD limits.

Consider first the expectation value of the electric charge density operator in the proton, in the kinematic $x \ll 1$ limit described above. A straightforward calculation yields³

$$\langle \rho(\vec{q}) \rangle_{\vec{K}_T} = \sum_f e_f \int dx_1 dx_2 dx_3 \delta(1 - x_1 - x_2 - x_3) \\ \times \int \frac{d^2p_1 d^2p_2 d^2p_3}{(16\pi^3)^2} \delta(\vec{p}_1 + \vec{p}_2 + \vec{p}_3) \\ \times \psi_3^*(\vec{p}_1 + (1 - x_1)\vec{K}_\perp, \vec{p}_2 - x_2\vec{K}_\perp, \vec{p}_3 - x_3\vec{K}_\perp) \\ \times \psi_3(\vec{p}_1, \vec{p}_2, \vec{p}_3) \\ \equiv f(K_T) \sum_f e_f. \quad (9)$$

Here, e_f denotes the fractional electric charge of quark $f = (u, u, d)$, so that $\sum_f e_f = 1$. For a lighter notation, we omit the arguments x_1 , x_2 , and x_3 of the wave functions here and in similar expressions below.

The function $f(K_T)$ is a leading twist GPD of quarks in the proton at $x \ll 1$, as it corresponds to the nonforward matrix element of the plus component of the quark current between single-quark states. The photon probe can only attach to one quark at a time, and so this matrix element does not probe correlations among the quarks. Also, for a single probe, its transverse momentum is equal to minus the recoil momentum of the proton, $\vec{q} = -\vec{K}_T$. For $K_T \rightarrow 0$, the wave function normalization in Eq. (2) implies $f(0) = 1$.

³ $\langle \dots \rangle_{\vec{K}_T}$ corresponds to $\langle K | \dots | P \rangle$ stripped of the δ -functions expressing conservation of transverse and plus momenta such as $\langle K | \rho(\vec{q}) | P \rangle = 16\pi^3 P^+ \delta(P^+ - K^+) \delta(\vec{K}_T + \vec{q}) \langle \rho(\vec{q}) \rangle_{\vec{K}_T}$, if we set $\vec{P}_T = 0$ for the incoming proton.

Next, we consider the correlator of two color charge density operators which enters the amplitude for $\mathcal{C}$ -even two-gluon exchange. It is given by [26]

$$\begin{aligned} \langle \rho^a(\vec{q}) \rho^b(-\vec{q} - \vec{K}_T) \rangle_{\vec{K}_\perp} &= \frac{1}{2} \delta^{ab} \int dx_1 dx_2 dx_3 \delta(1 - x_1 - x_2 - x_3) \int \frac{d^2 p_1 d^2 p_2 d^2 p_3}{(16\pi^3)^2} \delta(\vec{p}_1 + \vec{p}_2 + \vec{p}_3) \\ &\times [\psi_3^*(\vec{p}_1 + (1 - x_1)\vec{K}_T, \vec{p}_2 - x_2\vec{K}_T, \vec{p}_3 - x_3\vec{K}_T) - \psi_3^*(\vec{p}_1 + \vec{q} + (1 - x_1)\vec{K}_T, \\ &\vec{p}_2 - \vec{q} - x_2\vec{K}_T, \vec{p}_3 - x_3\vec{K}_T)] \psi_3(\vec{p}_1, \vec{p}_2, \vec{p}_3) \end{aligned} \quad (10)$$

$$\equiv \frac{1}{2} \delta^{ab} G(\vec{q}, -\vec{q} - \vec{K}_T). \quad (11)$$

$G(\vec{q}, -\vec{q} - \vec{K}_T)$ is real because the bound state wave functions ψ_3 are real. It is invariant under a simultaneous rotation of both $\vec{q}$ and $\vec{K}_T$ by the same angle and is also symmetric under exchange of its two arguments, $G(\vec{q}, -\vec{q} - \vec{K}_T) = G(-\vec{q} - \vec{K}_T, \vec{q})$. This color charge correlator vanishes when $\vec{q} \rightarrow 0$ or $\vec{q}_2 = -\vec{q} - \vec{K}_T \rightarrow 0$, which expresses the color charge neutrality of the proton. On the other hand, for large $|\vec{q}|$, $|\vec{q}_2|$, and large $|\vec{q} - \vec{q}_2|$, it is dominated by the first⁴ term in the square brackets of Eq. (10). That is the “one-body” diagram where both gluon probes attach to the same quark. In this GPD limit,

$$G(\vec{q}, \vec{q}_2) \rightarrow f(K_T) \quad (\text{for } \vec{q}^2, \vec{q}_2^2, (\vec{q} - \vec{q}_2)^2/4 \gg \Lambda_{\text{eff}}^2). \quad (12)$$

Here, $\Lambda_{\text{eff}} \sim \beta$ is a soft scale encoded in the light-cone wave function of the proton, Eqs. (4). Note that this

approximation does not necessarily require large K_T . However, at high momentum transfer, the contribution from the diagram where the gluons couple to different quarks is important not only for $\vec{q} \rightarrow 0$ or $\vec{q}_2 \rightarrow 0$ but also when $\vec{q} \sim \vec{q}_2 \sim -\vec{K}_T/2 \gg \Lambda_{\text{eff}}$. Here, the second term in Eq. (10) is actually much greater than the first one due to contributions from configurations where the two active quarks have large light-cone momenta: at $K_T \gg \Lambda_{\text{eff}}$, there is a large mismatch of the arguments of ψ_3^* and ψ_3 in the first term of Eq. (10), while there is strong overlap in the second term when $\vec{q} \sim -\vec{K}_T/2$ and $x_1 \sim x_2 \sim 0.5$. We shall see below that restricting to the single-quark matrix element leads to a too-rapid falloff of $d\sigma^{J/\Psi}/dt$ with $|t|$.

We now move on to the correlator of one electric charge with two color charge densities, which is relevant for $\mathcal{C}$ -odd exchange of a photon and two gluons. It is given by

$$\begin{aligned} \langle \rho(-\vec{q}_2 - \vec{q}_3 - \vec{K}_T) \rho^a(\vec{q}_2) \rho^b(\vec{q}_3) \rangle_{\vec{K}_T} &= \frac{1}{6} \delta^{ab} \sum_f e_f \int dx_1 dx_2 dx_3 \delta(1 - x_1 - x_2 - x_3) \int \frac{d^2 p_1 d^2 p_2 d^2 p_3}{(16\pi^3)^2} \delta(\vec{p}_1 + \vec{p}_2 + \vec{p}_3) \\ &\times [\psi_3^*(\vec{p}_1 + (1 - x_1)\vec{K}_\perp, \vec{p}_2 - x_2\vec{K}_\perp, \vec{p}_3 - x_3\vec{K}_\perp) \\ &+ 2\psi_3^*(\vec{p}_1 + \vec{q}_2 + \vec{q}_3 + (1 - x_1)\vec{K}_\perp, \vec{p}_2 - \vec{q}_2 - \vec{q}_3 - x_2\vec{K}_\perp, \vec{p}_3 - x_3\vec{K}_\perp) \\ &- \psi_3^*(\vec{p}_1 + \vec{q}_2 + (1 - x_1)\vec{K}_\perp, \vec{p}_2 - \vec{q}_2 - x_2\vec{K}_\perp, \vec{p}_3 - x_3\vec{K}_\perp) \\ &- \psi_3^*(\vec{p}_1 + \vec{q}_3 + (1 - x_1)\vec{K}_\perp, \vec{p}_2 - \vec{q}_3 - x_2\vec{K}_\perp, \vec{p}_3 - x_3\vec{K}_\perp) \\ &- \psi_3^*(\vec{p}_1 + \vec{q}_2 + \vec{q}_3 + (1 - x_1)\vec{K}_\perp, \vec{p}_2 - \vec{q}_2 - x_2\vec{K}_\perp, \vec{p}_3 - \vec{q}_3 - x_3\vec{K}_\perp)] \psi_3(\vec{p}_1, \vec{p}_2, \vec{p}_3) \end{aligned} \quad (13)$$

$$\equiv \frac{1}{2} \delta^{ab} \left(\frac{1}{3} \sum_f e_f \right) H(-\vec{q}_2 - \vec{q}_3 - K_T, \vec{q}_2, \vec{q}_3). \quad (14)$$

This correlator vanishes when the transverse momentum of either one of the attaching gluons vanishes, $\vec{q}_2 = 0$ or $\vec{q}_3 = 0$. When the transverse momentum of the photon vanishes, $\vec{q}_1 \equiv -\vec{q}_2 - \vec{q}_3 - \vec{K}_T = 0$, then it reduces to the correlator of two color charge operators from Eq. (10) times $\sum_f e_f = 1$.

⁴For $\psi_3^* = \psi_{\text{Power}}^*$ and large momentum transfer K_T , it dominates by a power of q^2/K_T^2 when $q^2 \gg K_T^2$.

For high transverse photon and gluon momenta, the first term in Eq. (13) dominates, and

$$H(\vec{q}_1, \vec{q}_2, \vec{q}_3) \rightarrow f(K_T) \left(\text{for } \vec{q}_1^2, \vec{q}_2^2, \vec{q}_3^2, \left(\vec{q}_i + \frac{1}{3} \vec{K}_T \right)^2 \gg \Lambda_{\text{eff}}^2 \right). \quad (15)$$

This is the leading twist GPD limit where both gluons and the photon attach to the same quark in the proton. Note, however, that this approximation does not apply when the exchanged bosons share the total momentum transfer $\vec{K}_T$ in approximately equal proportions.

Finally, we recall the correlator of three color charge densities obtained in Ref. [26],

$$\begin{aligned} \langle \rho^a(\vec{q}_1) \rho^b(\vec{q}_2) \rho^c(-\vec{q}_1 - \vec{q}_2 - \vec{K}_T) \rangle_{\vec{K}_\perp} &= \frac{1}{4} d^{abc} \int dx_1 dx_2 dx_3 \delta(1 - x_1 - x_2 - x_3) \int \frac{d^2 p_1 d^2 p_2 d^2 p_3}{(16\pi^3)^2} \delta(\vec{p}_1 + \vec{p}_2 + \vec{p}_3) \\ &\times [\psi_3^*(\vec{p}_1 + (1 - x_1)\vec{K}_\perp, \vec{p}_2 - x_2\vec{K}_\perp, \vec{p}_3 - x_3\vec{K}_\perp) \\ &- \psi_3^*(\vec{p}_1 - \vec{q}_1 - x_1\vec{K}_\perp, \vec{p}_2 + \vec{q}_1 + (1 - x_2)\vec{K}_\perp, \vec{p}_3 - x_3\vec{K}_\perp) \\ &- \psi_3^*(\vec{p}_1 + \vec{q}_2 + (1 - x_1)\vec{K}_\perp, \vec{p}_2 - \vec{q}_2 - x_2\vec{K}_\perp, \vec{p}_3 - x_3\vec{K}_\perp) \\ &- \psi_3^*(\vec{p}_1 - \vec{q}_1 - \vec{q}_2 - x_1\vec{K}_\perp, \vec{p}_2 + \vec{q}_1 + \vec{q}_2 + (1 - x_2)\vec{K}_\perp, \vec{p}_3 - x_3\vec{K}_\perp) \\ &+ 2\psi_3^*(\vec{p}_1 - \vec{q}_1 - x_1\vec{K}_\perp, \vec{p}_2 + \vec{q}_1 + \vec{q}_2 + (1 - x_2)\vec{K}_\perp, \vec{p}_3 - \vec{q}_2 - x_3\vec{K}_\perp)] \\ &\times \psi_3(\vec{p}_1, \vec{p}_2, \vec{p}_3) \end{aligned} \quad (16)$$

$$\equiv \frac{1}{4} d^{abc} G_O(\vec{q}_1, \vec{q}_2, -\vec{q}_1 - \vec{q}_2 - \vec{K}_T). \quad (17)$$

(We have redefined the normalization of G_O as compared to Ref. [26] by a factor of $N_c/4$ for convenience.) Here, on the rhs, we have written only the C -odd contribution, which is symmetric under the exchange of the color indices a, b , and c since this is the only piece that couples to a dipole. G_O is invariant under a simultaneous rotation of the three transverse momentum vectors $\vec{q}_1, \vec{q}_2$, and $\vec{K}_T$ and under permutations of its arguments. When either one of the three momenta $\vec{q}_1, \vec{q}_2$, or $\vec{q}_3 \equiv -\vec{q}_1 - \vec{q}_2 - \vec{K}_T$ is zero, then G_O vanishes. Once again, one recovers the leading twist GPD corresponding to the matrix element of the plus current between single-quark states in the limit of large transverse momenta of the gluon probes,

$$G_O(\vec{q}_1, \vec{q}_2, \vec{q}_3) \rightarrow f(K_T) \left(\text{for } \vec{q}_1^2, \vec{q}_2^2, \vec{q}_3^2, \left(\vec{q}_i + \frac{1}{3} \vec{K}_T \right)^2 \gg \Lambda_{\text{eff}}^2 \right). \quad (18)$$

This approximation again requires not only large transverse gluon momenta but also that the total momentum transfer is not being (approximately) shared equally.

In closing this section, we note that in our expressions for the color charge correlators $\langle \rho^a(\vec{q}) \rho^b(-\vec{q} - \vec{K}_T) \rangle_{\vec{K}_\perp}$, etc., we have ignored the appropriate gauge links connecting the sources [36–38]. These gauge links account for soft multiple scattering, while we retain only two or three “hard” gluon exchanges with the target. In this paper, we shall be interested mainly in the limit $K_T^2 \gg \Lambda_{\text{eff}}^2$.

IV. DIPOLE SCATTERING AMPLITUDE

The invariant amplitude $\mathcal{T}$ for elastic scattering of the $c\bar{c}$ pair off the fields in the target proton can be expressed as

$$\mathcal{T}(\vec{r}, \vec{b}_\perp; \vec{K}_\perp) = 2N_c \left[1 - \frac{1}{N_c} \text{tr} \left\langle U \left(\vec{b} + \frac{\vec{r}}{2} \right) U^\dagger \left(\vec{b} - \frac{\vec{r}}{2} \right) \right\rangle_{\vec{K}_\perp} \right], \quad (19)$$

$$\mathcal{T}(\vec{r}, \vec{K}_\perp) = \int d^2 b e^{i\vec{b} \cdot \vec{K}_T} \mathcal{T}(\vec{r}, \vec{b}_\perp; \vec{K}_\perp). \quad (20)$$

At $K_T = 0$, Eq. (19) is related to the so-called dipole gluon distribution evaluated in covariant gauge [36]. Here, U (and $U^\dagger$) are lightlike Wilson lines representing the eikonal scattering of the dipole of size $\vec{r}$ at impact parameter $\vec{b}$. Two of the diagrams that contribute to the C -odd part of this amplitude are shown in Fig. 1.

To account for photon exchange to the scattering amplitude (19), we use Wilson lines in the combined color and electromagnetic fields:

$$U^\dagger(\vec{x}_T) = \mathcal{P} e^{i \int dx^- [gA^{+a}(x^-, \vec{x}_T) t^a + e_Q eA^+(x^-, \vec{x}_T)]}. \quad (21)$$

Here, $e_Q = 2/3$ is the fractional charge of the c quark.

In what follows, we expand $\mathcal{T}(\vec{r}, \vec{K}_\perp)$ up to first order in the electromagnetic field and up to third order in the

FIG. 1. Two of the diagrams that contribute to the production of a pseudoscalar η_c meson via $\mathcal{C}$ -odd three-gluon exchange. The diagram on the left involves a matrix element between single-quark states in the proton, while the diagram on the right involves a matrix element in a three-quark state sensitive to multi-quark correlations.

color field. The relation of such a weak field expansion to a resummation of kinematic twists in Wandzura-Wilczek type approximations has been elucidated recently in Ref. [37]. Indeed, we do not expand the scattering amplitude about small dipole size r or small momentum exchange K_T . However, as already indicated at the end of the previous section, we do neglect the resummation of multiple soft scattering. For a proton target and $x \sim 0.1$, the weak field limit should provide a reasonable first approximation.

Expanding to first order in the fields, we obtain the amplitude for single-photon exchange,

$$\mathcal{T}_\gamma(\vec{r}, \vec{K}_\perp) = 16\pi N_c \alpha e_Q \sum_f e_f \frac{f(K_T)}{K_T^2} \sin\left(\frac{\vec{r} \cdot \vec{K}_T}{2}\right), \quad (22)$$

with $f(K_T)$ from Eq. (9). Note that the exchanged photon is off shell as it possesses transverse momentum but no light-cone momentum.

The contribution at second order in the color field gA^{+a} corresponds to $\mathcal{C}$ -even two-gluon (“Pomeron”) exchange [26],

$$\begin{aligned} \mathcal{T}_{gg}(\vec{r}, \vec{K}_\perp) = & -(4\pi\alpha_s)^2 N_c C_F \int_q \frac{1}{(\vec{q} - \frac{1}{2}\vec{K}_T)^2 (\vec{q} + \frac{1}{2}\vec{K}_T)^2} \\ & \times \left(\cos(\vec{r} \cdot \vec{q}) - \cos\left(\frac{\vec{r} \cdot \vec{K}_T}{2}\right) \right) \\ & \times G\left(\vec{q} - \frac{1}{2}\vec{K}_T, -\vec{q} - \frac{1}{2}\vec{K}_T\right). \end{aligned} \quad (23)$$

[We use the shorthand notation $\int_q = \int d^2q/(2\pi)^2$.] $\mathcal{T}_{gg}(\vec{r}, \vec{K}_\perp)$ is even under a sign flip of either $\vec{r}$ or $\vec{K}_T$. For forward scattering of a small dipole,

$$\begin{aligned} \mathcal{T}_{gg}(\vec{r}, K_\perp = 0) \\ \simeq \frac{1}{4} (4\pi\alpha_s)^2 N_c C_F r^2 \log \frac{1}{r\Lambda_{\text{eff}}} \quad (\text{for } r\Lambda_{\text{eff}} \ll 1) \end{aligned} \quad (24)$$

exhibits the well-known “color transparency” effect. The logarithm in the previous expression arises because the transverse momenta of the exchanged gluons are distributed from Λ_{eff} to the hard scale r^{-1} according to dq^2/q^2 .

At second order in gA^{+a} and first order in eA^+ , we have

$$\begin{aligned} \mathcal{T}_{\gamma gg}(\vec{r}, \vec{K}_\perp) = & \frac{1}{2} e_Q 4\pi\alpha (4\pi\alpha_s)^2 (N_c^2 - 1) \left(\frac{1}{3} \sum_f e_f \right) \int_{q_2, q_3} \frac{1}{q_2^2} \frac{1}{q_3^2} \frac{1}{(\vec{q}_2 + \vec{q}_3 + \vec{K}_T)^2} \left[\sin\left(\vec{q}_2 \cdot \vec{r} + \frac{1}{2}\vec{K}_T \cdot \vec{r}\right) \right. \\ & + \sin\left(\vec{q}_3 \cdot \vec{r} + \frac{1}{2}\vec{K}_T \cdot \vec{r}\right) - \sin\left(\frac{1}{2}\vec{K}_T \cdot \vec{r}\right) - \sin\left((\vec{q}_2 + \vec{q}_3) \cdot \vec{r} + \frac{1}{2}\vec{K}_T \cdot \vec{r}\right) \Big] \\ & \times H(-\vec{q}_2 - \vec{q}_3 - \vec{K}_T, \vec{q}_2, \vec{q}_3), \end{aligned} \quad (25)$$

with H as given in Eq. (13). The integrand does not exhibit any infrared divergences at $\vec{q}_2 = 0$ or $\vec{q}_3 = 0$ or $\vec{q}_2 + \vec{q}_3 + \vec{K}_T = 0$. $\mathcal{T}_{\gamma gg}(\vec{r}, \vec{K}_\perp)$ is odd under a sign flip of either $\vec{r}$ or $\vec{K}_T$.

Finally, at third order in gA^{+a} , we have the following scattering amplitude for $\mathcal{C}$ -odd three-gluon exchange⁵ [26]:

$$\mathcal{T}_{ggg}(\vec{r}, \vec{K}_\perp) = \frac{5}{3}(4\pi\alpha_s)^3 \int_{q_1, q_2} \frac{1}{q_1^2} \frac{1}{q_2^2} \frac{1}{q_3^2} G_O(\vec{q}_1, \vec{q}_2, \vec{q}_3) \times \left[\sin\left(\vec{r} \cdot \vec{q}_1 + \frac{1}{2}\vec{r} \cdot \vec{K}_T\right) - \frac{1}{3}\sin\left(\frac{1}{2}\vec{r} \cdot \vec{K}_T\right) \right]. \quad (26)$$

Here, $\vec{q}_3 \equiv -\vec{q}_1 - \vec{q}_2 - \vec{K}_T$. $\mathcal{T}_{ggg}(\vec{r}, \vec{K}_\perp)$ is also odd under a sign flip of either $\vec{r}$ or $\vec{K}_T$. The relation of the Odderon amplitude to the T -odd gluon generalized transverse momentum dependent parton distributions (GTMDs) and GPDs in the gluon dominated regime of very small x has been worked out in Ref. [38].

In the limit of nearly forward scattering of a small dipole, the Odderon behaves differently than the Pomeron. The second term in the equation above, for example, is

$$\sim \sin\left(\frac{1}{2}\vec{r} \cdot \vec{K}_T\right) \int_{q_1} \int_{q_2} \frac{1}{q_1^2} \frac{1}{q_2^2} \frac{1}{q_3^2} G_O(\vec{q}_1, \vec{q}_2, \vec{q}_3). \quad (27)$$

The integral in this expression is independent of the hard scale $1/r$ set by the size of the dipole. For nearly forward scattering, all three exchanged gluons will have transverse momenta of order of the soft color neutralization scale Λ_{eff} because the integrands drop faster than d^2q_i/q_i^2 . On the other hand, if one requires a large momentum transfer K_T^2 , then q_2 in Eq. (26) runs from Λ_{eff} to K_T , while q_1 extends from Λ_{eff} to $\min(K_T, 1/r)$. Hence, for large K_T but small r , with $\vec{r} \cdot \vec{K}_T \sim 1$ or greater, and neglecting the contribution from $\vec{q}_1 \sim \vec{q}_2 \sim \vec{q}_3 \sim -\vec{K}_T/3$, the leading logarithmic contribution to $\mathcal{T}_{ggg}$ is

$$\mathcal{T}_{ggg}(\vec{r}, \vec{K}_\perp) \simeq \frac{10}{9}(4\pi\alpha_s)^3 \sin\left(\frac{1}{2}\vec{r} \cdot \vec{K}_T\right) \times f(K_T) \frac{\log(K_T/\Lambda_{\text{eff}}) \log(1/r\Lambda_{\text{eff}})}{(2\pi)^2 K_T^2}. \quad (28)$$

This expression again involves the same GPD $f(K_T)$ encountered above.

We have indicated in the previous section that the correlators of multiple charge density operators in the proton reduce to the expectation value of a *single* such operator (a leading twist GPD) when the transverse momenta of the attached gauge bosons are large and not close to each other. In that regime, the correlators are dominated by the diagram where all gauge bosons couple to the same quark. We shall illustrate the importance of the diagrams involving multi-quark matrix elements as

⁵Reference [26] denotes this amplitude $iO(\vec{r}, \vec{K}_\perp)$. Our expression in Eq. (26) includes a factor of $-2N_c$ omitted in Ref. [26] in the step from their Eqs. (74)–(76).

follows.⁶ In Eqs. (10) and (16) for the two- and three-gluon exchange matrix elements,⁷ respectively, we drop all but the first contribution in the square brackets, which is the diagram where all exchanged gluons couple to a single quark. The respective correlators at fixed total momentum transfer $\vec{K}_T$ are then independent of the transverse momenta of the attached gluons. Hence, $G(\vec{q} - \frac{1}{2}\vec{K}_T, -\vec{q} - \frac{1}{2}\vec{K}_T) \rightarrow f(K_T)$ in Eq. (23), and $G_O(\vec{q}_1, \vec{q}_2, \vec{q}_3) \rightarrow f(K_T)$ in Eq. (26). However, now the contributions to the integrals in Eqs. (23) and (26) are no longer cut off at soft transverse momentum. To restore color screening at low transverse momentum, we therefore introduce by hand cutoffs of the form

$$\left(1 - e^{-\frac{(\vec{q} - \vec{K}_T/2)^2}{2\Lambda^2}}\right) \left(1 - e^{-\frac{(\vec{q} + \vec{K}_T/2)^2}{2\Lambda^2}}\right), \quad (29)$$

$$\left(1 - e^{-\frac{q_1^2}{2\Lambda^2}}\right) \left(1 - e^{-\frac{q_2^2}{2\Lambda^2}}\right) \left(1 - e^{-\frac{q_3^2}{2\Lambda^2}}\right)$$

in Eqs. (23) and (26). In particular, for three-gluon exchange, such *ad hoc* cutoffs are unavoidable if one restricts to the one-body diagram, as $\mathcal{T}_{ggg}$ is otherwise infrared divergent.

We shall use $\Lambda = 0.1$ GeV for numerical estimates. We have not attempted to “fine tune” the cutoff Λ to the color neutralization scale encoded in the light-cone wave function of the proton. However, we have checked that imposing such a cutoff on the complete set of diagrams does not affect the cross sections much. Below, we show the numerical accuracy of these “single-quark + cutoff” approximations for the J/Ψ and η_c cross sections.

V. EXCLUSIVE J/Ψ AND η_c PRODUCTION IN $\gamma^{(*)} - p$ SCATTERING

A. Light-cone wave function of the η_c

Before discussing the amplitude for exclusive meson production in $\gamma^* p \rightarrow Mp$, we need to derive the light-cone wave function of the η_c required in the dipole scattering approach.

We take the spinor part as $i\gamma^5$ sandwiched between $\bar{u}_h(z, \vec{k}_T)$ and $v_{\bar{h}}(1-z, -\vec{k}_T)$ spinors, in the transverse rest frame of the η_c . This is multiplied by a phenomenological scalar wave function $\phi^P(k_T, z)$ like for the J/Ψ meson, cf. Appendix:

$$\Psi_{h,\bar{h}}^{\eta_c}(\vec{k}_T, z) = -i \frac{\bar{u}_h(z, \vec{k}_T)}{\sqrt{z}} \gamma^5 \frac{v_{\bar{h}}(1-z, -\vec{k}_T)}{\sqrt{1-z}} \tilde{\phi}^P(k_T, z). \quad (30)$$

⁶As already mentioned, here, we restrict to the valence quark component of the proton wave function. For a discussion of correlations among quarks in the proton at $x \ll 0.1$, see, e.g., Ref. [39].

⁷We do not discuss $\mathcal{T}_{ggg}$ in this context as its contribution turns out to be very small.

Here, z , $\vec{k}_T$, and h denote the light-cone momentum fraction, the transverse momentum, and the helicity of the c quark; $1-z$, $-\vec{k}_T$, and $\bar{h}$ denote those of the $\bar{c}$ antiquark.

The spinor matrix element can be computed using the expressions summarized in Refs. [17,40]:

$$\begin{aligned} & -\bar{u}_h(z, \vec{k}_T) \gamma^5 v_{\bar{h}}(1-z, -\vec{k}_T) \\ &= \frac{1}{\sqrt{z(1-z)}} [-k_T^R \delta_{h-} \delta_{\bar{h}-} - k_T^L \delta_{h+} \delta_{\bar{h}+} \\ &+ m_c (\delta_{h+} \delta_{\bar{h}-} - \delta_{h-} \delta_{\bar{h}+})]. \end{aligned} \quad (31)$$

m_c denotes the mass of the charm quark, which we take as 1.4 GeV [12]. Here, the transverse momenta are written in complex representation as $k_T^{R,L} = k^1 \pm i k^2$. Written as operators in coordinate representation, $k_T^{R,L} \rightarrow \pm i e^{\pm i \phi_r} \partial_r$. Then,

$$\begin{aligned} \Psi_{h,\bar{h}}^{\eta_c}(\vec{r}, z) &= \frac{i}{z(1-z)} [i(\delta_{h+,\bar{h}+} e^{-i\phi_r} - \delta_{h-,\bar{h}-} e^{i\phi_r}) \partial_r \\ &- m_c (\delta_{h-,\bar{h}+} - \delta_{h+,\bar{h}-})] \phi^P(r, z). \end{aligned} \quad (32)$$

The scalar part $\phi^P(r, z)$ of the pseudoscalar meson wave function has to be modeled. In order to fix $\phi^P(r, z)$, we adopt a simple approach and use the same “boosted Gaussian” functional form of the scalar function as for the vector meson [12,41–43]:

$$\begin{aligned} \phi^P(r, z) &= \mathcal{N}_P z(1-z) \\ &\times \exp \left(-\frac{m_c^2 \mathcal{R}_P^2}{8z(1-z)} - \frac{2z(1-z)r^2}{\mathcal{R}_P^2} + \frac{m_c^2 \mathcal{R}_P^2}{2} \right). \end{aligned} \quad (33)$$

To fix the parameters $\mathcal{R}_P^2$ and $\mathcal{N}_P$, we impose the normalization condition

$$1 = N_c \sum_{h,\bar{h}} \int d^2 r \int_0^1 \frac{dz}{4\pi} |\Psi_{h\bar{h}}^{\eta_c}(\vec{r}, z, Q^2)|^2, \quad (34)$$

which, after substitution of (32), becomes

$$1 = \frac{N_c}{2\pi} \sum_{h,\bar{h}} \int d^2 r \int_0^1 \frac{dz}{z^2(1-z)^2} [(\partial_r \phi^P)^2 + (m_c \phi^P)^2]. \quad (35)$$

We have assumed that $\phi^P(r, z)$ is real.

A second constraint on the wave function arises from the requirement that it matches the coupling to the axial-vector current,

$$\langle 0 | \bar{c}(0) \gamma^\mu \gamma_5 c(0) | P \rangle = i f_P P^\mu. \quad (36)$$

The meson state with momentum P^μ is written as

$$\begin{aligned} |P\rangle &= \frac{\tilde{N}_P}{\sqrt{N_c}} \sum_{h,\bar{h},i} \int \frac{dx d^2 k_T}{16\pi^3 \sqrt{x(1-x)}} \Psi_{h\bar{h}}^{\eta_c}(x, \vec{k}_T) \\ &\times |x, \vec{k}_T - x\vec{P}_T, h, i; 1-x, -\vec{k}_T - (1-x)\vec{P}_T, \bar{h}, i\rangle. \end{aligned} \quad (37)$$

Here, $i = r, g, b$ denotes color, and h and $\bar{h}$ are the helicities of the c quarks. The normalization condition (34) requires $\tilde{N}_P = \sqrt{N_c}$ in order to make sure that

$$\langle K | P \rangle = 16\pi^3 P^+ \delta(P^+ - K^+) \delta(\vec{P}_T - \vec{K}_T). \quad (38)$$

We now introduce quark and antiquark creation and annihilation operators through the bare field expansions [17]

$$\begin{aligned} c_i(x^\mu) &= \int \frac{dx_p d^2 p_T}{16\pi^3 x_p} \\ &\times \sum_h [b_{hi}(p) u_h(p) e^{-ip \cdot r} + d_{hi}^\dagger(p) v_h(p) e^{ip \cdot r}], \end{aligned} \quad (39)$$

$$\begin{aligned} \bar{c}_i(x^\mu) &= \int \frac{dx_p d^2 p_T}{16\pi^3 x_p} \\ &\times \sum_h [b_{hi}^\dagger(p) \bar{u}_h(p) e^{ip \cdot r} + d_{hi}(p) \bar{v}_h(p) e^{-ip \cdot r}]. \end{aligned} \quad (40)$$

These satisfy the anticommutation relations

$$\begin{aligned} \{b_{hi}(p), b_{h'i'}^\dagger(k)\} &= \{d_{hi}(p), d_{h'i'}^\dagger(k)\} \\ &= 16\pi^3 p^+ \delta(p^+ - k^+) \delta(\vec{p}_T - \vec{k}_T) \delta_{ii'} \delta_{hh'}. \end{aligned} \quad (41)$$

A straightforward calculation of the plus component of Eq. (36), using again the spinor matrix elements summarized in Ref. [40], now leads to

$$f_P = \tilde{N}_P \sqrt{N_c} \int \frac{dz}{\pi} \frac{m_c}{z(1-z)} \Phi^P(z, r=0). \quad (42)$$

The leading order decay rate of the η_c to two photons is related to f_P through (see, e.g., Ref. [44])

$$\Gamma_{\eta_c \rightarrow \gamma\gamma} = 4\pi e_c^4 \alpha^2 \frac{f_P^2}{M_{\eta_c}}. \quad (43)$$

Experimentally [45],

$$M_{\eta_c} = 2.984 \text{ GeV}, \quad \Gamma_{\eta_c \rightarrow \gamma\gamma} = 5.04 \times 10^{-6} \text{ GeV}. \quad (44)$$

Equations (43), (44) give $f_P = 0.337 \text{ GeV}$. We employ $m_c = 1.4 \text{ GeV}$ for both the η_c and J/Ψ wave functions. The two constraints (34) and (42) determine the two parameters of the η_c wave function to be $\mathcal{N}_P = 0.547$

and $\mathcal{R}_p = 2.48 \text{ GeV}^{-2}$. As expected, these values are close to those for the J/Ψ meson given in the Appendix.

The products of photon and η_c wave functions are

$$[(\Psi^{\eta_c})^* \Psi_L^{\gamma^*}]_{h=\pm}(\vec{r}, z, Q^2) = i \text{sign}(h) \frac{m_c e_c e Q}{\pi} K_0(\epsilon r) \phi^P(z, r), \quad (45)$$

$$\begin{aligned} & [(\Psi^{\eta_c})^* \Psi_{T,\lambda=\pm}^{\gamma^*}]_{h\bar{h}}(\vec{r}, z, Q^2) \\ &= -\frac{\sqrt{2}e_c e}{2\pi z(1-z)} m_c e^{i\lambda\phi_r} \{K_0(\epsilon r) \partial_r \phi^P \delta_{\lambda h=+} \delta_{\lambda \bar{h}=+} \\ &+ \epsilon K_1(\epsilon r) \phi^P [z \delta_{\lambda h=+} \delta_{\lambda \bar{h}=-} + (1-z) \delta_{\lambda h=-} \delta_{\lambda \bar{h}=+}]\}, \end{aligned} \quad (46)$$

where $\epsilon \equiv \sqrt{z(1-z)Q^2 + m_c^2}$. Summing the amplitude over quark helicities gives

$$[(\Psi^{\eta_c})^* \Psi_L^{\gamma^*}](\vec{r}, z, Q^2) = 0, \quad (47)$$

$$\begin{aligned} & [(\Psi^{\eta_c})^* \Psi_{T,\lambda=\pm}^{\gamma^*}](\vec{r}, z, Q^2) \\ &= -\frac{\sqrt{2}e_c e}{2\pi z(1-z)} m_c e^{i\lambda\phi_r} \{K_0(\epsilon r) \partial_r \phi^P + \epsilon K_1(\epsilon r) \phi^P\}. \end{aligned} \quad (48)$$

The transverse amplitude corresponds to an average over $\lambda = \pm$,

$$\begin{aligned} & [(\Psi^{\eta_c})^* \Psi_T^{\gamma^*}](\vec{r}, z, Q^2) = -\frac{\sqrt{2}e_c e}{2\pi z(1-z)} m_c \cos(\phi_r) \\ & \times \{K_0(\epsilon r) \partial_r \phi^P + \epsilon K_1(\epsilon r) \phi^P\}. \end{aligned} \quad (49)$$

B. Amplitudes and cross sections for exclusive J/Ψ and η_c production

The amplitude for exclusive production of a J/Ψ meson is given by

$$\begin{aligned} & \mathcal{A}_{T,L}^{\gamma^* p \rightarrow J/\Psi p}(Q^2, K_T) \\ &= i \int d^2 r \int_0^1 \frac{dz}{4\pi} (\Psi_{\gamma^*}^* \Psi_{J/\Psi}^*)_{T,L}(\vec{r}, z, Q^2) \\ & \times e^{-i\frac{(1-2z)}{2}\vec{r} \cdot \vec{K}_T} \mathcal{T}_{gg}(\vec{r}, \vec{K}_\perp). \end{aligned} \quad (50)$$

This is independent of the direction of the transverse momentum transfer $\vec{K}_T$ and satisfies

$$\begin{aligned} & [\mathcal{A}_{\lambda}^{\gamma^* p \rightarrow J/\Psi p}(Q^2, \vec{K}_T)]^* = -\mathcal{A}_{\lambda}^{\gamma^* p \rightarrow J/\Psi p}(Q^2, -\vec{K}_T) \\ &= -\mathcal{A}_{-\lambda}^{\gamma^* p \rightarrow J/\Psi p}(Q^2, -\vec{K}_T) \end{aligned} \quad (51)$$

since it does not depend on the sign of λ either. This last expression verifies the analyticity property of the S-matrix, $S = 1 + \mathcal{A}_{\lambda}(\vec{K}_T)$, for elastic scattering.

In Eq. (50), $\Psi_{J/\Psi}$ and Ψ_{γ^*} denote the J/Ψ and virtual photon light-cone wave functions (for longitudinal or transverse polarization); their product is summed over the helicities of the c and $\bar{c}$ quarks. We use the expressions given in Ref. [12] for numerical estimates; see Appendix.

$\mathcal{T}_{gg}(\vec{r}, \vec{K}_\perp)$ is independent of x due to us neglecting QCD evolution. On physical grounds, the resulting amplitude $\mathcal{A}_{T,L}^{\gamma^* p \rightarrow J/\Psi p}$ should provide a first approximation for a collision energy such that $x \sim 0.1$. Greater x are not accessible due to our assumption of eikonal scattering with negligible longitudinal momentum transfer. In fact, kinematic finite x corrections may be substantial even at $x \sim 0.1$ and should be accounted for in the future. From

$$x = \frac{Q^2 + M_{J/\Psi}^2}{2p \cdot q} = \frac{Q^2 + M_{J/\Psi}^2}{2m_p E_{\gamma^*}}, \quad (52)$$

we estimate that $E_{\gamma^*} \simeq 25\text{--}50 \text{ GeV}$ (on a fixed target), or $W^2 \simeq 50\text{--}100 \text{ GeV}^2$. For such energy, and $Q^2 \lesssim 1 \text{ GeV}^2$, the minimal value of $|t|$ due to longitudinal momentum exchange is

$$\begin{aligned} -t_{\min} &= \left[m_p \frac{Q^2 + M_{J/\Psi}^2}{W^2} \right]^2 \\ &= \left[m_p \frac{Q^2 + M_{J/\Psi}^2}{2m_p E_{\gamma^*} - Q^2 + m_p^2} \right]^2 \lesssim 0.05 \text{ GeV}^2. \end{aligned} \quad (53)$$

Since we neglect longitudinal momentum transfer, we restrict to $|t - t_{\min}| \gtrsim 0.1 \text{ GeV}^2$, and our values for t should be understood as corresponding to $t - t_{\min}$.

In the high-energy limit, the differential cross section is given by [12]

$$\frac{d\sigma}{dt} = \frac{1}{16\pi} \sum_{T,L} |\mathcal{A}_{T,L}^{\gamma^* p \rightarrow M p}|^2. \quad (54)$$

On the rhs of this equation, the squared amplitude can be evaluated for an arbitrary direction of $\vec{K}_T$ as it is invariant under rotations.

In Fig. 2, we plot the resulting cross section for J/Ψ production. Fitting by an exponential falloff over the range $-0.5 \text{ GeV}^2 > t - t_{\min} > -1 \text{ GeV}^2$, we obtain a slope of $B \simeq 3 \text{ GeV}^{-2}$, which is close to data at comparable energies [46]. On the other hand, a fit with a (squared) dipole form factor [47], $d\sigma/dt \sim (1 - (t - t_{\min})/m_g^2)^{-4}$, can be performed over the entire range $-0.1 > t - t_{\min} > -1 \text{ GeV}^2$. It results in $m_g^2 \simeq 0.6 \text{ GeV}^2$, which is close to m_ρ^2 but somewhat smaller than $m_g^2 \simeq 1 \text{ GeV}^2$ suggested in Ref. [47].

FIG. 2. The differential cross section for exclusive J/Ψ production. The bands indicate the variation due to the two-proton wave functions used here and also cover the range $Q^2 < 0.5 \text{ GeV}^2$. The flatter curve is two-gluon exchange with up to two quarks in the proton, while the steeper curve corresponds to the $c\bar{c}$ dipole scattering from a single quark in the proton. This figure is for a $\gamma^{(*)} - p$ collision energy of approximately $W \simeq 7\text{--}10 \text{ GeV}$.

Our expressions for the scattering amplitude have been obtained in a fixed coupling approximation. Assuming $\alpha_s = 0.35$, the integral of $d\sigma/dt$ over $|t - t_{\min}| > 0.1 \text{ GeV}^2$ is $\sigma \approx 2.0 \text{ nb}$; it increases by about a factor of 1.7 if we extrapolate the integral all the way down to $t = t_{\min}$. Then, the differential cross section at $t = t_{\min}$ is $d\sigma/dt \approx 20 \text{ nb/GeV}^2$.

The extrapolation to $t = t_{\min}$ of course suffers from some uncertainty due to neglecting longitudinal momentum transfer. Also, there are uncertainties as to the values of α_s and m_Q . Lastly, for more accurate results, one should account for the real part of the amplitude, too (see, e.g., Refs. [12,41,42,48,49]). However, $d\sigma/dt \approx 20 \text{ nb/GeV}^2$ is not very far from previous estimates for the J/Ψ cross section at $W \simeq 7\text{--}10 \text{ GeV}$ [49]. The J/Ψ cross section scales with the coupling as $\sim \alpha_s^4$.

Figure 2 also shows the cross section obtained from only the one-body diagram where both exchanged gluons couple to the same quark in the proton. This is a fair approximation to within a factor of about 2 for $|t - t_{\min}| \lesssim 1 \text{ GeV}^2$. The figure illustrates the effect on the slope of $d\sigma/dt$ of the diagrams where the $c\bar{c}$ dipole scatters from multiple quarks in the proton.⁸ They lead to a harder slope due to the contribution from configurations where the exchanged gluons have similar transverse momenta.

The amplitude for elastic exclusive production of a η_c meson is given by

⁸Charm production in photon scattering off multiple quarks has also been advocated in an unrelated setting for the $x \rightarrow 1$ threshold region [50].

$$\begin{aligned} \mathcal{A}_\lambda^{\gamma^* p \rightarrow \eta_c p}(Q^2, \vec{K}_T) &= i \int d^2 r \int_0^1 \frac{dz}{4\pi} (\Psi_\lambda^{\gamma^*} (\Psi^{\eta_c})^*)(\vec{r}, z, Q^2) e^{-i\frac{(1-2z)}{2} \vec{r} \cdot \vec{K}_T} \\ &\times (\mathcal{T}_\gamma + \mathcal{T}_{\gamma g g} + \mathcal{T}_{g g g})(\vec{r}, \vec{K}_\perp). \end{aligned} \quad (55)$$

Here, $\lambda = \pm$ denotes the polarization of the transverse photon, and the longitudinal photon does not contribute.

The product $\Psi_\lambda^{\gamma^*} (\Psi^{\eta_c})^*$ changes sign under $\vec{r} \rightarrow -\vec{r}$, and therefore

$$\mathcal{A}_\lambda^{\gamma^* p \rightarrow \eta_c p}(Q^2, \vec{K}_T) = -\mathcal{A}_\lambda^{\gamma^* p \rightarrow \eta_c p}(Q^2, -\vec{K}_T). \quad (56)$$

The complex-conjugate amplitude is

$$[\mathcal{A}_\lambda^{\gamma^* p \rightarrow \eta_c p}(Q^2, \vec{K}_T)]^* = \mathcal{A}_{-\lambda}^{\gamma^* p \rightarrow \eta_c p}(Q^2, -\vec{K}_T). \quad (57)$$

This confirms that the S-matrix, $S(\vec{K}_T, \lambda) = 1 + \mathcal{A}(\vec{K}_T, \lambda)$, exhibits the proper analytical properties for elastic scattering. We note that $\sum_\lambda |\mathcal{A}_\lambda^{\gamma^* p \rightarrow \eta_c p}|^2$ is invariant under rotations of $\vec{K}_T$.

In Fig. 3, we present our results for exclusive production of η_c mesons. We extend all curves down to $K_T = 0$ in order to show that the differential cross sections due to $\gamma g g$ or $g g g$ exchange vanish in this limit. However, as mentioned above, we do neglect longitudinal momentum transfer, and so we do not expect that these curves are reliable for $t \simeq t_{\min}$.

The left panel of Fig. 3 shows the cross section due to three-gluon exchange. We compare the full cross section, which includes matrix elements between two- and three-quark states to the one-body approximation with *ad hoc* infrared cutoff (29). They exhibit very different behavior for $|t| \lesssim 1.5 \text{ GeV}^2$. It is interesting to note that the sum of all three-gluon-exchange diagrams achieves its maximum at rather hard $|t| \gtrsim 1 \text{ GeV}^2$. The dominant contribution corresponds to approximate sharing of the momentum transfer among the three gluons, $\vec{q}_i \sim -\vec{K}_T/3$. At the same time, gluons with transverse momentum less than Λ_{eff} are screened so that the cross section below $|t| \sim 1 \text{ GeV}^2$ decreases with $|t|$.

On the other hand, the η_c cross section at $|t| \lesssim 1.5 \text{ GeV}^2$ is anyhow dominated by single-photon exchange as seen in Fig. 3 on the right. This appears to present the cleanest opportunity for measuring the quark GPD $f(K_T)$ in this process. The photon-plus-two-gluon exchange is negligible over the entire range of $|t|$. Odderon exchange dominates for $|t| \gtrsim 1.5 \text{ GeV}^2$. Hence, discovery of the three-gluon QCD exchange in this process requires measurements at fairly large momentum transfer. Since the dominant dipole scale for charmonium production is about $r \sim 1 \text{ GeV}^{-1}$, at large K_T , one requires the dipole scattering amplitude to all orders in $\vec{r} \cdot \vec{K}_T$.

FIG. 3. The differential cross section for exclusive η_c production. The bands indicate the variation due to the two-proton wave functions used here and also cover the range $Q^2 < 0.5 \text{ GeV}^2$. This figure is for a $\gamma^{(*)} - p$ collision energy of approximately $W \simeq 7\text{--}10 \text{ GeV}$. Left: the cross section due to three-gluon exchange alone. The flatter curve accounts for all diagrams, while the steeper curve corresponds to the $c\bar{c}$ dipole scattering from a single quark in the proton. Right: individual contributions due to single-photon, photon-plus-two-gluon, and three-gluon exchanges and the complete η_c cross section summed (at the amplitude level) over all these exchanges.

Our predictions for the η_c cross section due to three-gluon exchange in $\gamma - p$ scattering are substantially lower than earlier estimates⁹ [7,8]. In these references the cross section peaks at 10–25 pb/GeV² at $|t| \approx 0.5 \text{ GeV}^2$. In contrast, we find that the cross section due to three-gluon exchange increases with the momentum transfer up until $|t| \sim 1 \text{ GeV}^2$, where its magnitude is about 50 fb/GeV², and then decreases rather slowly for $|t|$ up to 3 GeV². Aside from using a different approach for the three-gluon exchange amplitude, which here is related to the light-cone wave function of the proton and perhaps more realistic models for the wave function of the η_c , we note the following differences. References [7,8] employ a coupling of the exchanged gluons to the valence quarks of the proton of $\alpha_s = 1$. We use the same $\alpha_s = 0.35$ obtained from the cross section for J/Ψ production for all gluon-quark vertices.¹⁰ This amounts to a suppression factor of $0.35^3 = 0.04$. Furthermore, we find that the “nonrelativistic approximation,” which sets the momentum fractions of the charm quarks in the η_c to 1/2, overestimates the cross section by as much as a factor of 4 (this agrees with the findings in Ref. [51]). We also use a more up-to-date value of $\Gamma(\eta_c \rightarrow \gamma\gamma) = 5 \text{ keV}$ [45] to normalize the η_c wave function, and this is smaller than the value used in

Refs. [7,8] by a factor of 1.4. Lastly, as the maximum of the differential cross section is shifted to higher $|t|$, it is natural that its magnitude would be substantially lower.

VI. SUMMARY

Exclusive η_c production in $\gamma - p$ scattering could provide clean evidence for the semihard Odderon in QCD. Moreover, this process would also provide valuable insight into the light-cone wave function of the proton, which determines the coupling to both the Pomeron and Odderon simultaneously [26]. Accordingly, we first applied our approach to J/Ψ production, and from the magnitude of the cross section, $\sigma^{J/\Psi} \sim \alpha_s^4$, we determined the effective quark-gluon coupling, $\alpha_s \simeq 0.35$. We also obtain a reasonable slope of $d\sigma^{\gamma^* p \rightarrow J/\Psi p}/dt$ without having to tune any parameters. Most importantly, we find that it is a fair approximation to consider J/Ψ production (up to intermediate $|t| \simeq 1 \text{ GeV}^2$) as due to two-gluon exchange with single quarks in the proton. This involves the matrix element of a product of two color charge densities between single-quark states and can be expressed in terms of a leading twist GPD.

$\mathcal{C}$ -parity even states like the η_c can be produced only via $\mathcal{C}$ -odd exchanges such as a single photon; a photon with two gluons; three gluons, etc. Exchange of a single photon dominates at low transverse momentum K_T , while the contribution from the exchange of a photon and two gluons is very small for any K_T . Diagrams involving coupling of the three gluons to multiple quarks are important for the cancellation of infrared divergences due to color neutrality of the proton. In addition, as our analysis indicates, such many-body contributions are also numerically very

⁹Those papers aimed at much higher HERA energies. However, their numerical estimates did not account for small- x evolution of the Odderon and exhibited no energy dependence (for W well above threshold).

¹⁰F. E. Low estimated more than 40 years ago that $\alpha_s \approx \frac{1}{3}$ for two-gluon Pomeron exchange [13], so our qualitative extraction of α_s from the expected J/Ψ cross section at $x \sim 0.1$ is not a new result.

important for $|t| \lesssim 1 \text{ GeV}^2$ (however, single-photon exchange dominates over three-gluon exchange in that regime) and for $|t| \gg 1 \text{ GeV}^2$. At high momentum transfer $K_T^2 = |t|$, one may not expand the dipole scattering amplitude in powers of $\vec{r} \cdot \vec{K}_T$.

A rather interesting outcome of our numerical analysis using the proton light-cone wave functions by Brodsky and Schlumpf [27] is that the differential cross section for $\gamma^{(*)}p \rightarrow \eta_c p$ via three-gluon exchange achieves its maximum for $|t| \simeq 1\text{--}3 \text{ GeV}^2$. This is remarkable since older estimates from the literature [7,8]—using proton impact factors dominated by the scale m_p^2 —obtained the peak at a much lower $|t| < 0.5 \text{ GeV}^2$. A cross section on the order of (at least) $10\text{--}100 \text{ fb/GeV}^2$ at $|t| > 0.5 \text{ GeV}^2$ would represent good evidence for $\mathcal{C}$ -odd three-gluon exchange. On the other hand, the best opportunity to measure the quark GPD in this process is through single-photon exchange at $|t| \lesssim 1.5 \text{ GeV}^2$.

We estimate the cross section for η_c production at $|t| = 1.5\text{--}3 \text{ GeV}^2$ to be $30\text{--}150 \text{ fb/GeV}^2$, for $\alpha_s = 0.35$ and photon-proton collision energy $W \sim 7\text{--}10 \text{ GeV}$. Experimental detection therefore requires very high luminosities. For other values of the coupling, the J/Ψ and η_c cross sections scale like α_s^4 and α_s^6 , respectively.

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APPENDIX: PHOTON AND J/Ψ LIGHT-CONE WAVE FUNCTIONS

This Appendix presents the expressions for the light-cone wave functions of the photon and J/Ψ meson used here. These have been discussed in many papers [12,41,47,53]; we follow the approach of Ref. [12]. (For more recent numerical solutions of quarkonium light-cone wave functions, see Ref. [54].)

The wave function of a longitudinally polarized virtual photon is given by [12]

$$\Psi_{h\bar{h},\lambda=0}^*(\vec{r}, z, Q^2) = e_f e \delta_{h,-\bar{h}} 2Qz(1-z) \frac{K_0(\epsilon r)}{2\pi}. \quad (\text{A1})$$

For transverse polarization,

$$\begin{aligned} \Psi_{h\bar{h},\lambda=\pm}^*(\vec{r}, z, Q^2) &= \lambda e_Q e \sqrt{2} \{ i e^{i\phi_r} [z \delta_{h,\lambda} \delta_{\bar{h},-\lambda} - (1-z) \delta_{h,-\lambda} \delta_{\bar{h},\lambda}] \partial_r \\ &\quad + m_Q \delta_{h,\lambda} \delta_{\bar{h},\lambda} \} \frac{K_0(\epsilon r)}{2\pi}. \end{aligned} \quad (\text{A2})$$

In the above expressions, K_0 is a modified Bessel function of the second kind, ϕ_r is the azimuthal angle of the $\vec{r}$ vector, and $\epsilon^2 \equiv z(1-z)Q^2 + m_f^2$. Note that in Eqs. (A1) and (A2) we do not include a factor of $\sqrt{N_c}$ as in Ref. [12] because we have explicitly included this factor in our expression for the scattering amplitude. Reference [41] also includes a factor of $1/\sqrt{4\pi}$ in the photon (and meson) wave function, which we write explicitly in Eqs. (50) and (55) for the amplitudes.

For the vector meson, we employ the following model wave functions [12,41,51,53]:

$$\Psi_{h\bar{h},\lambda=0}^V(r, z) = \delta_{h,-\bar{h}} \left[M_V + \frac{m_f^2 - \nabla_r^2}{M_V z(1-z)} \right] \phi_L^V(r, z) \quad (\text{A3})$$

and

$$\begin{aligned} \Psi_{h\bar{h},\lambda=\pm 1}^V(\vec{r}, z) &= \pm \frac{\sqrt{2}}{z(1-z)} \{ i e^{\pm i\phi_r} [z \delta_{h,\pm} \delta_{\bar{h},\mp} - (1-z) \delta_{h,\mp} \delta_{\bar{h},\pm}] \partial_r \\ &\quad + m_f \delta_{h,\pm} \delta_{\bar{h},\pm} \} \phi_T^V(r, z). \end{aligned} \quad (\text{A4})$$

Several phenomenological models of the scalar part are available in the literature. Here, we use the boosted Gaussian model [12,41–43]

$$\begin{aligned} \phi_{T,L}^V(r, z) &= \mathcal{N}_{T,L} z(1-z) \exp \left(-\frac{m_f^2 \mathcal{R}^2}{8z(1-z)} - \frac{2z(1-z)r^2}{\mathcal{R}^2} + \frac{m_f^2 \mathcal{R}^2}{2} \right). \end{aligned} \quad (\text{A5})$$

The parameters have been obtained in Ref. [12] from the normalization condition and from the electronic decay width. They are $M_{J/\Psi} = 3.097 \text{ GeV}$, $m_c = 1.4 \text{ GeV}$, $\mathcal{N}_T = 0.578$, $\mathcal{N}_L = 0.575$, and $\mathcal{R}^2 = 2.3 \text{ GeV}^{-2}$.

The product of photon and meson wave functions summed over quark helicities are given by

$$[(\Psi^M)_\lambda^* \Psi_{\lambda'}^*](\vec{r}, z, Q^2) \equiv \sum_{h, \bar{h}=\pm} \Psi_{h\bar{h}, \lambda}^M(\vec{r}, z)^* \Psi_{h\bar{h}, \lambda'}^*(\vec{r}, z, Q^2). \quad (\text{A6})$$

For a J/Ψ meson, after averaging over $\lambda = \lambda' = +$ and $\lambda = \lambda' = -$, one obtains explicitly [12]

$$\begin{aligned} & [(\Psi^{J/\psi})^* \Psi^*]_T(\vec{r}, z, Q^2) \\ &= e_c e \frac{1}{\pi z(1-z)} \{m_c^2 K_0(\epsilon r) \phi_T^V(r, z) \\ &\quad - [z^2 + (1-z)^2] \epsilon K_1(\epsilon r) \partial_r \phi_T^V(r, z)\}, \quad (\text{A7}) \end{aligned}$$

$$\begin{aligned} & [(\Psi^{J/\psi})^* \Psi^*]_L(\vec{r}, z, Q^2) \\ &= e_c e \frac{1}{\pi} 2Qz(1-z) K_0(\epsilon r) \\ &\quad \times \left[M_{J/\psi} \phi_L^V(r, z) + \frac{m_c^2 - \nabla_r^2}{M_{J/\psi} z(1-z)} \phi_L^V(r, z) \right]. \quad (\text{A8}) \end{aligned}$$

In Eq. (A7), the polarizations $\lambda = \lambda'$ of the photon and the J/Ψ are equal; the off-diagonal overlap for $\lambda = -\lambda'$ is proportional to $e^{2i\phi_r}$ and gives no contribution to the amplitude (50) for J/Ψ production as $\mathcal{T}_{gg}(\vec{r}, \vec{K}_T)$ is invariant under a simultaneous rotation of $\vec{r}$ and $\vec{K}_T$. Also, the overlaps $(\Psi_L^{J/\psi})^* \Psi_T^*$ and $(\Psi_T^{J/\psi})^* \Psi_L^*$ do not vanish either but change sign under $\vec{r} \rightarrow -\vec{r}$ and so do not contribute to Eq. (50).

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