Effective-field theory analysis of the $\tau^- \rightarrow (K\pi)^-\nu_\tau$ decays

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We analyze the $\tau^- \rightarrow (K\pi)^-\nu_\tau$ decays within an effective field theory description of heavy new physics (NP) modifying the SM left-handed weak charged current and include refined SM input for the participant meson form factors exploiting chiral symmetry, dispersion relations and (lattice) data. We include the leading dimension six operators and work at linear order in the effective couplings. Within this setting we (i) follow the derivation in [Phys. Rev. Lett. **120**, 141803 \(2018\)](#) where it was proved unambiguously that it is impossible to understand the *BABAR* anomaly in the CP asymmetry measurement within this framework. We allow for reasonable variations of the hadronic input involved and study the associated uncertainty. (ii) We first show that the anomalous bump present in the published Belle data for the $K_S\pi^-$ invariant mass distribution close to threshold cannot be due to heavy NP. (iii) We first bind the heavy NP effective couplings using $\tau^- \rightarrow (K\pi)^-\nu_\tau$ decays and show that they are competitive with those found in hyperon semileptonic decays [but clearly not with those obtained in kaon (semi)leptonic decays for NP scalar currents]. We put forward that the comparison of the considered tau decays with (semi)leptonic kaon and hyperon decays provides meaningful tests of lepton universality for (NP) tensor interactions. We also compare the SM predictions with the possible deviations caused by NP in a couple of Dalitz plot distributions, in the forward-backward asymmetry and in the dimeson invariant mass distribution, and we discuss the most interesting measurements to be performed at Belle-II using these decay data.

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I. INTRODUCTION

Even before its discovery [1] by the group lead by Martin Perl at SLAC, the tau lepton was identified as a very useful probe for precision electroweak studies and a clean QCD laboratory at low energies, thanks to the pioneering work of Tsai [2]. The precise measurements achieved mainly thanks to the Z and B factories (see, for instance, Refs. [3,4]) have promoted tau physics to precision tests of the Standard Model (SM) implying nontrivial restrictions on its possible extensions [5].

In this article we focus on the $\tau^- \rightarrow (K\pi)^-\nu_\tau$ decays, which we study within the SM and considering the effects of heavy new physics (NP) on a number of phenomenologically interesting observables. A clear motivation for this is the *BABAR* anomalous measurement of the CP asymmetry in the $K_S\pi^-$ channel [6]. The CP violation present in the SM [7] is clearly insufficient to understand

the baryon asymmetry of the Universe [8–10] so that any hint of NP involving CP violation becomes a candidate for providing us with a clue to understand the enormous matter-antimatter imbalance. With respect to this *BABAR* anomaly, however, the related Belle measurement [11] of a binned CP asymmetry in the same decay channel analyzing the decay angular distributions is compatible with zero, as expected in the SM with a per-mille level precision. On the theoretical side, the authors of Ref. [12] proved that heavy NP cannot explain this anomaly, as we discuss in detail when we list the main motivations for this work later in this Introduction.

After the discovery [13,14] and precise characterization [15,16] of the Higgs boson at the LHC, the electroweak sector of the SM does not show any hint of NP close to the top quark mass scale [17]. This confirms the usefulness of considering the SM as a low-energy effective field theory (EFT) of the underlying dynamics at some TeVs, the so-called SMEFT [18,19] (when the spontaneous electroweak symmetry breaking is linearly realized). In this spirit, and for weak charged current decays involving light quarks, the low-energy limit of SMEFT is a Fermi-type theory with straightforward connection to the TeV physics if this is weakly coupled [20–30]. In this context, several recent

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works [31–33] have put forward that semileptonic tau decays can be complementary to other low-energy semileptonic probes considered before, such as the corresponding pion and kaon decays, nuclear beta or hyperon decays. In this paper we extend this latter effort to a complete phenomenological analysis of the $\tau^- \rightarrow (K\pi)^- \nu_\tau$ decays, which has not been done, within this framework, before.

Specifically, the main motivations for this study are

- (i) The authors of Ref. [12] disproved earlier claims [34–36] that tensor interactions could explain the *BABAR* CP anomaly in $\tau \rightarrow K_S \pi \nu_\tau$ decays. This corresponds to the measurement of $A_{CP} = -3.6(2.3)(1.1) \times 10^{-3}$ [6],¹ which disagrees remarkably with the SM prediction $A_{CP} = 3.32(6) \times 10^{-3}$, driven by neutral kaon mixing [37,38], probed with high accuracy in semileptonic kaon decays [39]. In fact, the SM prediction is slightly modified by the experimental conditions corresponding to the reconstruction of the K_S at the B-factory, yielding $A_{CP} = 3.6(1) \times 10^{-3}$ [40], which increases the discrepancy at the 2.8σ level. Although we will follow closely the results of Ref. [12], as a novelty of our treatment, we will discuss the uncertainty induced on A_{CP} by the error of the tensor form factor modulus.
- (ii) Three data points at the beginning of the $K_S \pi^-$ spectra measured by Belle [41] have been excluded from the reference fits or signaled as controversial in the dedicated analyses [42–49] and are at odds with the prediction [50]. To our knowledge, only the authors of Ref. [51] were able to describe these data points due to the effect on the scalar form factor of the longitudinal correction to the $K^*(892)$ propagator induced by flavor symmetry breaking.² We will study if it is possible to explain these conflicting data points by the most general description of heavy NP contributions modifying the $\tau^- \rightarrow \bar{u} s \nu_\tau$ decays in the SM.
- (iii) Within an effective field theory analysis of possible nonstandard charged current interactions, semileptonic tau decays [31–33] have been proved competitive with the traditional semileptonic decays involving light quarks [20–30], like nuclear beta or leptonic and radiative pion decays. In this context, for the Cabibbo-suppressed sector, hyperon semileptonic decays [24,27] cannot compete with (semi) leptonic kaon decays [26] probing the scalar sector, given the (very accurately measured) dominant branching fractions of the latter and the suppressed ones (at most at the per-mille level) of the former.

This intuitive reasoning suggests that strangeness-changing tau decays can also give nontrivial bounds on nonstandard charged current interactions, although it is difficult at a competitive level with $K_{\ell(2,3)}$ decays.³ The present work will make these statements precise.

We will answer the previous questions throughout the paper and summarize our conclusions in Sec. VII. The outline of the paper is as follows. In Sec. II we present the effective Lagrangian describing the considered transitions, which is obtained from the general SMEFT Lagrangian after integrating nondynamical degrees of freedom at $E \leq M_\tau$ and particularizing it for the Cabibbo-suppressed tau decays. In Sec. III we introduce the lepton and hadron currents for the different SM and NP contributions and relate those of spins zero and one. For the meson part, we define the form factors that will be discussed at length in Sec. IV next. These are built using chiral perturbation theory input into a dispersive framework and exploiting available measurements in order to reduce the associated uncertainties, which are important in our phenomenological analysis carried out in the remainder of this work. We focus on the tensor form factor, which turns out to be crucial in understanding the strong CP phase, through its interference with the dominant (accurately known) vector form factor. Decay observables are discussed in Sec. V, starting with the doubly differential Dalitz plot distributions (that we discuss for the first time, both in a pair of Mandelstam variables and also trading one of them by a measurable angle) in Secs. VA and VB and continuing with the single differential meson spectrum (in Sec. VC), which was measured by Belle [41]. Instead of presenting the angular distribution, we prefer to show the forward-backward asymmetry in Sec. VD. The limits on the NP effective couplings are derived in Sec. VE, using first only the branching ratio measurements of the $\tau^- \rightarrow (K\pi)^- \nu_\tau$ decays and then also the measured spectrum for the $K_S \pi^-$ decay channel. Our phenomenological analysis ends in Sec. VI, where we revisit [12] the A_{CP} *BABAR* anomaly within the EFT framework.

II. EFFECTIVE THEORY ANALYSIS OF $\tau^- \rightarrow \nu_\tau \bar{u} s$

The lepton number conserving effective Lagrangian density constructed with dimension six operators and invariant under the local $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ SM gauge group has the following form [18,19]:

$$\mathcal{L}^{(\text{eff})} = \mathcal{L}_{\text{SM}} + \frac{1}{\Lambda^2} \sum_i \alpha_i O_i \rightarrow \mathcal{L}_{\text{SM}} + \frac{1}{v^2} \sum_i \hat{\alpha}_i O_i, \quad (1)$$

¹This observable is defined in Sec. VI.

²As we will recall in Sec. IV, the scalar form factor contribution that we employ [52] was obtained as a result of analyzing strangeness-changing meson-meson scattering [53] within chiral perturbation theory [54,55] with resonances [56,57], accounting for the leading flavor symmetry breaking.

³As we will discuss in Sec. V, the quite limited momentum transfer of kaon and hyperon semileptonic decays makes them much cleaner theoretically than hadronic tau decays.

with $\hat{\alpha}_i = (v^2/\Lambda^2)\alpha_i$ the dimensionless couplings encoding NP at a scale of some TeV.

In this framework we can explicitly construct the leading⁴ low-scale $\mathcal{O}(1)$ effective Lagrangian [which has $SU(3)_C \otimes U(1)_{em}$ local gauge symmetry] for semileptonic transitions upon integrating the heavy degrees of freedom out [20,21],

$$\begin{aligned} \mathcal{L}_{cc} = & \frac{-4G_F}{\sqrt{2}} V_{us} [(1 + [v_L]_{\ell\ell}) \bar{\ell}_L \gamma_\mu \nu_{\ell L} \bar{u}_L \gamma^\mu s_L \\ & + [v_R]_{\ell\ell} \bar{\ell}_L \gamma_\mu \nu_{\ell L} \bar{u}_R \gamma^\mu s_R \\ & + [s_L]_{\ell\ell} \bar{\ell}_R \gamma_\mu \nu_{\ell L} \bar{u}_R s_L + [s_R]_{\ell\ell} \bar{\ell}_R \gamma_\mu \nu_{\ell L} \bar{u}_L s_R \\ & + [t_L]_{\ell\ell} \bar{\ell}_R \sigma_{\mu\nu} \nu_{\ell L} \bar{u}_R \sigma^{\mu\nu} s_L] + \text{H.c.}, \end{aligned} \quad (2)$$

where G_F is the tree-level definition of the Fermi constant, L (R) stands for left-handed (right-handed) chiral projections and $\sigma^{\mu\nu} = i[\gamma^\mu, \gamma^\nu]/2$. Note that if we set $v_L = v_R = s_L = s_R = t_L = 0$, we recover the SM Lagrangian for the strangeness-changing semileptonic tau decays, with momentum transfer much smaller than the M_W scale. Right-handed and wrong-flavor neutrino contributions were neglected in Eq. (2) since they do not interfere with the SM amplitudes and do not contribute at leading order in the NP effective coefficients. The couplings v_L , v_R , s_L , s_R , and t_L are related to the α_i couplings of Eq. (1) in Appendix A of Ref. [21] (assuming a weakly coupled scenario at the few-TeV scale). Besides Lorentz invariance, the only assumptions behind Eq. (2) are the local gauge symmetries at low energies [$U(1)_{em}$ and $SU(3)_C$ of quantum electrodynamics and chromodynamics, respectively] and the absence of light non-SM particles.

It is convenient to recast the spin-zero contributions in terms of currents with defined parity (scalar and pseudo-scalar) in the following way:

$$\begin{aligned} \mathcal{L}_{cc} = & -\frac{G_F V_{us}}{\sqrt{2}} (1 + \epsilon_L + \epsilon_R) [\bar{\tau} \gamma_\mu (1 - \gamma_5) \nu_\ell \\ & \cdot \bar{u} [\gamma^\mu - (1 - 2\hat{\epsilon}_R) \gamma^\mu \gamma_5] s \\ & + \bar{\tau} (1 - \gamma_5) \nu_\ell \cdot \bar{u} [\hat{\epsilon}_S - \hat{\epsilon}_P \gamma_5] s \\ & + 2\hat{\epsilon}_T \bar{\tau} \sigma_{\mu\nu} (1 - \gamma_5) \nu_\ell \cdot \bar{u} \sigma^{\mu\nu} s] + \text{H.c.}, \end{aligned} \quad (3)$$

where $\epsilon_{L,R} = v_{L,R}$, $\epsilon_S = s_L + s_R$, $\epsilon_P = s_L - s_R$, and $\epsilon_T = t_L$. In Eq. (3) we have particularized the Lagrangian for the tau lepton case ($\ell = \tau$),⁵ and we have also introduced the notation $\hat{\epsilon}_i = \epsilon_i/(1 + \epsilon_L + \epsilon_R)$ [31] for $i = R, S, P, T$.⁶ In this way, our Lagrangian in Eq. (3) is equivalent to the one in Eq. (9) of Ref. [12] working at

linear order in the epsilon Wilson coefficients. These are complex, although we will assume them real in first approximation in all subsequent analyses, but in the study of the CP asymmetry in Sec. VI, where a nonvanishing relative (weak) phase between the SM and tensor contributions is necessary to modify the SM result.

III. SEMILEPTONIC τ DECAY AMPLITUDE

In this section we calculate the decay amplitudes corresponding to the $\tau^- \rightarrow \bar{K}^0 \pi^- \nu_\tau$ and the $\tau^- \rightarrow K^- \pi^0 \nu_\tau$ decays. The first thing to note is that due to the parity of pseudoscalar mesons, only the vector, scalar and tensor currents give a nonzero contribution to the decay amplitude, as shown in the following equation:^{7,8}

$$\begin{aligned} \mathcal{M} = & \mathcal{M}_V + \mathcal{M}_S + \mathcal{M}_T \\ = & \frac{G_F V_{us} \sqrt{S_{EW}}}{\sqrt{2}} (1 + \epsilon_L + \epsilon_R) \\ & \times [L_\mu H^\mu + \hat{\epsilon}_S L H + 2\hat{\epsilon}_T L_{\mu\nu} H^{\mu\nu}], \end{aligned} \quad (4)$$

where the leptonic currents have the structure (p and p' are the momenta of the tau lepton and its neutrino, respectively)

$$\begin{aligned} L_\mu = & \bar{u}(p') \gamma_\mu (1 - \gamma_5) u(p), \\ L = & \bar{u}(p') (1 + \gamma_5) u(p), \\ L_{\mu\nu} = & \bar{u}(p') \sigma_{\mu\nu} (1 + \gamma_5) u(p), \end{aligned} \quad (5)$$

and the vector, scalar and tensor hadronic matrix elements for the case of the $\tau^- \rightarrow \bar{K}^0 \pi^- \nu_\tau$ decay are defined as follows:

$$H^\mu = \langle \pi^- \bar{K}^0 | \bar{s} \gamma^\mu u | 0 \rangle = Q^\mu F_+(s) + \frac{\Delta_{K\pi}}{s} q^\mu F_0(s), \quad (6)$$

$$H = \langle \pi^- \bar{K}^0 | \bar{s} u | 0 \rangle = F_S(s), \quad (7)$$

$$H^{\mu\nu} = \langle \pi^- \bar{K}^0 | \bar{s} \sigma^{\mu\nu} u | 0 \rangle = i F_T(s) (p_K^\mu p_\pi^\nu - p_\pi^\mu p_K^\nu), \quad (8)$$

⁴We note that this reshuffling is not convenient when comparing neutral and charged current processes and also when analyzing different semileptonic tau decays with an odd and an even number of pseudoscalar mesons, respectively [33]. Since $\epsilon_i = \hat{\epsilon}_i$ at linear order in these coefficients, we may use ϵ_i instead of $\hat{\epsilon}_i$ when comparing to works which use the former instead of the latter.

⁵Equation (4) displays clearly that the renormalization scale dependence of the Wilson coefficients $\hat{\epsilon}_i$ needs to be canceled by the one of the hadron matrix elements. As is conventional, both are defined in the $\overline{\text{MS}}$ scheme at $\mu = 2$ GeV.

⁶For convenience, the short-distance electroweak correction factor S_{EW} [58–65] is written as an overall constant, although it only affects the SM contribution. The error of this simplification is negligible working at leading order in the $\hat{\epsilon}_i$ coefficients [31,32].

⁴We assume that higher-dimensional operator contributions, suppressed by powers of M_τ/M_W , are negligible.

⁵This implicit flavor dependence must be remembered for lepton universality tests, however.

where $q^\mu = (p_\pi + p_K)^\mu$, $Q^\mu = (p_K - p_\pi)^\mu - \frac{\Delta_{K\pi}}{s} q^\mu$, $s = q^2$, and $\Delta_{ij} = m_i^2 - m_j^2$. The hadron matrix elements H , H^μ and $H^{\mu\nu}$ were decomposed in terms of the allowed Lorentz structures, taking into account the discrete symmetries of the strong interactions, and a number of scalar functions of the invariant mass of the $K\pi$ system: the $F_S(s)$, $F_+(s)$, $F_0(s)$ and $F_T(s)$ form factors, which encode the details of the hadronization procedure.

The $\tau^- \rightarrow K^- \pi^0 \nu_\tau$ decay is completely analogous. Neglecting (tiny) isospin corrections, the only difference is given by the Clebsch-Gordan flavor symmetry factor of $\sqrt{2}$ between both decay channels, that is, $\sqrt{2} F_{0,+T}^{K^- \pi^0}(s) = F_{0,+T}^{\bar{K}^0 \pi^-}(s)$.

From Eqs. (5) one can easily see that the vector and the scalar currents are related through the Dirac equation in the following way:

$$L = \frac{L_\mu q^\mu}{M_\tau}. \quad (9)$$

Similarly, one can find a relation between the vector and the scalar hadronic matrix elements by taking the four-divergence of Eq. (6). This yields

$$F_S(s) = \frac{\Delta_{K\pi}}{m_s - m_u} F_0(s). \quad (10)$$

Taking into account the previous two equations, we conclude that the scalar and vector contributions in Eq. (4) can be treated jointly by doing the convenient replacement

$$\frac{\Delta_{K\pi}}{s} \rightarrow \frac{\Delta_{K\pi}}{s} \left[1 + \frac{s \hat{e}_s}{M_\tau(m_s - m_u)} \right]. \quad (11)$$

Obtaining the three independent form factors [$F_0(s)$, $F_+(s)$ and $F_T(s)$] using as much experimental and theoretical knowledge as possible is the subject of the next section.

IV. HADRONIZATION OF THE SCALAR, VECTOR AND TENSOR CURRENTS

In this section we focus on the scalar, vector and tensor form factors. These are needed SM inputs for binding the nonstandard interactions. Consequently, it is fundamental to obtain them reliably (including associated errors) so as to enable robust NP limits. This is conveniently done by using a dispersive framework (which warrants analyticity and elastic unitarity) that employs chiral perturbation theory input (as given by low-energy QCD) and exploits available data to guide the dispersive integral and balance the precise theoretical knowledge at low energies with the very accurate intermediate-energy measurements (especially near the resonance peaks). This approach is discussed minutely in Refs. [44,45,49,52] for the scalar and vector

$K\pi$ form factors. We will devote special attention to the corresponding dispersive tensor form factor [12] following our earlier work [32] for the $\pi\pi$ case.

We will start with a brief reminder of the approach employed for the scalar form factor, $F_0(s)$. In a series of papers [52,53,66–68] Jamin *et al.* studied strangeness-changing coupled-channel meson-meson scattering within chiral perturbation theory with resonances and obtained very precise information on the corresponding scalar form factors, light quark masses and related chiral low-energy constants. We benefit from that analysis here.⁹ In particular, we employ the update presented in Ref. [68] for the dispersive representation of the $K\pi$ channel, together with its corresponding uncertainties.¹⁰

We turn next to the vector form factor. A dispersion relation for $F_+(s)$ was formulated in the references quoted previously. It is seen that a thrice-subtracted dispersion relation is optimal:

$$F_+(s) = \exp \left[\alpha_1 s + \frac{\alpha_2}{2} s^2 + \frac{s^3}{\pi} \int_{s_{\pi K}}^{\infty} ds' \frac{\delta_1^{1/2}(s')}{(s')^3 (s' - s - i\epsilon)} \right], \quad (12)$$

where α_1 , α_2 , and the one to set $F_+(0) = 1$ are the three subtraction constants, and $s_{\pi K} = (m_{\bar{K}^0} + m_{\pi^-})^2$. Equation (12) shows that each additional subtraction in the dispersion relation gives rise to a further suppression factor $1/s'$ in the integrand, enhancing the relative importance of the low-energy input.

In Eq. (12) Watson's final-state interactions theorem [70] was used. It states that below inelasticities the phase of the form factor equals the scattering phase of the $K\pi$ system [$\delta_1^{1/2}(s)$ in this case, as it has spin one and isospin one half]. For this decay channel, departures are expected above $(m_K + m_\eta)^2 \sim 1.022 \text{ GeV}^2$. These are accounted for in the analyses cited above and are included in our study. We consider as reference input the results obtained in Sec. 3 of Ref. [45] (without using constraints from kaon decays [45,47] or information from $\tau^- \rightarrow K^- \eta \nu_\tau$ decays [49,71]). The corresponding systematic and statistical errors of $F_+(s)$ that we use can be traced back to the results in Table 1 of Ref. [45]: the (correlated) statistical errors of the fitted parameters characterizing $F_+(s)$ are those coming from the fit and given in this table and the systematic errors are estimated from the differences induced by changing s_{cut} between the different columns of that table for a given

⁹P. Roig thanks Matthias Jamin for providing him with these data.

¹⁰For the analysis of the $K\pi$ spectra near threshold it is particularly important to employ a scalar form factor that is consistent with the information coming from S-wave $K\pi$ scattering (including the coupled channels $K\eta$ and $K\eta'$). The scalar form factor obtained in Ref. [68] is included in the RChL version of TAUOLA [69], but not in other releases.

FIG. 1. Modulus and phase, $|F_T(s)|$ (left) and $\delta_T(s) = \delta_+(s)$ (right), of the tensor form factor, $F_T(s)$. In the left plot, the dotted line corresponds to $s_{\text{cut}} = 9 \text{ GeV}^2$, the dashed one to $s_{\text{cut}} = 4 \text{ GeV}^2$, and the solid one to $s_{\text{cut}} = M_\pi^2$.

$F_0(s)$. The phase $\delta_1^{1/2}(s)$ is confronted to data [72,73] in Fig. 2 of Ref. [45]. The fine agreement of $|F_+(s)|$ with the corresponding measurements can be appreciated in various plots of the papers quoted above.

A comment is in order with respect to the dependence on s_{cut} : having perfect knowledge of the form factor phase (including inelasticities) from the threshold on, the dispersive integral should be performed with $s_{\text{cut}} \rightarrow \infty$. In this ideal case, a once-subtracted dispersion relation would suffice and $\alpha_{1,2,\dots}$ in Eq. (12) could be obtained via sum rules and would perfectly match the coefficients of the polynomial terms of the low-energy expansion of $F_+(s)$. In practice, this is not the case, and one needs to increase the number of subtractions (three are optimal in this case, as we mentioned before). When this is done, a subtle cancellation (impossible to achieve exactly) is needed between this polynomial part and the one coming from the dispersive integral, to render a form factor that vanishes as $1/s$ asymptotically. That is why, even when making the phase go to π for $s \rightarrow \infty$, the asymptotic behavior of the form factor does not comply exactly with the Brodsky-Lepage [74] behavior. Because of this shortcoming, performing the dispersive integral with $s_{\text{cut}} \rightarrow \infty$ is not unambiguously the best choice (this is discussed already in Ref. [53]) as one would have expected. That is why we decided to follow Refs. [49,71] and include the variation on s_{cut} as the main source of systematic theory errors.

To finish this section, we sketch the hadronization of the tensor current, which was presented in Eq. (8); further details are given in Ref. [32]. Its normalization at zero-momentum transfer is obtained using chiral perturbation theory with tensor sources [75]. Its energy dependence is obtained by solving numerically the dispersion relation, where the input phase corresponds to the one of the vector form factor in the elastic region [12].

Proceedings analogously as in Ref. [32], we find that¹¹

$$i\langle \pi^0 K^- | \frac{\delta \mathcal{L}}{\delta \bar{t}_{a\beta}} | 0 \rangle = \frac{\Lambda_2}{\sqrt{2}F^2} (p_K^\alpha p_0^\beta - p_0^\alpha p_K^\beta), \quad (13)$$

and verify the relative factor of $\frac{1}{\sqrt{2}}$ between the matrix elements for both decay channels.

The value of Λ_2 is not restricted by symmetry requirements and cannot be fixed from phenomenology. Fortunately, the lattice QCD evaluation of Ref. [77] found $f_T^{K^0\pi^-}(0) = 0.417(15)$. This, together with the fact that $F_T^{K^0\pi^-}(0) = \frac{\Lambda_2}{F^2}$, implies that $\Lambda_2 = (11.1 \pm 0.4) \text{ MeV}$, which we will use in our numerical analysis. This value is consistent within one sigma with the one employed in Ref. [32] for the $\pi\pi$ channel.

Unlike the vector and scalar form factor cases, there are no experimental data that can help us with constructing $F_T(s)$ so that we must rely only on theory.

We calculate the energy dependence of the tensor form factor $F_T(s)$ using again a phase dispersive representation as it is shown in Refs. [12,32]:

$$\frac{F_T(s)}{F_T(0)} = \exp \left[\frac{s}{\pi} \int_{s_{\pi K}}^{\infty} ds' \frac{\delta_T(s')}{s'(s' - s - i\epsilon)} \right], \quad (14)$$

where $s_{\pi K} = (m_{\bar{K}^0} + m_{\pi^-})^2$. As in the scalar case we have included one subtraction. It is clear that, lacking precise low-energy information, we cannot increase the number of subtractions of $F_T(s)$. This, in turn, implies a sizable sensitivity to the upper limit of the integral that is used numerically (s_{cut}), which is illustrated in the left plot of our Fig. 1, where we consider the cases $s_{\text{cut}} = M_\pi^2, 4, 9 \text{ GeV}^2$ [32].¹² We take the differences between these curves as an estimate of our systematic theoretical error on $F_T(s)/F_T(0)$. In the right panel of Fig. 1 we show the tensor form factor phase corresponding to $\delta_T(s) = \delta_+(s)$, with $\delta_+(s)$ from the fits in Table 1 of Ref. [45]. In the

¹¹ $\pi^0 - \eta - \eta'$ mixing (see e. g. Ref. [76]) is neglected.

¹²In principle, one could try to reduce this sensitivity following the strategies employed in Ref. [78], but the procedure will again be limited in this case by the absence of measurements sensitive to $F_T(s)$.

inelastic region, our curve plotted for $\delta_T(s)$ lies within the error band shown in Fig. 2 of Ref. [12].¹³

The phases of $F_T(s)$ and $F_+(s)$ can be related as shown in Ref. [12]. We will not repeat their argument here, but only quote their main result: in the elastic region, $\delta_T(s) = \delta_+(s) = \delta_1^{1/2}(s)$. We will also estimate violations of this equation in the inelastic region (with their corresponding uncertainties) following again Ref. [12] (see Fig. 2 in that reference).

$$t^\pm(s) = \frac{1}{2s} \left[2s(M_\tau^2 + m_{K^0}^2 - s) - (M_\tau^2 - s)(s + m_{\pi^-}^2 - m_{K^0}^2) \pm (M_\tau^2 - s)\sqrt{\lambda(s, m_{\pi^-}^2, m_{K^0}^2)} \right], \quad (16)$$

with $\lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz$ being the usual Källén function and $t = (P_\tau - p_\pi)^2$.

A. Dalitz plots

Putting together the equations of Sec. III, the amplitude (we will omit from now on the indices identifying the $K_S\pi^-$ charge channel) can be written as follows:

$$\mathcal{M} = \frac{G_F}{\sqrt{2}} V_{us} \sqrt{S_{EW}} (1 + \epsilon_L + \epsilon_R) (M_0 + M_+ + M_T), \quad (17)$$

where it is straightforward to identify the precise form of the scalar (M_0), vector (M_+) and tensor (M_T) contributions from Sec. III. All contributions to the squared matrix element (including interferences) are nonvanishing. Their expressions can be obtained from Eqs. (5.4) in [32] taking into account the corresponding $SU(3)$ Clebsch-Gordan coefficients and replacing $\Delta_{\pi^-\pi^0} \rightarrow -\Delta_{K\pi}$.

In the study of the Dalitz plots it is convenient to define the following observable introduced in Ref. [32]:

$$\tilde{\Delta}(\hat{\epsilon}_S, \hat{\epsilon}_T) = \frac{||\mathcal{M}(\hat{\epsilon}_S, \hat{\epsilon}_T)|^2 - |\mathcal{M}(0, 0)|^2|}{|\mathcal{M}(0, 0)|^2}, \quad (18)$$

which is sensitive to the relative difference between the squared matrix element in the presence/absence of NP contributions [the SM case corresponds to $\mathcal{M}(0, 0)$].

In the left panel of Fig. 2 we show the Dalitz plot for the SM case in the (s, t) variables, and in the left panels of Figs. 3 and 4 we show the corresponding plots for the values $(\hat{\epsilon}_S = -0.5, \hat{\epsilon}_T = 0)$ and $(\hat{\epsilon}_S = 0, \hat{\epsilon}_T = 0.6)$, respectively. The election of these particular values of the $\hat{\epsilon}_{S,T}$ is discussed in Sec. V E.

In the SM plots (Fig. 2) it is clearly appreciated that the dynamics is dominated by the $K^*(892)$ vector resonance but the effect of its excitation $K^*(1410)$ and of the

V. DECAY OBSERVABLES

In the rest frame of the τ lepton, the doubly differential decay width for the $\tau^- \rightarrow K_S\pi^-\nu_\tau$ process is

$$\frac{d^2\Gamma}{dsdt} = \frac{1}{32(2\pi)^3 M_\tau^3} |\overline{\mathcal{M}}|^2, \quad (15)$$

where s is the invariant mass of the π^-K_S system taking values in the $(m_{K^0} + m_{\pi^-})^2 \leq s \leq M_\tau^2$ interval, and

dynamically generated $K_0^*(700)$ [79], of the $K_0^*(1430)$ and heavier states cannot be appreciated from the figure, although it is visible both in $F_+(s)$ and the decay spectrum [44] and in $F_0(s)$ [68], respectively. The left panels of Figs. 3 and 4 show the relative modification of the squared matrix element for nonzero reasonable values of $\hat{\epsilon}_S$ and $\hat{\epsilon}_T$ in the (s, t) plane. Although large variations are seen in a couple of regions close to the border of the Dalitz plot in Fig. 3 (left), these correspond to zones with very suppressed probability, as can be seen in Fig. 2 (left). On the contrary, the regions with larger probability have a small relative change, according to Fig. 3 (left). In Fig. 4 (left) the region with the most noticeable change (though still smaller than those seen in Fig. 3) is located very close to the s minimum of the Dalitz plot, which has a very small probability density in Fig. 2 (left). This region quite overlaps with one of the two mentioned for the left plot in Fig. 3. Because of this feature, observing a deviation from the SM result in this region could be due to both tensor and nonstandard scalar interactions. On the contrary, a deviation in the region of small t values would be signaling spin-zero NP contribution. In any case, changes are very small in the region most densely populated by measured events in the left plots of Figs. 3 and 4. Due to this, we conclude that it will be extremely challenging to identify NP contributions in the (s, t) Dalitz plot even with the large data samples accumulated by the end of operation of Belle-II [80].

B. Angular distribution

In this section we are going to study the angular dependence of the decay distribution. It is convenient to work in the rest frame of the hadronic system, in which we have $\vec{p}_\pi + \vec{p}_K = \vec{p}_\tau - \vec{p}_\nu = \vec{0}$; consequently the tau lepton and the pion energies are given by $E_\tau = (s + M_\tau^2)/(2\sqrt{s})$ and $E_\pi = (s + m_\pi^2 - m_K^2)/(2\sqrt{s})$.

We will study the decay distribution in terms of the $(s, \cos\theta)$ variables, where θ is the angle between the three-momenta of the pion and the three-momenta of the tau lepton; this angle is related to the invariant t variable by

¹³Our phase is given in degrees while theirs is in radians.

FIG. 2. Dalitz plot distribution $|\overline{\mathcal{M}}|^2_{00}$ in the SM: differential decay distribution for $\tau^- \rightarrow K_S \pi^- \nu_\tau$ in the (s, t) variables (left). The right-hand panel shows the differential decay distribution in the $(s, \cos \theta)$ variables. The Mandelstam variables, s and t , are normalized to M_τ^2 .

FIG. 3. Dalitz plot distribution $\tilde{\Delta}(\hat{e}_S, \hat{e}_T)$ [Eq. (18)] in the $\tau^- \rightarrow K_S \pi^- \nu_\tau$ decays. The left-hand side corresponds to the differential decay distribution in the (s, t) variables and the right-hand side to the one in the $(s, \cos \theta)$ variables, both with $(\hat{e}_S = -0.5, \hat{e}_T = 0)$. The Mandelstam variables, s and t , are normalized to M_τ^2 .

FIG. 4. Dalitz plot distribution $\tilde{\Delta}(\hat{e}_S, \hat{e}_T)$ [Eq. (18)] in the $\tau^- \rightarrow K_S \pi^- \nu_\tau$ decays. The left-hand side corresponds to the differential decay distribution in the (s, t) variables and the right-hand side to the one in the $(s, \cos \theta)$ variables, both with $(\hat{e}_S = 0, \hat{e}_T = 0.6)$. The Mandelstam variables, s and t , are normalized to M_τ^2 .

$$t = M_\tau^2 + m_\pi^2 - 2E_\tau E_\pi + 2|\vec{p}_\pi||\vec{p}_\tau|\cos\theta, \quad \text{where} \quad |\vec{p}_\pi| = \sqrt{E_\pi^2 - m_\pi^2} \quad \text{and} \quad |\vec{p}_\tau| = \sqrt{E_\tau^2 - M_\tau^2}.^{14}$$

¹⁴The tau lifetime and decay width (τ_τ and Γ_τ , respectively) are defined in the τ rest frame. Consequently, their values are boosted in the reference frame considered in this subsection.

The analytical results for the angular distribution can be obtained trivially from Eqs. (5.6) in Ref. [32], as mentioned above.

The Dalitz plots for the $(s, \cos \theta)$ variables are shown in the right panels of Figs. 2–4 [in these last two the observable $\tilde{\Delta}(\hat{e}_S, \hat{e}_T)$ is plotted]. In Fig. 2 we plot the SM

FIG. 5. The $\bar{K}^0\pi^-$ hadronic invariant mass distribution for the SM (solid line) and $\hat{e}_S = -0.5$, $\hat{e}_T = 0$ (dashed line) and $\hat{e}_S = 0$, $\hat{e}_T = 0.6$ (dotted line). The decay distributions are normalized to the tau decay width.

case, and in Figs. 3 and 4 we show Dalitz plots for the values $(\hat{e}_S = -0.5, \hat{e}_T = 0)$ and $(\hat{e}_S = 0, \hat{e}_T = 0.6)$, respectively. The SM plot gives equivalent information in the $(s, \cos\theta)$ variables as the one seen in the (s, t) variables (right versus left plot of Fig. 2). Comparing both panels of Fig. 3 one can see that one of the enhanced regions in the (s, t) plot (the one at very low s values) is washed away in the $(s, \cos\theta)$ diagram, while the other is slightly further enhanced in a limited region ($0 \leq \cos\theta \leq 0.5$). The comparison of the left and right plots of Fig. 4 shows that the enhanced area for large t values is a bit more prominent in the $(s, \cos\theta)$ distribution (for nearly maximal $\cos\theta$) although again it will be very hard to disentangle these possible deviations from the SM patterns in near future data.

Assuming approximate lepton universality, using the bounds from Ref. [26] [obtained analyzing kaon (semi) leptonic decays] $\hat{e}_S \sim -8 \times 10^{-4}$, $\hat{e}_T \sim 6 \times 10^{-3}$ (maximum allowed absolute values at one standard deviation) minimizes the deviations from the SM to an unobservable level in both the (s, t) and $(s, \cos\theta)$ Dalitz plots.

C. Decay rate

Integrating Eq. (15) upon the t variable we obtain the invariant mass distribution, whose analytical form can be easily obtained from Eqs. (5.7) and (5.8) in Ref. [32].

It is important to mention that the only possible source of CP violation coming from the hadronic part is due to the vector-tensor interference; we will comment about this in Sec. VI.

In Fig. 5, we plot the invariant mass distribution of the $K\pi$ system for $\tau^- \rightarrow K_S\pi^-\nu_\tau$ decays for the SM case and for $(\hat{e}_S = -0.5, \hat{e}_T = 0)$ and $(\hat{e}_S = 0, \hat{e}_T = 0.6)$ which

would be realistic values for these couplings, according to their impact on the decay width. Despite the logarithmic scale of the plot, the deviations from the SM curve shown in Fig. 5 are too large when they are confronted with the Belle measurements of this spectrum, as we will see in the fits of Sec. 5.5. This will allow us to set better bounds on $\hat{e}_{S,T}$ than those used in this subsection.

D. Forward-backward asymmetry

The forward-backward asymmetry is defined in analogy to the di-pion mode [32]

$$\mathcal{A}_{K\pi}(s) = \frac{\int_0^1 d\cos\theta \frac{d^2\Gamma}{ds d\cos\theta} - \int_{-1}^0 d\cos\theta \frac{d^2\Gamma}{ds d\cos\theta}}{\int_0^1 d\cos\theta \frac{d^2\Gamma}{ds d\cos\theta} + \int_{-1}^0 d\cos\theta \frac{d^2\Gamma}{ds d\cos\theta}}. \quad (19)$$

Integrating the angular distribution upon the $\cos\theta$ variable we obtain its analytical expression¹⁵

$$\begin{aligned} \mathcal{A}_{K\pi} = & \frac{3\sqrt{\lambda(s, m_\pi^2, m_K^2)}}{2s^2[X_{VA} + \hat{e}_S X_S + \hat{e}_T X_T + \hat{e}_S^2 X_{S^2} + \hat{e}_T^2 X_{T^2}]} \\ & \times \left(1 + \frac{s\hat{e}_S}{M_\tau(m_s - m_u)}\right) \Delta_{\pi K} \\ & \times \left[-\text{Re}[F_0(s)F_+^*(s)] + \frac{2s\hat{e}_T}{M_\tau} \text{Re}[F_T(s)F_0^*(s)]\right]. \quad (20) \end{aligned}$$

Compared to the $\pi\pi$ case, the main difference is that the very small parameter there $\Delta_{\pi\pi}$ is replaced here by $\Delta_{\pi K}$,

¹⁵In Eq. (20) we use $\mathcal{A}_{K\pi}$ to emphasize the decay channel under consideration and compare it next to our previous result for the $\pi\pi$ decay mode. Otherwise we will also be using the most common notation A_{FB} for this observable.

FIG. 6. Forward-backward asymmetry in $\tau^- \rightarrow K_S \pi^- \nu_\tau$ decays for the SM case.

FIG. 7. Forward-backward asymmetry in $\tau^- \rightarrow K_S \pi^- \nu_\tau$ decays compared with the SM prediction (solid line). The dashed line corresponds to $\hat{e}_S = -0.5$, $\hat{e}_T = 0$, and the dotted line corresponds to $\hat{e}_S = 0$, $\hat{e}_T = 0.6$.

which is not that small. The forward-backward asymmetry for the case in which $\epsilon_R = \epsilon_L = \hat{e}_S = \hat{e}_T = 0$, corresponding to the SM, is plotted in Fig. 6. It should not be difficult to measure a nonzero forward-backward asymmetry around $\sqrt{s} \sim 0.6$ GeV. Above the onset of inelasticities ($\sqrt{s} \gtrsim 1.05$ GeV) the theory uncertainty starts to increase up to the kinematical upper limit of $\sqrt{s}$. It was already emphasized long ago that a measurement of the forward-backward asymmetry in this decay channel would be crucial in improving our knowledge of both vector and scalar form factors [81].¹⁶

In Fig. 7, we plot $\mathcal{A}_{K\pi}$ for the values ($\hat{e}_S = -0.5, \hat{e}_T = 0$) and ($\hat{e}_S = 0, \hat{e}_T = 0.6$), and we compare those plots with

¹⁶We note that in this reference, and also later on in Refs. [46,82], the angle θ used to compute A_{FB} is defined between the three-momenta of the tau lepton and the K_S in the dimeson rest frame. Taking into account the different sign conventions, it can be checked that there is reasonable agreement with these works in the elastic region.

FIG. 8. Deviations from the SM forward-backward asymmetry, $\Delta \mathcal{A}_{K\pi}$, in $\tau^- \rightarrow K_S \pi^- \nu_\tau$ decays using the bounds from Ref. [26]. The solid line corresponds to $\hat{e}_S = -8 \times 10^{-4}$, $\hat{e}_T = 0$ and the dashed line to $\hat{e}_S = 0$, $\hat{e}_T = 6 \times 10^{-3}$.

the SM case. For quite large $\hat{e}_T$ values some difference is appreciated for the tensor case; otherwise it may not be possible to disentangle it from the standard contribution. On the contrary, for nonstandard scalar interaction, $\mathcal{A}_{K\pi}$ flips sign with respect to the SM and it gets smaller in magnitude as s increases. If it is possible to measure $\mathcal{A}_{K\pi}$ in a low-energy bin, this would ease the identification of this type of NP in $\mathcal{A}_{K\pi}$. When the more realistic limits obtained in Ref. [26] are considered (under the assumption of approximate lepton universality), it is impossible to identify any departures from the SM prediction in this observable. For this reason, we follow Ref. [32] and use

$$\Delta \mathcal{A}_{K\pi} = \mathcal{A}_{K\pi}(s, \hat{e}_S, \hat{e}_T) - \mathcal{A}_{K\pi}(s, 0, 0), \quad (21)$$

instead. The corresponding (unmeasurably small) deviations from the SM result are plotted in Fig. 8.

E. Limits on $\hat{e}_S$ and $\hat{e}_T$

One of the main purposes in the search for NP using the channel $\tau^- \rightarrow \bar{K}^0 \pi^- \nu_\tau$ is to set bounds on the couplings $\hat{e}_S$ and $\hat{e}_T$, which are the effective couplings responsible for NP effects in this case. For this task we compare the decay width (Γ) for nonvanishing NP effective couplings with respect to the SM width (Γ^0) where NP is absent. We take the observable Δ defined in the following equation as the appropriate one to enhance the sensitivity to nonvanishing values of $\hat{e}_S$ and $\hat{e}_T$:

$$\Delta \equiv \frac{\Gamma - \Gamma^0}{\Gamma^0} = \alpha \hat{e}_S + \beta \hat{e}_T + \gamma \hat{e}_S^2 + \delta \hat{e}_T^2. \quad (22)$$

Here we obtained the following results for the coefficients: $\alpha \in [0.30, 0.34]$; $\beta \in [-2.92, -2.35]$; $\gamma \in [0.95, 1.13]$; and $\delta \in [3.57, 5.45]$. These values correspond to a slightly larger sensitivity to the possible tensor contribution than

FIG. 9. Δ as a function of $\hat{\epsilon}_S$ for $\hat{\epsilon}_T = 0$ (left hand) and of $\hat{\epsilon}_T$ for $\hat{\epsilon}_S = 0$ (right hand) for $\tau^- \rightarrow K_S \pi^- \nu_\tau$ decays. Horizontal lines represent the values of Δ according to the current measurement and theory errors (at three standard deviations) of the branching ratio (dashed line) and in the hypothetical case where the measured branching ratio at Belle-II has a three-times-reduced uncertainty (dotted line).

to the effects of nonstandard scalar interactions. We note that $|\beta| \sim \delta$, which corresponds to the fact that the interference between the spin-one SM amplitude and the tensor contribution is proportional to $\text{Re}[F_T(s)F_+^*(s)]$, which in the elastic region is $\sim |F_T(s)||F_+(s)|$ and should be of similar magnitude than $|F_T(s)|^2$.

The values for $\hat{\epsilon}_S$ and $\hat{\epsilon}_T$ are calculated from Eq. (22) in two different ways, as it is done in Refs. [31,32]. First we set one of the couplings to zero and obtained bounds for the other; these results are shown in Fig. 9. Then, we also obtained constraints for $\hat{\epsilon}_S$ and $\hat{\epsilon}_T$ in the general case where both are nonvanishing. We show these results in Fig. 10, where the bounds on both couplings are limited by an ellipse in the $\hat{\epsilon}_S - \hat{\epsilon}_T$ plane.

The information for the couplings obtained here was used in the previous subsections, where we took the values

$\hat{\epsilon}_S \sim -0.5$ and $\hat{\epsilon}_T \sim 0.6$ as representative of realistic maximum absolute values of these coefficients when only the branching ratio (and not the decay spectrum) is considered.

Our results for the bounds in the $\hat{\epsilon}_S$ and $\hat{\epsilon}_T$ couplings are summarized in the Table I.

The fact that the interval of allowed values for $\hat{\epsilon}_S$ is larger than that for $\hat{\epsilon}_T$ (this is slightly more clear in the last two columns of the Table I, when both of them are floated simultaneously) agrees with our results for the β and δ coefficients being larger (of similar magnitude, though, as the bounds on $\hat{\epsilon}_S$ and $\hat{\epsilon}_T$ are) than the α and γ ones in Eq. (22). According to the Table I, the allowed region is not symmetric and has a mild preference for $\hat{\epsilon}_S < 0$ and $\hat{\epsilon}_T > 0$.

Next we will consider fits to the branching ratio and decay spectrum¹⁷ of the $\tau^- \rightarrow K_S \pi^- \nu_\tau$ decays as measured by Belle [41]. We will pay special attention to the possible explanation of the conflicting data points (bins 5, 6 and 7) by the nonstandard interactions. Therefore, we will consider fits with and without these data points. In all our fits, as explained e.g., in Ref. [49], we will not consider the first data point (as it lies below the threshold for physical K_S and π^- masses) and will disregard the data from the last 10 bins, as suggested by the Belle Collaboration.

The χ^2 function minimized in our fits is

$$\sum_i \left(\frac{\mathcal{N}_i^{\text{exp}} - \mathcal{N}_i^{\text{th}}}{\sigma_{\mathcal{N}_i}} \right)^2 + \left(\frac{\text{BR}^{\text{exp}} - \text{BR}^{\text{th}}}{\sigma_{\text{BR}}^{\text{exp}}} \right)^2, \quad (23)$$

where the sum over the i bins may or may not include the $i = 5, 6, 7$ bins. The theoretical expression for the differential decay rate (whose integration yields $\text{BR}^{\text{th}} \times \Gamma_\tau$) is related to the distribution of the measured number of events as indicated in Eq. (3.1) of Ref. [49] and in the subsequent explanation. We will consider the measurement of BR^{exp} reported in the Belle paper [41] (and not the PDG [39] or

FIG. 10. Constraints on the scalar and tensor couplings obtained from $\Delta(\tau^- \rightarrow K_S \pi^- \nu_\tau)$ using theory and the measured value reported in the PDG, with their corresponding uncertainties at three standard deviations (solid line). The dashed line ellipse corresponds to the case where the measurements error was reduced to a third of the current uncertainty.

¹⁷P.R. thanks Denis Epifanov for providing him with these data.

TABLE I. Constraints on the scalar and tensor couplings obtained through the limits on the current branching ratio at three standard deviations using the current theory and experimental errors and assuming the latter be reduced to a third (“Future bounds”). This last case should be taken only as illustrative of the improvement that can be achieved thanks to higher-statistics measurements, even in the absence of any progress on the theory side. It is clear that the knowledge of $\hat{e}_{S,T}$ using $\tau^- \rightarrow K_S \pi^- \nu_\tau$ decay data is limited by theory uncertainties.

Δ limits	$\hat{e}_S(\hat{e}_T = 0)$	$\hat{e}_T(\hat{e}_S = 0)$	$\hat{e}_S$	$\hat{e}_T$
Current bounds	$[-0.57, 0.27]$	$[-0.059, 0.052] \cup [0.60, 0.72]$	$[-0.89, 0.58]$	$[-0.07, 0.72]$
Future bounds	$[-0.52, 0.22]$	$[-0.047, 0.036] \cup [0.62, 0.71]$	$[-0.87, 0.56]$	$[-0.06, 0.71]$

TABLE II. Best fit values to the Belle spectrum and branching ratio of the $\tau^- \rightarrow K_S \pi^- \nu_\tau$ decays [41]. The cases where the $i = 5, 6, 7$ bins are excluded/included are considered. We display the reference results obtained by floating $\hat{e}_S$ and $\hat{e}_T$ simultaneously. In the last two columns the χ^2 of these fits is compared to the SM result.

Best fit values	$\hat{e}_S$	$\hat{e}_T$	χ^2	χ^2 in the SM
Excluding $i = 5, 6, 7$ bins	$(1.3 \pm 0.9) \times 10^{-2}$	$(0.7 \pm 1.0) \times 10^{-2}$	[72, 73]	[74, 77]
Including $i = 5, 6, 7$ bins	$(0.9 \pm 1.0) \times 10^{-2}$	$(1.7 \pm 1.7) \times 10^{-2}$	[83, 86]	[91, 95]

the HFLAV [83] values), as discussed in Ref. [49]. Along our fits we float the meson form factors within their estimated uncertainty bands and our quoted results take these errors into account. As in the EFT analysis of the $\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$ decays [32], the data are not precise enough (and also are only for the dimeson spectrum and branching fraction, and not for the other observables) to allow a simultaneous fit of the parameters entering the dispersive representation of $F_+(s)$ and the NP coefficients $\hat{e}_{S,T}$ that improves the bounds obtained previously for the latter using only the branching ratio measurement.

First we have tried fits excluding the $i = 5, 6, 7$ bins: initially varying $\hat{e}_S$ and $\hat{e}_T$ simultaneously,¹⁸ with the results $\hat{e}_S = (1.3 \pm 0.9) \times 10^{-2}$ and $\hat{e}_T = (0.7 \pm 1.0) \times 10^{-2}$ and $\chi^2 \sim [72, 73]$, which improves very little with respect to the SM description, with a $\chi^2 \sim [74, 77]$.¹⁹ Then we have set one in turn to values of the order of those found in Ref. [26] ($|\hat{e}_S| \leq 1 \times 10^{-3}$ and $|\hat{e}_T| \leq 6 \times 10^{-3}$, respectively) and fit the other one, with the results $\hat{e}_S = (1.4 \pm 1.6) \times 10^{-2}$ (with fixed $\hat{e}_T$) and $\hat{e}_T = (0.9^{+1.0}_{-1.2}) \times 10^{-2}$ (with fixed $\hat{e}_S$) with a χ^2 of [74, 75] and [75, 84] respectively. These values are in accord with those of the joint fit, which we take as our reference results.

When we include the $i = 5, 6, 7$ bins the previous results change in the following way: the reference SM result χ^2

increases up to $\sim [91, 95]$, and a simultaneous fit of both effective couplings yields $\hat{e}_S = (0.9 \pm 1.0) \times 10^{-2}$ and $\hat{e}_T = (1.7 \pm 1.7) \times 10^{-2}$, with a $\chi^2 \sim [83, 86]$. When we restrict $|\hat{e}_S| \leq 1 \times 10^{-3}$ ($|\hat{e}_T| \leq 6 \times 10^{-3}$), the best fit value of the floated coupling is $\hat{e}_T = (1.2^{+1.2}_{-1.0}) \times 10^{-2}$ ($\hat{e}_S = (1.8 \pm 1.6) \times 10^{-2}$) with a χ^2 of [86, 89] ([84, 87]). Again the single-parameter fit results agree with those obtained in the two-parameter fits, which we take as our central results.

All these fits favor slightly $\hat{e}_S > 0$ and $\hat{e}_T > 0$ (although negative $\hat{e}_T$ is possible within one sigma and $\hat{e}_S < 0$ at less than 1.5 standard deviations).²⁰ We summarize our main results in the Table II.

In view of these results it is clear that the narrow peak structure constituted by the $i = 5, 6, 7$ bins cannot be understood either in the SM (with a dispersive scalar form factor coming from the S-wave of a coupled channel analysis of meson-meson scattering [53]) [42–50] or in the EFT analysis considered in this work. This conclusion agrees with the later preliminary data of BABAR [85] and a Belle posterior measurement [86], where such a bump near threshold is absent.

It would be desirable to compare these bounds to those obtained in (semi)leptonic kaon [26] and hyperon [24] decays. In these cases, the momentum transfer corresponding to the hadronic transition is small enough that a dispersive treatment with chiral perturbation theory and lattice QCD input limits the systematic theory errors in a controlled way. Despite our careful treatment of the hadronic input used in this work, this cannot be the case for semileptonic tau decays because of the large range of

¹⁸The correlation of $\hat{e}_S$ and $\hat{e}_T$ is always smaller than 0.4 (in some fits it is even smaller than 0.1) irrespective of including the $i = 5, 6, 7$ bins in the fit or not. This is in agreement with the very different energy dependence of $F_0(s)$ and $F_T(s)$.

¹⁹The quoted intervals for the best χ^2 include the systematic theoretical uncertainties as estimated by varying s_{cut} in Ref. [44]. We are not following a more complete treatment of these uncertainties, as proposed in Ref. [78] for the $\pi^- \pi^0$ and $K^- K^0$ tau decays.

²⁰We recall that $\hat{e}_S < 0$ and $\hat{e}_T > 0$ were marginally preferred using only the branching ratio information.

the invariant mass of the hadron system. While our framework is able to provide an excellent control of the uncertainties in the elastic region, this is not the case out of it and we cannot discard those shortcomings of our approach (like for instance assuming that inelastic effects can be captured in the phase shift that we employ, which is inherent to the elastic approximation) that could mimic NP effects. This may be the case especially in the absence of alternative/complementary measurements to the Belle spectrum in $\tau^- \rightarrow K_S \pi^- \nu_\tau$ decays. With multidimensional information on these decays (through Dalitz plot analyses for instance) it should be possible—maybe by requiring an improved theoretical description—to tell a NP scenario from a not sufficiently accurate description of the hadronic input. Taking this situation into account, our comparisons to (semi)leptonic kaon and hyperon [24] decays should only be seen presently as tests of lepton universality.

The global fit of (semi)leptonic kaon decays in Ref. [26] yielded the results $\hat{e}_S = (-3.9 \pm 4.9) \times 10^{-4}$ and $\hat{e}_T = (0.5 \pm 5.2) \times 10^{-3}$ for decays involving muons.²¹ If we assume (at least approximate) lepton universality we can compare the previous limits with the ones we have just obtained for the tau flavor which are $-0.89 \leq \hat{e}_S \leq 0.58$ and $-0.07 \leq \hat{e}_T \leq 0.72$ (when only the branching ratio is used) and $\hat{e}_S = (1.3 \pm 0.9) \times 10^{-2}$ and $\hat{e}_T = (-0.7 \pm 1.0) \times 10^{-2}$ (when the decay spectrum is also fitted). We verify that the $\tau^- \rightarrow (K\pi)^- \nu_\tau$ decays are not competitive with weak kaon decays in binding $\hat{e}_S$ and that the corresponding lepton universality test is quite modest. In the case of $\hat{e}_T$ (where the limits from both types of decays differ only by a factor 3) the comparison agrees with lepton universality. Our bounds are at the same level of hyperon decays [24,27], where $|\epsilon_S| < 4 \times 10^{-2}$ and $|\epsilon_T| < 5 \times 10^{-2}$ are found at a 90% confidence level. Therefore, while $\tau^- \rightarrow (K\pi)^- \nu_\tau$ decays cannot complement kaon decays concerning the bounds on $\hat{e}_S$ (neither hyperon decays do), the use of the three types of decays could allow us to improve the knowledge of $\hat{e}_T$ [provided more complete and better measurements of $\tau^- \rightarrow (K\pi)^- \nu_\tau$ decays become available and these improvements are matched on the theory side].

Our results $\hat{e}_S = (1.6 \pm 0.9) \times 10^{-2}$ and $\hat{e}_T = (0.9 \pm 1.0) \times 10^{-2}$ translate into bounds on the corresponding NP scale $\Lambda \sim 2\text{--}5$ TeV,²² assuming effective couplings of natural value at $\mu = \Lambda$ and accounting for the running of these coefficients on the renormalization scale μ [22,87]. These results are, of course, quite restricted compared to the NP reach of (semi)leptonic kaon decays, which can probe related scales as high as $\mathcal{O}(500)$ TeV [26] for nonstandard scalar interactions.

VI. CP VIOLATION

The observable A_{CP} , measured by BABAR [6] at odds with the SM prediction (tiny corrections from direct CP violation are neglected along this section), is defined as

$$A_{CP} = \frac{\Gamma(\tau^+ \rightarrow \pi^+ K_S \bar{\nu}_\tau) - \Gamma(\tau^- \rightarrow \pi^- K_S \nu_\tau)}{\Gamma(\tau^+ \rightarrow \pi^+ K_S \bar{\nu}_\tau) + \Gamma(\tau^- \rightarrow \pi^- K_S \nu_\tau)}. \quad (24)$$

In the SM, A_{CP} is saturated by the neutral kaon mixing contribution. Because of that, it is given by the analogous asymmetry measured in semileptonic kaon decays [12] ($\ell = e, \mu$),

$$\frac{\Gamma(K_L \rightarrow \pi^- \ell^+ \nu_\ell) - \Gamma(K_L \rightarrow \pi^+ \ell^- \bar{\nu}_\ell)}{\Gamma(K_L \rightarrow \pi^- \ell^+ \nu_\ell) + \Gamma(K_L \rightarrow \pi^+ \ell^- \bar{\nu}_\ell)} = 3.32(6) \times 10^{-3}, \quad (25)$$

up to small corrections given by the fact that the K_S is reconstructed at the B-factories through its two-pronged pion decay mode with a decay time of the order of the K_S lifetime, which modify the previous value to $A_{CP}^{\text{SM}} = 3.6(1) \times 10^{-3}$ [40], which is 2.8σ away from the BABAR measurement, $A_{CP} = -3.6(2.3)(1.1) \times 10^{-3}$.

In Ref. [34] it is shown that, in the presence of beyond-SM (BSM) interactions, A_{CP} is modified to

$$A_{CP} = \frac{A_{CP}^{\text{SM}} + A_{CP}^{\text{BSM}}}{1 + A_{CP}^{\text{SM}} \times A_{CP}^{\text{BSM}}}, \quad (26)$$

where, in our case [12]²³

$$A_{CP}^{\text{BSM}} = \frac{2 \sin \delta_T^W |\hat{e}_T| G_F^2 |V_{us}|^2 S_{\text{EW}}}{256 \pi^3 M_\tau^2 \Gamma(\tau \rightarrow K_S \pi \nu_\tau)} \int_{s_{\pi K}}^{M_\tau^2} ds |f_+(s)| |F_T(s)| \sin(\delta_+(s) - \delta_T(s)) \frac{\lambda^{3/2}(s, m_\pi^2, m_K^2)(M_\tau^2 - s)^2}{s^2}, \quad (27)$$

²¹We note that both signs for both coefficients are still possible at one standard deviation, so we find it natural that our analysis cannot be conclusive on these signs either.

²²Explicitly, $\Lambda \sim v(V_{us} \hat{e}_{S,T})^{-1/2}$, with $v = (\sqrt{2} G_F)^{-1/2} \sim 246$ GeV.

²³We recall that c_T in this reference equals $2\hat{e}_T$ in our notation.

where δ_T^W stands for the relative weak phase between the SM V-A and the tensor contributions. In Ref. [12], using the $SU(2)_L$ invariance of the weak interactions and the EFT machinery, stringent constraints were imposed on $\Im m[\hat{e}_T]$ by exploiting the measurements of $D - \bar{D}$ mixing and the upper limit on the electric dipole moment of the neutron. This results in the bound $2\Im m[\hat{e}_T] \lesssim 10^{-5}$, which we will use. To see that δ_T^W is a small parameter, we recall the limits

from the global EFT analysis of NP in kaon (semi)leptonic decays [26], according to which $|\epsilon_T| = (0.5 \pm 5.2) \times 10^{-3}$. Considering this, $\sin \delta_T^W |\hat{\epsilon}_T| \sim \Im m[\hat{\epsilon}_T]$ and the numerical evaluation of Eq. (27) is straightforward with the inputs at hand.

We have computed Eq. (27) using $|F_T(s)|$ obtained with $s_{\text{cut}} = M_\tau^2, 4, 9 \text{ GeV}^2$ (shown in the left panel of Fig. 1) and with $\delta_T(s)$ varying (smoothly) within the band shown in Fig. 2 of Ref. [12], as we find perfectly reasonable the estimation of this uncertainty.²⁴ The errors on $|F_+(s)|$ and $\delta_+(s)$ are negligible compared to the uncertainties on $F_T(s)$. Among these two types of uncertainties, the error on $\delta_T(s)$ dominates completely: changing s_{cut} for a given $\delta_T(s)$ can modify A_{CP}^{BSM} by a factor 3, at most, while, with a fixed s_{cut} , A_{CP}^{BSM} can be vanishing for $\delta_T(s) \rightarrow \delta_+(s)$ also in the inelastic region. In this way, we find

$$A_{CP}^{\text{BSM}} \lesssim 8 \times 10^{-7}, \quad (28)$$

which is a slightly weaker bound than the one reported in Ref. [12]: $A_{CP}^{\text{BSM}} \lesssim 3 \times 10^{-7}$. This small difference comes mainly from our accounting for the variation in s_{cut} and also for the slightly different phase $\delta_+(s)$ in both analyses. In any case, it is clear that heavy BSM interactions can only modify A_{CP} at the 10^{-6} level at most,²⁵ which is at least 3 orders of magnitude smaller than the theoretical uncertainty in its prediction (which is, in turn, some 25 times smaller than the error of the *BABAR* measurement). Therefore, any conclusive anomaly in A_{CP} must be explained outside the framework considered in this paper (and in Ref. [12]), for instance, by BSM effects of very light particles.

VII. SUMMARY AND CONCLUSIONS

In this paper, besides giving an extended set of SM predictions for a bunch of observables (Dalitz plots, decay spectrum, forward-backward asymmetry) and comparisons with effects of heavy NP parametrized in terms of a couple of Wilson coefficients for the nonstandard scalar and tensor interactions ($\hat{\epsilon}_{S,T}$) in $\tau^- \rightarrow (K\pi)^-\nu_\tau$ decays, we have focused on the three questions highlighted in Sec. I, whose answers we summarize in the following:

- (i) In agreement with Ref. [12], we confirm that it is not possible to understand within the low-energy limit of the SMEFT framework the *BABAR* measurement [6] of the CP asymmetry, which disagrees at 2.8σ with the SM prediction [40]. As a consequence of our

dedicated treatment of the uncertainties on the tensor form factor, we find a slightly weaker bound than in Ref. [12], $A_{CP}^{\text{BSM}} \lesssim 8 \times 10^{-7}$, which is anyway some 3 (5) orders of magnitude smaller than the theoretical uncertainty in its prediction (the error of the *BABAR* measurement). If the *BABAR* anomaly is confirmed, its explanation must be due to light NP. A determination of this quantity with Belle-I data, together with the future measurement at Belle-II [80], will shed light on this puzzle.

- (ii) Bins 5, 6 and 7 of the Belle measurement [41] of the $K_S\pi^-$ mass spectrum in $\tau^- \rightarrow K_S\pi^-\nu_\tau$ decays could not find an explanation using a scalar form factor obtained from the corresponding partial wave of a meson-meson scattering coupled channels analysis [43,53].²⁶ We have shown here, for the first time, that nonstandard scalar or tensor interactions produced by heavy NP are not capable of explaining these data points either. Again a caveat remains with respect to light NP effects, which are beyond the scope of this paper.
- (iii) Current branching ratio and spectrum measurements of the $\tau^- \rightarrow K_S\pi^-\nu_\tau$ decays restrict the NP effective couplings, ϵ_S and ϵ_T , as we have studied in this work for the first time. Our results are consistent with naive expectations: while the considered decays set bounds similar to those coming from hyperon semi-leptonic decays (which are at the level of a few TeV NP energy scale under reasonable assumptions), they are not competitive with (semi)leptonic kaon decays that could probe $\mathcal{O}(500)$ TeV NP generating nonstandard scalar interactions. However, we put forward that tensor interactions are probed in $\tau^- \rightarrow (K\pi)^-\nu_\tau$ decays with similar NP energy reach as in (semi)leptonic kaon and hyperon decays. Therefore, the corresponding comparisons for ϵ_T are meaningful tests of lepton universality and—under this assumption—tau decays can complement kaon and hyperon physics in restricting tensor interactions.

We expect that the analysis carried out in this work can serve as a motivation for the experimental tau physics groups at Belle(-II) to undertake the very dedicated work leading to the measurement of the observables considered in this paper. According to our discussion (see also Ref. [80]) the CP asymmetry and the $K\pi$ spectrum are the most attractive measurements but it should never be forgotten that the power of correlations among observables can enhance the limits on (most interestingly light) NP coming from only these two measurements. As an example, a measurement of the corresponding forward-backward asymmetry can reduce the uncertainties of the SM input for both meson form factors, thus allowing us to improve the limits in NP analyses.

²⁴The authors of Ref. [88] disagree with this and with the bound on $2\Im m[\hat{\epsilon}_T]$. See, however, Ref. [89], where NP bounds obtained from $\tau^- \rightarrow K^-\nu_\tau$ decays are also discussed.

²⁵A similar upper limit was set in Ref. [90] in the context of SUSY contributions to this observable. The recent paper [91] finds that a charged Higgs contribution can enhance CP violation in the $K^\pm\pi^0$ tau decay modes up to $A_{CP} \lesssim 2 \times 10^{-6}$, which is much larger than the corresponding (negligible) SM contribution [38].

²⁶The effect of the otherwise dominant vector form factor is kinematically suppressed in this region and can never give such a strong enhancement as observed in these data points.

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