Rapid Communications | Editors' Suggestion

Isospin breaking decays as a diagnosis of the hadronic molecular structure of the $P_c(4457)$

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The LHCb Collaboration announced the observation of three narrow structures consistent with hidden-charm pentaquark states. They are candidates of hadronic molecules formed of a pair of a charmed baryon and an anticharmed meson. Among them, the $P_c(4457)$ mass is consistent with earlier predictions of a $\Sigma_c\bar{D}^*$ molecule with $I = 1/2$. We point out that if such a picture were true, one would have $\mathcal{B}(P_c(4457) \to J/\psi\Delta^+)/\mathcal{B}(P_c(4457) \to J/\psi p)$ at the level ranging from a few percent to about 30%. Such a large isospin breaking decay ratio is two to three orders of magnitude larger than that for normal hadron resonances. It is a unique feature of the $\Sigma_c\bar{D}^*$ molecular model, and can be checked by LHCb.

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Four years after the discovery of the hidden-charm pentaquark-like states $P_c(4380)$ and $P_c(4450)$ [1], with the full set of Run-1 and Run-2 data the LHCb Collaboration announced the observation of more structures consistent with hidden-charm pentaquark states with masses and widths given by [2,3]

$$\begin{aligned}
M_{P_c(4312)^+} &= 4311.9 \pm 0.7^{+6.8}_{-0.6}\ \text{MeV},\\
\Gamma_{P_c(4312)^+} &= 9.8 \pm 2.7^{+3.7}_{-4.5}\ \text{MeV},\\
M_{P_c(4440)^+} &= 4440.3 \pm 1.3^{+4.1}_{-4.7}\ \text{MeV},\\
\Gamma_{P_c(4440)^+} &= 20.6 \pm 4.9^{+8.7}_{-10.1}\ \text{MeV},\\
M_{P_c(4457)^+} &= 4457.3 \pm 0.6^{+4.1}_{-1.7}\ \text{MeV},\\
\Gamma_{P_c(4457)^+} &= 6.4 \pm 2.0^{+5.7}_{-1.9}\ \text{MeV}.
\end{aligned} \tag{1}$$

That is, the $P_c(4450)$ reported earlier is split into two peaks corresponding to the $P_c(4440)$ and the $P_c(4457)$, and the small spike (sticking out in a single bin) at slightly above 4.3 GeV in the 2015 measurement is resolved into a pronounced peak with a 7.3σ significance. Pentaquark states with hidden-charm as hadronic molecules of a pair of a charmed baryon and an anticharmed meson were predicted to exist in this mass region prior to the LHCb observations [4–11]. In particular, the masses of the $P_c(4312)$ and $P_c(4457)$ are in a remarkable agreement with the predictions for the isospin $I = 1/2$ $\Sigma_c\bar{D}$ ($J^P = 1/2^-$) and $\Sigma_c\bar{D}^*$ ($J^P = 1/2^-$ or $3/2^-$) S-wave bound states in Ref. [8] where a coupled-channel formalism with the vector-meson exchange potential is used. The first observation in Ref. [1] inspired a flood of models for the P_c structures, such as the baryon–meson molecules [12–27], compact pentaquark states [28–35] and baryocharmonia [36], while the importance of triangle singularities, in particularly for the $P_c(4450)$, has also been discussed [37–40].[1] Reviews of these models can be found in Refs. [41–48]. Of particular interest here is the interpretation of the $P_c(4450)$ as an $I = 1/2$ $\Sigma_c\bar{D}^*$ molecular with $J^P = 3/2^-$ [14,15,17–19,23,24,26,27] (see also early predictions in Ref. [8]), which is adopted as the interpretation for the $P_c(4457)^+$ in Refs. [49,50]. We notice that such an interpretation will lead to large isospin breaking effects

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[1]It could be that the triangle singularities enhance the production of the P_c states at around 4.45 GeV.

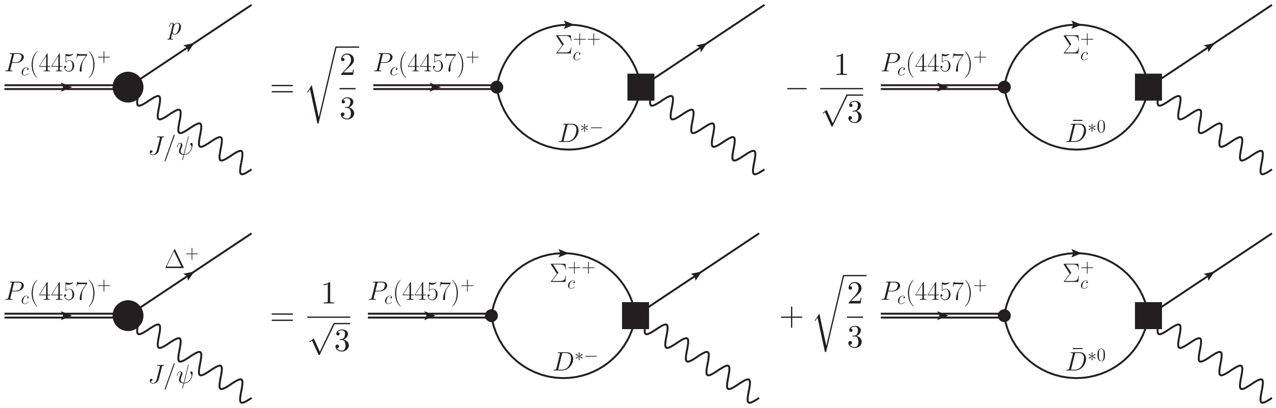

FIG. 1. Illustration of the decays of the $P_c(4457)^+ \to J/\psi p$ and $P_c(4457)^+ \to J/\psi \Delta^+$ through $\Sigma_c \bar{D}^*$ loops. Here the double lines represent the physical $P_c(4457)^+$ state.

in the decays. We have the following nearby $\Sigma_c \bar{D}^*$ thresholds:[2]

$$\begin{aligned} M_{\Sigma_c^+} + M_{\bar{D}^{*0}} &= 4459.8 \pm 0.4 \text{ MeV}, \\ M_{\Sigma_c^{++}} + M_{D^{*-}} &= 4464.23 \pm 0.15 \text{ MeV}. \end{aligned} \tag{2}$$

Thus, the binding energy of the $P_c(4457)^+$ with respect to the $\Sigma_c^+ \bar{D}^{*0}$ threshold, $2.5^{+1.8}_{-4.2}$ MeV, is sizably smaller than that with respect to the $\Sigma_c^{++} D^{*-}$ threshold, $6.9^{+1.8}_{-4.1}$ MeV. As a result, one would expect sizeable isospin breaking effects in the decays, similar to the case of the $X(3872)$ which decays with comparable rates into the $I = 0$ $J/\psi \pi^+\pi^-\pi^0$ and $I = 1$ $J/\psi \pi^+\pi^-$ final states,[3] though with a much more modest magnitude as will be shown in the following.

Since the isospin of the Σ_c is 1 and that of the $\bar{D}^*$ is $1/2$, one can form $I = 3/2$ and $I = 1/2$ states out of them,

$$\begin{aligned} \left|\Sigma_c \bar{D}^*; I = \frac{1}{2}, I_3 = \frac{1}{2}\right\rangle &= \sqrt{\frac{2}{3}} |\Sigma_c^{++} D^{*-}\rangle - \frac{1}{\sqrt{3}} |\Sigma_c^+ \bar{D}^{*0}\rangle, \\ \left|\Sigma_c \bar{D}^*; I = \frac{3}{2}, I_3 = \frac{1}{2}\right\rangle &= \frac{1}{\sqrt{3}} |\Sigma_c^{++} D^{*-}\rangle + \sqrt{\frac{2}{3}} |\Sigma_c^+ \bar{D}^{*0}\rangle. \end{aligned} \tag{3}$$

In the $\Sigma_c \bar{D}^*$ molecular picture, the decays of the $P_c(4457)^+$ into the $J/\psi p$ and $J/\psi \Delta^+$ dominantly proceed through the $\Sigma_c \bar{D}^*$ loops with the intermediate states carrying different electric charges, as shown in Fig. 1. We denote the S-wave coupling constant for the $P_c(4457)^+ \to \Sigma_c^+ \bar{D}^{*0}$ vertex as $g_{+,0}$ and that for the $P_c(4457)^+ \to \Sigma_c^{++} D^{*-}$ vertex as $g_{++,-}$. Assuming the $P_c(4457)$ to be a hadronic molecule generated from the $I = 1/2$ S-wave interaction between the $\Sigma_c \bar{D}^*$ pair, from the Lippmann-Schwinger equation (LSE), we have

$$T^{-1}_{1/2} = V^{-1}_{1/2} - G^{\Lambda}_{\Sigma_c \bar{D}^*} = 0 \tag{4}$$

when the energy equals to the mass of the P_c. Here, $T_{1/2}$ is the $I = 1/2$ $\Sigma_c \bar{D}^*$ scattering T-matrix, $V_{1/2}$ is the corresponding nonrelativistic potential, and $G^{\Lambda}_{\Sigma_c \bar{D}^*}$ is the $\Sigma_c \bar{D}^*$ two-body Green's function whose form is irrelevant here (it will be given below when it is used). In $G^{\Lambda}_{\Sigma_c \bar{D}^*}$, the isospin averaged masses for the Σ_c and $\bar{D}^*$ should be used. Now let us switch on isospin breaking and consider the two-channel ($\Sigma_c^{++} D^{*-}$, $\Sigma_c^+ \bar{D}^{*0}$) nonrelativistic system. Because the products of couplings are the residues of the T-matrix elements, i.e., $g^2_{++,-} = \text{Res}\, T_{++,-\to++,-}$ and $g_{++,-} g_{+,0} = \text{Res}\, T_{++,-\to+,0}$, we get from the two-channel LSE the following ratio (the energy is at the P_c mass),

$$\frac{g_{++,-}}{g_{+,0}} = \frac{2V_{1/2} + V_{3/2} - 3V_{1/2}V_{3/2}G^{\Lambda}_{+,0}}{-\sqrt{2}(V_{1/2} - V_{3/2})}, \tag{5}$$

where $V_{3/2}$ is the potential for the $I = 3/2$ $\Sigma_c \bar{D}^*$ scattering, $G^{\Lambda}_{+,0}$ is the two-body Green's function for $\Sigma_c^+ \bar{D}^{*0}$, and we have used Eq. (3) to express the particle-basis potentials in terms of the isospin-basis ones $V_{1/2}$ and $V_{3/2}$. From Eq. (4), we get $V_{1/2} G^{\Lambda}_{+,0} = 1 - V_{1/2}(G^{\Lambda}_{+,0} - G^{\Lambda}_{\Sigma_c \bar{D}^*})$, where the second term is an isospin breaking effect and is much smaller than 1. Therefore, Eq. (5) becomes

$$g_{++,-} \simeq -\sqrt{2} g_{+,0}. \tag{6}$$

Then from Fig. 1 one sees that in the isospin limit when all the masses in the same isospin multiplet are degenerate, the two loops exactly cancel with each other for the decay into the $I = 3/2$ final state $J/\psi \Delta^+$. The isospin splittings of the

[2] As noticed in Ref. [2], the mass of the $P_c(4457)^+$ coincides with the $\Lambda_c(2595)^+ \bar{D}^0$ threshold, 4457.09 ± 0.28 MeV.

[3] Several interesting similarities between the $P_c(4450)$ and the $X(3872)$, including the possibility of a sizeable isospin symmetry breaking, were discussed in Ref. [17].

intermediate particles make the transition possible. In order to estimate the size of the isospin breaking effect, we make use of the method of Ref. [51] which was developed for the $X(3872)$ (see also Refs. [52,53]).

The magnitudes of the three-momenta for the decays of the $P_c(4457)^+$ into $J/\psi p$ and $J/\psi\Delta^+$ are about 0.83 GeV and 0.52 GeV, respectively. They are much larger than the binding momenta which are 73 MeV and 124 MeV for the $\Sigma_c^+\bar{D}^{*0}$ and $\Sigma_c^{++}D^{*-}$, respectively (here the central values of all involved masses are used). Thus, these decays are short-distance processes, and the decay rates would be determined by the wave function at the origin.

The wave function at the origin for a two-body component (labeled by i) of a physical state with a mass M is given by

$$\psi_i(r=0) = \int \frac{d^3\vec{q}}{(2\pi)^3}\langle\vec{q}|\psi\rangle = -2\mu_i\int\frac{d^3\vec{q}}{(2\pi)^3}\frac{\langle\vec{q}|\hat{V}_i|\psi\rangle}{\gamma_i^2+\vec{q}^2}, \tag{7}$$

where we have used the Schrödinger equation $(\vec{q}^2/(2\mu_i)+\hat{V}_i)|\psi\rangle = (M-m_1-m_2)|\psi\rangle$, and the binding momentum is defined as $\gamma_i = \sqrt{2\mu_i(m_1+m_2-M)}$, with $m_{1,2}$ the masses of the constituents and $\mu_i = m_1m_2/(m_1+m_2)$ the reduced mass. Since the physical state is nearby the threshold, one can approximate the S-wave vertex form factor $\langle\vec{q}|\hat{V}_i|\psi\rangle$ by the coupling constant g_i. Then, one gets

$$\psi_i^\Lambda(r=0) = -2\mu_i g_i\int\frac{d^3\vec{q}}{(2\pi)^3}\frac{\exp(-2\vec{q}^2/\Lambda^2)}{\gamma_i^2+\vec{q}^2} \equiv g_i G_i^\Lambda, \tag{8}$$

where a Gaussian form factor with a cutoff Λ is introduced to regularize the ultraviolet divergence, and G_i^Λ is simply the nonrelativistic two-point scalar loop integral evaluated at the mass of the state. When $M < m_1+m_2$, it is given by

$$G_i^\Lambda = -\frac{\mu_i\Lambda}{(2\pi)^{3/2}} - \frac{\mu_i\gamma_i}{2\pi}e^{2\gamma_i^2/\Lambda^2}\left[\mathrm{erf}\left(\frac{\sqrt{2}\gamma_i}{\Lambda}\right)-1\right], \tag{9}$$

where $\mathrm{erf}(x)$ is the error function.

Thus, for the $P_c(4457)^+$ we have

$$\psi_{++,-}^\Lambda(r=0) = g_{++,-}G_{++,-}^\Lambda, \quad \text{and}$$
$$\psi_{+,0}^\Lambda(r=0) = g_{+,0}G_{+,0}^\Lambda, \tag{10}$$

for the $\Sigma_c^{++}D^{*-}$ and $\Sigma_c^+\bar{D}^{*0}$ components, respectively. From Eq. (3), the isospin $I=1/2$ and $I=3/2$ components are

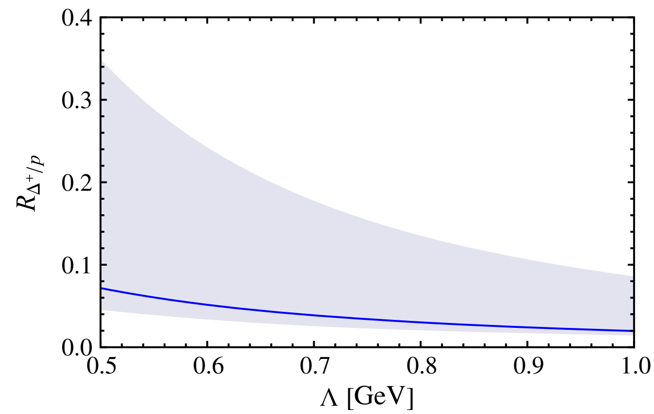

FIG. 2. Dependence of the ratio $R_{\Delta^+/p}$ on the cutoff. The solid line corresponds to the result calculated using the central values of all the involved masses, and the band reflects the uncertainties of the masses.

$$\psi_{1/2}^\Lambda(r=0) = \sqrt{\frac{2}{3}}\psi_{++,-}^\Lambda(r=0) - \frac{1}{\sqrt{3}}\psi_{+,0}^\Lambda(r=0),$$
$$\psi_{3/2}^\Lambda(r=0) = \frac{1}{\sqrt{3}}\psi_{++,-}^\Lambda(r=0) + \sqrt{\frac{2}{3}}\psi_{+,0}^\Lambda(r=0). \tag{11}$$

In view that the Δ resonances and the nucleons are in the same spin-flavor multiplet in the large N_c limit (see, e.g., Ref. [54]), one gets the following relation for the decay amplitudes

$$\frac{|\mathcal{A}(P_c(4457)^+\to J/\psi\Delta^+)|}{|\mathcal{A}(P_c(4457)^+\to J/\psi p)|} \simeq \sqrt{10}\left|\frac{\psi_{3/2}^\Lambda(r=0)}{\psi_{1/2}^\Lambda(r=0)}\right| = 2\sqrt{5}\left|\frac{G_{++,-}^\Lambda - G_{+,0}^\Lambda}{2G_{++,-}^\Lambda + G_{+,0}^\Lambda}\right|, \tag{12}$$

where the factor of $\sqrt{10}$ comes from the spin-flavor matrix elements worked out in Ref. [17] [see Eqs. (17,18) therein], and Eq. (6) has been used. We have further assumed that $V_{3/2}$ is much smaller than $V_{1/2}$ so that we can neglect contribution from the isospin breaking effect in Eq. (6) here. This is plausible in the molecular picture as the $I=1/2$ interaction needs to be strong to produce the $P_c(4457)$ as a $\Sigma_c\bar{D}^*$ bound state.[4] From this equation, and taking into account the S-wave phase spaces for the decays of the $P_c(4457)^+$ into $J/\psi p$ and $J/\psi\Delta^+$, one can predict the isospin breaking ratio

$$R_{\Delta^+/p} \equiv \frac{\mathcal{B}(P_c(4457)^+\to J/\psi\Delta^+)}{\mathcal{B}(P_c(4457)^+\to J/\psi p)}, \tag{13}$$

[4]For the $X(3872)$ in the $D\bar{D}^*+$c.c. hadronic molecular picture, the $I=1$ potential is indeed much weaker than the $I=0$ one, see, e.g., Ref. [55].

and the result is shown in Fig. 2 with the cutoff Λ in the region from 0.5 GeV to 1 GeV. One sees that the ratio ranges from a few percent to as large as 30% with the large uncertainty mainly from the uncertainty of the $P_c(4457)^+$ mass. It is two to three orders of magnitude larger than the isospin breaking effects for the decays of normal hadron resonances. In order to see that, one notices that there are two sources of isospin breaking: the up and down quark mass difference, and the electromagnetic interactions (virtual photons). They give amplitudes of the order of $(m_d - m_u)/\Lambda_{\mathrm{QCD}}$ and α, respectively, where Λ_{QCD} is the nonperturbative scale in quantum chromodynamics and α is the fine structure constant. Both of them are of $\mathcal{O}(10^{-2})$, and thus lead to a suppression for the branching fractions of $\mathcal{O}(10^{-4})$. To give an example, the ratio of the branching fraction of the decay of an isoscalar state into another isoscalar and a π^0 over that into the same isoscalar and an η is given by $\epsilon^2_{\pi^0\eta}$ up to the phase space factor. The isospin breaking $\pi^0 - \eta$ mixing angle is

$$\begin{aligned}\epsilon_{\pi^0\eta} &= \frac{\sqrt{3}}{2}\frac{m_d - m_u}{2m_s - m_u - m_d}\\ &\simeq \frac{\sqrt{3}}{2}\frac{M^2_{K^0} - M^2_{K^\pm} - M^2_{\pi^0} + M^2_{\pi^\pm}}{M^2_{K^0} + M^2_{K^\pm} - M^2_{\pi^0} - M^2_{\pi^\pm}} \simeq 0.01, \qquad (14)\end{aligned}$$

where the combinations of meson masses are constructed such that the virtual photon effects are canceled out.

To summarize, in this paper we propose that the structure of the $P_c(4457)$ can be diagnosed using isospin breaking decays. If the $P_c(4457)^+$ is an S-wave $\Sigma_c\bar{D}^*$ hadronic molecule with $I = 1/2$, which implies that it couples most strongly to the $\Sigma_c\bar{D}^*$ channels, then because its mass is closer to the $\Sigma_c^+\bar{D}^{*0}$ threshold than to the $\Sigma_c^{++}D^{*-}$ one, one expects large isospin breaking effects in its decays. A quantitative estimate of the ratio $\mathcal{B}(P_c(4457)^+ \to J/\psi\Delta^+)/\mathcal{B}(P_c(4457)^+ \to J/\psi p)$ gives a value ranging from $\mathcal{O}(10^{-2})$ to about 30%, where the large uncertainty comes mainly from the mass of the $P_c(4457)^+$. It is two to three orders of magnitude higher than the isospin breaking effects for the decays of normal hadron resonances. It is worthwhile to mention that the large isospin breaking effect is a key to unveiling the nature of the $D^*_{s0}(2317)$, whose isospin breaking decay width is about 100 keV [56–60] in the DK molecular picture and is one order of magnitude smaller [61,62] if it couples weakly to the DK (for detailed discussions, see Ref. [44]). Therefore, we suggest to search for the $P_c(4457)^+$ ($P_c(4457)^0$) in the $J/\psi\Delta^+(J/\psi\Delta^0)$ mode. Given the large ratio, it is feasible at the LHCb experiment.

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