Study of unflavored light mesons with $J^{PC} = 2^{--}$

Dan Guo,^{1,3,‡,¶} Cheng-Qun Pang,^{2,§,¶} Zhan-Wei Liu,^{1,3,*} and Xiang Liu^{1,3,†}

¹*School of Physical Science and Technology, Lanzhou University, Lanzhou 730000, China*

²*College of Physics and Electronic Information Engineering, Qinghai Normal University, Xining 810000, China*

³*Research Center for Hadron and CSR Physics, Lanzhou University and Institute of Modern Physics of CAS, Lanzhou 730000, China*

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The unflavored light meson families, namely, ω_2 , ρ_2 , and ϕ_2 , are studied systematically by investigating the spectrum and the two-body strong decays allowed by the Okubo-Zweig-Iizuka rule. Including the four experimentally observed states and other predicted states, phenomenological analysis of the partial decay widths can verify the corresponding assignments of these states into the families. Moreover, we provide typical branching ratios of the dominant decay channels, especially for missing ground states, which is helpful to search for or confirm them and explore more properties of these families in experiments.

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I. INTRODUCTION

Since the quark model was proposed in 1964 [1,2], much progress on the study of the light hadron spectrum has been made by joint efforts from both theorists and experimentalists [3]. Among the light hadron families, light mesons are an indispensable part and are still unclear, which is the main reason why investigating and exploring light mesons has become an interesting physical aim of present running experiments like BESIII and COMPASS and forthcoming experiments like GlueX and PANDA.

In the past years, there exist abundant phenomenological works that are involved in pseudoscalar mesons associated with these observed $X(1835)$, $X(2120)$, $X(2370)$, and $X(2500)$ [4,5]; vector mesons with the reported $Y(2175)$ [6,7]; ρ and ρ_3 mesons [7,8]; axial vector mesons [9,10]; tensor mesons [11–13]; pseudotensor mesons [14,15]; kaons [16]; and higher spin mesons [17]. These studies have played a crucial role not only in establishing the light meson spectrum, but also in enlarging our understanding of these new hadron states like $X(1835)$ and $Y(2175)$.

When further checking the experimental status of light mesons [3], we notice an interesting phenomenon; i.e., the ground states of unflavored light meson families with $J^{PC} = 2^{--}$ are still absent, their higher excited states are not well established yet, and they are only collected into “further states” in the Particle Data Group (PDG) [3]. This phenomenon stimulates our interest in exploring unflavored light mesons with $J^{PC} = 2^{--}$ thoroughly.

According to the isospin, unflavored light mesons with $J^{PC} = 2^{--}$ can be categorized into three groups, which are the isovector ρ_2 meson family and the isoscalar ω_2 and ϕ_2 meson families. Though experimental information about 2^{--} unflavored light ground mesons is totally barren, their counterparts, the 1^{--} and 3^{--} mesons, are properly recognized. By comparing the similarity of them, the mass spectrum of ω_2 , ρ_2 , and ϕ_2 ground states is successfully constructed. In addition, we predict the behaviors of two-body strong decays allowed by the Okubo-Zweig-Iizuka (OZI) rule, which are the key information explaining the reason for their disappearance. Here, potential decay channels and typical branching ratios which are valuable to explore in future experiments are also provided. On the other hand, combining with the theoretical investigation of the K_2^* meson [17], we can find an integrated nonet and then systematically compare with 1^{--} and 3^{--} light mesons.

In PDG [3], we can find $\rho_2(1940)$, $\omega_2(1975)$, $\omega_2(2195)$, and $\rho_2(2225)$ as further states in the $p\bar{p}$ reaction. Based on the experimental information of these four 2^{--} states, we continue to discuss their possible assignments into ρ_2 and ω_2 meson families with the mass spectrum analysis and a two-body OZI-allowed strong decay calculation. Also, we predict the properties of the corresponding undiscovered ϕ_2 mesons. Additionally, masses and decay behaviors of third

*Corresponding author.
liuzhanwei@lzu.edu.cn

†Corresponding author.
xiangliu@lzu.edu.cn

‡guod13@lzu.edu.cn

§xuehua45@163.com

¶These authors contributed equally to this work and should be considered co-first authors.

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radial excitations are also calculated based on previous results. We hope that our effort will be helpful to establish ρ_2 , ω_2 , and ϕ_2 meson families.

Comparing with other theoretical calculations [18–20], not only a more detailed analysis of the missing ground states but also a theoretical study of the four newly observed resonances are included in this work. Moreover, based on the consistency of experimental information, we predict the decay properties of the third excitation states.

We organize this paper as follows. In Sec. II, the mass spectrum of the unflavored light meson is studied, where one can fix the mass range of the ground state in unflavored light meson families with $J^{PC} = 2^{--}$ and give the possible assignments of radial excitation states. In Sec. III, we first introduce the quark-pair-creation (QPC) model, and then we discuss the strong decay behaviors of these discussed states with a fixed mass spectrum. In the end, Sec. IV is devoted to a summary and discussion.

II. ANALYSIS OF MASS SPECTRUM

Although the light isoscalar and isovector states with $^{2S+1}L_J = ^3D_1$ or 3D_3 are relatively well established and categorized because of many discovered candidates [6,8], only four unflavored light 3D_2 mesons have been discovered in experiments. By using the Crystal Barrel detector's data, four 2^{--} resonances were reported for $p\bar{p}$ collision in 2002 [21,22], whose masses and widths are listed as follows:

$$\begin{aligned}\rho_2(1940): & m = 1940 \pm 40 \text{ MeV}, \Gamma = 155 \pm 40 \text{ MeV}; \\ \rho_2(2225): & m = 2225 \pm 35 \text{ MeV}, \Gamma = 335_{-50}^{+100} \text{ MeV}; \\ \omega_2(1975): & m = 1975 \pm 20 \text{ MeV}, \Gamma = 175 \pm 25 \text{ MeV}; \\ \omega_2(2195): & m = 2195 \pm 30 \text{ MeV}, \Gamma = 225 \pm 40 \text{ MeV}.\end{aligned}$$

According to our previous works [6,8] and the review of PDG [3], the accompanying isoscalar $\omega(1650)$ and isovector $\rho(1700)$ are generally considered as 3D_1 ground states and the isoscalar $\omega_3(1670)$ and isovector $\rho_3(1690)$ as 3D_3 ground states. Those are consistent with the rough estimations as follows. Because of the same spin angular momentum and orbital angular momentum, the masses of ω_2 and ρ_2 ground states should be around 1.7 GeV by comparing the masses of the 3D_1 state and 3D_3 state. Similarly, the mass of the ϕ_2 ground state is about 1.9 GeV by comparing that of $\phi_3(1850)$.

In most situation, the Godfrey-Isgur (GI) model [23] is used for mass spectrum analysis and has achieved great success since it was proposed. Here, we employ this model to obtain masses of ground states.

The Regge trajectory is another effective approach to quantitatively study the mass spectrum of the radial excited light mesons [24,25]. As to higher excitations, masses of GI model calculation generally are larger than experimental results because of the coupled channel effect and the

relativity effect. Thus, we use the Regge trajectory to obtain the masses of the radical excitation states. A general expression for the Regge trajectory is

$$M^2 = M_0^2 + (n-1)\mu^2, \quad (2.1)$$

where M_0 is the mass of ground state, M is the mass of state with radical excitation number n , and μ^2 denotes the trajectory slope.

A general mass assignment of unflavored light mesons with 3D_2 is shown in Fig. 1, and the four experimental states are well arranged as the $n = 2$ or 3 state. Here, we need to specify that the masses of $\rho_2(1D)$, $\omega_2(1D)$, and $\phi_2(1D)$ are taken from the GI model calculation [23]. By combining these theoretical inputs and experimental data, we can construct three Regge trajectories as shown in Fig. 1, by which we can further predict the masses of other missing states. As shown in Fig. 1, linear typical Regge trajectories of the ω_2 and ρ_2 family are observed, and we can obtain the slope $\mu^2 = 1.0 \text{ GeV}^2$, which is gotten by fitting the experimental and the GI ground state data. Moreover, when we take the mass of the ground state $\phi_2(1D)$ from the GI model, the masses of the ϕ_2 family can also be obtained with the extrapolation from the Regge trajectory using the same slope, $\mu^2 = 1.0 \text{ GeV}^2$.

Besides these ω_2 , ρ_2 , and ϕ_2 mesons, there exist the corresponding K_2 partners. In our previous work [16], the resonance parameter and partial decay width of these K_2^* mesons were calculated. An integrated nonet is established as shown in Fig. 2 and thus we can have a complete comprehension of 2^{--} light mesons. The masses of unflavored mesons with different radical number n are summarized in Table I, and these masses are employed in

FIG. 1. Regge trajectories of the ω_2 , ρ_2 , and ϕ_2 families. Slope μ^2 of trajectories is 1.0 GeV^2 . Here, the empty circles (or diamonds) represent the theoretical values of ground states (or radical excitations) calculated by the GI model (or Regge trajectory). The cyan triangles denote the experiment data.

FIG. 2. The 2^{--} nonet.

the following study for the decay widths. On the other hand, an analysis of decay behaviors will examine the reasonability of our assignments.

Presently, in addition to the study of ground state masses from Godfrey and Isgur [23] and Ebert *et al.* [20], there are also two other works [18,19] indicating that the ground state masses of ω_2 and ρ_2 are ~ 1.7 GeV, comparable with ours. Furthermore, Ebert *et al.* also predicted the masses of the first and second excitations, and the masses of the ω_2 and ρ_2 families are generally consistent with ours, but the masses of ϕ_2 excitations are higher than ours. In the next section, we give a complete analysis of strong decay behaviors based on the well-established mass assignments.

III. STRONG DECAY BEHAVIORS

In this section, we firstly introduce the phenomenological model applied to obtain the information for the OZI-allowed hadronic strong decays. We then illustrate strong decay behaviors of ω_2 , ρ_2 , and ϕ_2 families in detail, respectively.

A. A brief introduction to the QPC model

The QPC model was first proposed by Micu [26] in 1968 and developed by the Orsay group [27–31]. The QPC model assumes that a quark-antiquark pair $q\bar{q}$ is created from vacuum with $J^{PC} = 0^{++}$ and then rearranged with the initial hadron to form two daughter hadrons. For instance, the transition operator $\mathcal{T}$ of a meson decay progress $A \rightarrow B + C$ can be expressed as

$$\mathcal{T} = -3\gamma \sum_m \langle 1m; 1-m | 00 \rangle \int d^3\mathbf{p}_3 d^3\mathbf{p}_4 \delta^3(\mathbf{p}_3 + \mathbf{p}_4) \times \mathcal{Y}_{1m} \left(\frac{\mathbf{p}_3 - \mathbf{p}_4}{2} \right) \chi_{1,-m}^{34} \phi_0^{34} \omega_0^{34} b_3^\dagger(\mathbf{p}_3) d_4^\dagger(\mathbf{p}_4), \quad (3.1)$$

TABLE I. The masses of the ω_2 , ρ_2 , and ϕ_2 families in units of MeV.

n	ω_2	ρ_2	ϕ_2
1	1696	1696	1904
2	1975	1940	2151
3	2195	2225	2372
4	2424	2424	2574

where $\mathcal{Y}_{lm}(\mathbf{p}) = |\mathbf{p}|^l Y_{lm}(\theta_p, \phi_p)$ is the solid spherical harmonic polynomial, and $\mathbf{p}_3$ and $\mathbf{p}_4$ depict the momenta of the quark and antiquark created from vacuum. $b_3^\dagger$ ($d_4^\dagger$) denotes the quark (antiquark) creation operator. $\chi_{1,-m}^{34}$, ϕ_0^{34} , and ω_0^{34} denote the spin triplet, flavor singlet, and color singlet wave functions, respectively. The dimensionless parameter γ describes the quark pair creation strength, which can be fitted by the experimental width data. From our fitted results of 2^{--} light states with minimal $\chi^2 = 17.8$ [$\chi^2 = \sum_i (\text{Theo} - \text{Exp})^2 / \text{Error}^2$], $\gamma = 7.1$ for the $u\bar{u}$ ($d\bar{d}$) pair creation, and the $s\bar{s}$ quark pair creation strength sets as $7.1/\sqrt{3}$ [31].

Then the transition matrix of the decay process in the rest frame of particle A reads as

$$\langle BC | \mathcal{T} | A \rangle = \delta^3(\mathbf{P}_B + \mathbf{P}_C) \mathcal{M}^{M_{J_A} M_{J_B} M_{J_C}}, \quad (3.2)$$

where $\mathbf{P}_B$ and $\mathbf{P}_C$ depict the momenta of mesons B and C , respectively, and $\mathcal{M}^{M_{J_A} M_{J_B} M_{J_C}}$ is the decay amplitude with M_{J_i} ($i = A, B, C$) describing the magnetic quantum number of the meson.

The helicity amplitude $\mathcal{M}^{M_{J_A} M_{J_B} M_{J_C}}$ can be converted into partial wave amplitude $\mathcal{M}^{JL}$ by the Jacob-Wick formula [32], i.e.,

$$\mathcal{M}^{JL}(\mathbf{P}) = \frac{\sqrt{2L+1}}{2J_A+1} \sum_{M_{J_B} M_{J_C}} \langle L0; JM_{J_A} | J_A M_{J_A} \rangle \times \langle J_B M_{J_B}; J_C M_{J_C} | J_A M_{J_A} \rangle \mathcal{M}^{M_{J_A} M_{J_B} M_{J_C}}. \quad (3.3)$$

To sum up, the general decay width can be written as

$$\Gamma_{BC} = \frac{\pi}{4} \frac{|\mathbf{P}| \mathcal{S}}{m_A^2} \sum_{J,L} |\mathcal{M}_{BC}^{JL}(\mathbf{P})|^2, \quad (3.4)$$

where $\mathcal{S} \equiv 1/(1 + \delta_{BC})$ denotes the statistic factor which is responsible for a situation if B and C are identical particles, and m_A is the mass of the initial particle.

Furthermore, the meson wave function is defined as a mock state, i.e.,

$$|D(n^{2S+1} L_{JM_J})(\mathbf{P}_D)\rangle = \sqrt{2E_D} \sum_{M_S, M_L} \langle LM_L; SM_S | JM_J \rangle \times \int d^3\mathbf{p}_D \chi_{S, M_S}^D \phi^D \omega^D \Psi_{nLM_L}^D(\mathbf{p}_D) \times \left| q_1 \left(\frac{m_1}{m_1 + m_2} \mathbf{P}_D + \mathbf{p}_D \right) \bar{q}_2 \left(\frac{m_1}{m_1 + m_2} \mathbf{P}_D - \mathbf{p}_D \right) \right\rangle, \quad (3.5)$$

and here the spacial wave function $\Psi_{nLM_L}(\mathbf{p})$ of the meson adopts the simple harmonic oscillator (SHO) wave function which has the explicit form

$$\Psi_{nLM_L}(\mathbf{p}) = R_{nL}(p, \beta) Y_{LM_L}(\Omega_p), \quad (3.6)$$

$$R_{nL}(p, \beta) = \frac{(-1)^n (-i)^L}{\beta^{3/2}} e^{-\frac{p^2}{2\beta^2}} \sqrt{\frac{2n!}{\Gamma(n+L+3/2)}} \left(\frac{p}{\beta}\right)^L \times L_n^{L+1/2}\left(\frac{p^2}{\beta^2}\right), \quad (3.7)$$

where $Y_{LM_L}(\Omega)$ is the spherical harmonic function and $L_{n-1}^{L+1/2}(x)$ is the associated Laguerre polynomial. The parameters $\beta = 1/R$ and R are obtained by reproducing the realistic root mean square radius via solving the Schrödinger equation (see more details in Ref. [33], though we proceed with some improvements of the method and results).

In the following subsections, we perform phenomenological analysis of the total widths and partial widths calculated by the QPC model by comparing our results with experimental data. This analysis is helpful to explain why no information about 2^{--} unflavored light ground states exists and to reveal the underlying properties of these

ground states and their radical excitations for future experimental measurements, especially in the BESIII detector, CMD-3, SND, and KEDR detector.

B. ω_2 family

As shown in Fig. 3, the calculated total widths of ω_2 family states are illustrated by contour lines which depend on R and γ . At present, experiments have only observed $\omega_2(1975)$, $\omega_2(2195)$, $\rho_2(1940)$, and $\rho_2(2225)$, where their widths were measured. Thus, we may take these experimental data to fix the γ value to be 7.1. Considering the uncertainty, we set $\gamma = 7.1 \pm 0.3$ in our concrete calculation. Generally, the total widths of $\omega_2(2D)$ and $\omega_2(3D)$ in our results agree well with those of $\omega_2(1975)$ and $\omega_2(2195)$ in experiments. It demonstrates the reasonability of setting the two states as $\omega_2(2D)$ and $\omega_2(3D)$, respectively, and also predicts the existence of an undiscovered ground state.

By assuming the ground state $\omega_2(1D)$ with mass equal to 1696 MeV, the total decay width of the missing $\omega_2(1D)$ reaches up to 220 MeV when choosing the fitted $\gamma = 7.1$ and recommended R value [33] shown in the top left of Fig. 3. We present the branching ratios of $\omega_2(1D)$ at the top of Fig. 4. The corresponding partial decay width can be obtained by multiplying the total width and branching ratio. The largest branching ratio comes from the $\pi\rho$ channel and

FIG. 3. The total widths of the ω_2 family ($n = 1$ to 4) with dependence on R and γ . Different colors correspond to different width values and the contour curves are marked with the width values. In particular, the red dashed contour lines mean the experimental widths, and the red longitudinal solid lines depict the R dependence when $\gamma = 7.1$.

FIG. 4. The branching ratios of $\omega_2(1D)$ and $\omega_2(1975)$ with R dependence. Decay channels with branching ratios less than 1% are neglected.

is about 71%. The $\pi b_1(1235)$, $\omega\eta$, and KK^* channels are also important. Also, all branching ratios are not strongly dependent on R , and those of the $\pi b_1(1235)$ and $\omega\eta$ channels are close to each other.

The information of stable partial width ratios will be helpful for experimental searches, and we list them for $\omega_2(1D)$: $\Gamma(\pi\rho)/\Gamma(\pi b_1(1235)) = 5.58\text{--}6.33$, $\Gamma(\pi\rho)/\Gamma(\omega\eta) = 5.93\text{--}6.19$, $\Gamma(\pi\rho)/\Gamma(KK^*) = 12.9\text{--}13.4$, and $\Gamma(\omega\eta)/\Gamma(KK^*) = 2.16\text{--}2.18$. In Refs. [18,19], the similar branching ratios of $\pi\rho$ are obtained, where the branching ratio of this $\pi\rho$ decay channel for $\omega_2(1D)$ is 74% and 60%, given by Refs. [18,19], respectively, and the other branching ratios are also analogous. As mentioned in Ref. [19], $\pi_2(1670)$ is with the mass and total width similar to $\omega_2(1D)$, and also decays to $\pi\rho$, so that $\omega_2(1D)$ is possibly masked by $\pi_2(1670)$ in the $\pi\rho$ channel. Therefore, $\omega\eta$ and KK^* are the ideal channels suggested to search for $\omega_2(1D)$. The process $\omega_2(1D) \rightarrow b_1(1235)\pi \rightarrow \omega\pi\pi$ is also valuable since $\omega_2(1975)$ and $\omega_2(2195)$ are discovered in the $\omega\pi\pi$ channel.

The structures of $\omega_2(1975)$ and $\omega_2(2195)$ are well determined by both the $p\bar{p} \rightarrow \omega\eta$ and $p\bar{p} \rightarrow \omega\pi\pi$ processes with ω decaying to $\pi^+\pi^-\pi^0$ [22]. The two resonances are classified as further states in PDG, which means that these states are not confirmed well in experiments and more measurements are needed. Thus, a detailed analysis of their categorization in nonet and decay properties is valuable to confirm them.

According to the analysis of the Regge trajectory in the previous section, $\omega_2(1975)$ is assigned to $n = 2$, whose total width and branching ratios of calculated two-body decays are presented in the top right of Fig. 3 and at the bottom of Fig. 4, respectively. The experimental total width of $\omega_2(1975)$ is 175 ± 25 MeV, which is well reproduced in our results.

As to the partial widths of $\omega_2(1975)$, our calculation shows that $\pi\rho(1450)$ is dominant decay channel and the branching ratio reaches up to 38%–49%. $\pi\rho$, $\pi b_1(1235)$, and $\pi\rho_3(1690)$ are also main modes, and $\omega\eta$ is also considerable. Corresponding to our calculation, the $\pi\rho(1450)$, $\pi b_1(1235)$, and $\pi\rho_3(1690)$ modes can contribute to the $\omega\pi\pi$ final state, and the $\omega\eta$ mode also shows a significant proportion. It can explain why $\omega_2(1975)$ has been discovered in the $p\bar{p} \rightarrow \omega\eta$ and $p\bar{p} \rightarrow \omega\pi\pi$ processes by the Crystal Barrel Collaboration. Moreover, we can give partial decay width ratios which are stable against the R value: $\Gamma(\pi\rho(1450))/\Gamma(\pi\rho) = 1.27\text{--}2.22$, $\Gamma(\pi b_1(1235))/\Gamma(\pi\rho) = 0.23\text{--}0.52$, $\Gamma(\pi\rho_3(1690))/\Gamma(\pi b_1(1235)) = 0.11\text{--}2.76$, $\Gamma(\omega\eta)/\Gamma(\pi\rho) = 0.140\text{--}0.145$, $\Gamma(\omega\eta)/\Gamma(KK^*) = 1.71\text{--}2.06$, $\Gamma(\omega\eta)/\Gamma(\omega\eta') = 2.37\text{--}4.74$, and $\Gamma(\pi\rho(1700))/\Gamma(KK^*(1410)) = 1.05\text{--}1.32$. In particular, the ratio $\Gamma(\omega\eta)/\Gamma(\omega\eta')$ is promising to explore. We believe that more experimental tests of these ratios can examine the rationality of setting $\omega_2(1975)$ as a 2^3D_2 state.

Another $J^{PC} = 2^{--}$ state $\omega_2(2195)$ observed in Ref. [22] is categorized as $n = 3$, and its experimental total width is 225 ± 40 MeV. A good agreement of our total width calculation with experimental data is observed in the lower left of Fig. 3. The R dependence of branching ratios for $\omega_2(3D)$ is illustrated in the left of Fig. 5. The main decay channels are $\pi\rho(1450)$, $\pi\rho$, and $\pi b_1(1235)$, whose branching ratios are 27.5%–33.6%, 25.2%–25.4%, and 14.5%–21.6%, respectively. The $\pi\rho(1450)$ and $\pi b_1(1235)$ channels give contributions to $\omega\pi\pi$, and $\omega\eta$ also has a moderate percentage share. Thus, it can be understood that $\omega_2(2195)$ is observed in these two final states. Here the decay width ratios weakly related to R are listed: $\Gamma(\pi\rho(1450))/\Gamma(\pi\rho) = 1.08\text{--}1.33$, $\Gamma(\pi\rho)/\Gamma(\pi b_1(1235)) = 1.18\text{--}1.74$, $\Gamma(\pi\rho(1450))/\Gamma(\pi b_1(1235)) = 1.27\text{--}2.31$, $\Gamma(\omega\eta)/\Gamma(\omega(1420)\eta) = 0.94\text{--}1.59$, $\Gamma(\omega\eta)/\Gamma(KK^*(1410)) = 1.34\text{--}1.85$, $\Gamma(KK^*(1410))/\Gamma(\pi\rho(1700)) = 1.06\text{--}1.28$, $\Gamma(\omega\eta)/\Gamma(KK^*) = 1.84\text{--}2.04$, $\Gamma(\omega\eta)/\Gamma(\omega\eta') = 1.91\text{--}2.38$, $\Gamma(KK^*)/\Gamma(\omega\eta') = 1.04\text{--}1.16$, and $\Gamma(\eta h_1(1170))/\Gamma(KK_2^*(1430)) = 1.03\text{--}1.06$. Among these ratios, $\Gamma(\omega\eta)/\Gamma(\omega\eta')$ is suggested to study first in experiments. Owing to the main channels contributing to the 5π final state, these modes are inefficient to reconstruct in experiments. Generally speaking, the total and partial widths depend more sharply on R for higher radial excited states, and a larger radial quantum number takes a larger R value.

Since there is no experimental signal of $\omega_2(4D)$, we predict its mass as 2424 MeV, and we plot the total width

FIG. 5. The branching ratios of $\omega_2(2195)$ and $\omega_2(4D)$ with R dependence. Decay channels with branching ratios less than 1% are neglected.

and branching ratios in the bottom right corner of Fig. 3 and the right column of Fig. 5, respectively. When choosing the recommended R and γ values, the total width of $\omega_2(4D)$ is about 130 MeV. Among the three main decay channels, $\pi\rho(1450)$ linearly relies on the R value whose branching ratio reaches up to 20.7%–26.9%, while those of $\pi\rho$ and $\pi b_1(1235)$ are 24.9%–25.7% and 23.0%–24.0%, respectively. Here, the corresponding relative ratios are also given, $\Gamma(\pi\rho)/\Gamma(\pi b_1(1235)) = 1.00\text{--}1.12$, $\Gamma(\pi\rho)/\Gamma(\pi\rho(1450)) = 0.95\text{--}1.20$, $\Gamma(\pi b_1(1235))/\Gamma(\pi\rho(1450)) = 0.85\text{--}1.21$, $\Gamma(\rho\pi(1300))/\Gamma(\pi\rho) = 0.12\text{--}0.23$, $\Gamma(\omega\eta)/\Gamma(\eta h_1(1170)) = 0.97\text{--}1.29$, $\Gamma(\omega\eta)/\Gamma(\omega(1420)\eta) = 0.91\text{--}0.94$, $\Gamma(\omega\eta)/\Gamma(KK^*(1410)) = 1.41\text{--}1.64$, $\Gamma(\omega\eta)/\Gamma(\omega\eta') = 2.17\text{--}2.26$, and $\Gamma(\omega\eta')/\Gamma(KK^*) = 1.03\text{--}1.09$. Advisable invariant mass spectra that should be investigated in experiments are $\omega\pi\pi$, $\omega\eta$, and KK^* . This decay information will be helpful for future experimental searches.

C. ρ_2 family

As the isovector counterpart of the ω_2 family, the ρ_2 family takes the same R ranges. The contour lines of the total widths from $n = 1$ to 4 are illustrated in Fig. 6. Corresponding branching ratios are illustrated in Figs. 7 and 8. We can roughly conclude that the higher radial excitations rely more intensively on R variation and take larger R values.

First, because no signal of $\rho_2(1D)$ has been observed, a mass the same as that of $\omega_2(1D)$ is obtained via the

FIG. 6. The total widths of the ρ_2 family ($n = 1$ to 4) with dependence on R and γ . Different colors correspond to different width values and the contour curves are marked with the width values. In particular, the red longitudinal solid lines depict the R dependence when $\gamma = 7.1$.

FIG. 7. The branching ratios of $\rho_2(1D)$ and $\rho_2(1940)$ with R dependence. Decay channels with branching ratios less than 1% are neglected.

GI model. Shown as the top left diagram of Fig. 6, the total width of $\rho_2(1D)$ is around 390 MeV when $\gamma = 7.1$, consistent with Ref. [18] but little less than the result of Ref. [19]. From the left of Fig. 7, the $\pi a_2(1320)$ mode of $\rho_2(1D)$ decay is predicted to obtain the largest branching ratio of $\sim 55\%$. $a_2(1320)$ sequentially decays to $\rho\pi$, and $\rho_2(1D)$ eventually decays into the 4π final state through this mode. Furthermore, the secondary modes, $\rho\rho$, $\pi\omega$, $\pi a_1(1260)$, and $\pi h_1(1170)$ also contribute to the 4π final state. Therefore, why no signal of the ρ_2 ground state is observed can be explained by the fact that it is difficult to find a broad width structure and complicated to reconstruct a 4π final state in experiments. Although $\rho_2(1D)$ can be produced in principle, $\rho_2(1D)$ is difficult to determine using the Crystal Barrel detector because of quite low statistics. Actually, the mass and width of $\rho_2(1940)$ already are poorly determined with low statistics, being close to the bottom of the available energy range. However, $\rho_2(1D)$ also couples significantly to $\rho\eta$ and KK^* , so it may be observable in both channels. The $\pi\omega$ mode should also be of concern since $\rho_2(1940)$ and $\rho_2(2225)$ are observed in this mode and the reconstruction efficiency is higher than $\omega\eta\pi$. We also list some ratios that are weakly dependent on R of the partial decay width: $\Gamma(\rho\rho)/\Gamma(\pi a_2(1320)) = 0.22\text{--}0.27$, $\Gamma(\pi\omega)/\Gamma(\pi a_2(1320)) = 0.23\text{--}0.26$, $\Gamma(\pi\omega)/\Gamma(\rho\rho) = 0.84\text{--}1.16$, $\Gamma(\rho\eta)/\Gamma(\pi\omega) = 0.49\text{--}0.51$, $\Gamma(\pi h_1(1170))/\Gamma(\rho\eta) = 0.56\text{--}0.60$, and $\Gamma(\rho\eta)/\Gamma(KK^*) = 2.18\text{--}2.19$.

According to mass spectrum analysis, the two resonances $\rho_2(1940)$ and $\rho_2(2225)$ observed in the processes

FIG. 8. The branching ratios of $\rho_2(2225)$ and $\rho_2(4D)$ with R dependence. Decay channels with branching ratios less than 1% are neglected.

$p\bar{p} \rightarrow \omega\pi^0$ and $p\bar{p} \rightarrow \omega\eta\pi^0$ [21] are assigned as 2^3D_2 and 3^3D_2 states. From the upper right of Fig. 6, $\rho_2(1940)$ has a total width range 127–144 MeV as R varies from 4.1 to 4.7 GeV^{-1} when $\gamma = 7.1$, and that well overlaps with the experimental width 155 ± 40 MeV. The strong two-body decay behavior of $\rho_2(1940)$ as the 2^3D_2 state is presented in the right of Fig. 7. The main decay channels include $\pi a_2(1320)$, $\pi\omega(1420)$, and $\pi\omega$ with branching ratios 20.6%–25.5%, 20.8%–23.1%, and 9.6%–15.0%, respectively. However, whether $\rho\rho$ is a main channel depends on the R value. $\pi\omega(1420)$ also contributes to the $\omega\eta\pi^0$ final state, and it makes sense for $\rho_2(1940)$ to be discovered in the $p\bar{p} \rightarrow \omega\eta\pi^0$ process. Apart from the channels contributing to the 4π final state, $\rho_2(2D)$ obtains sizable branching ratios of kaons, $\rho\eta$ and $\rho\eta'$, and these modes can be carefully inspected in experiments. Further information on the typical ratios of partial decay widths is provided: $\Gamma(\pi a_2(1320))/\Gamma(\pi\omega(1420)) = 0.89\text{--}1.22$, $\Gamma(\pi a_2(1320))/\Gamma(\pi\omega) = 1.70\text{--}2.14$, $\Gamma(\rho\rho)/\Gamma(\pi\omega(1420)) = 0.23\text{--}0.76$, $\Gamma(\pi h_1(1170))/\Gamma(\pi a_2(1320)) = 0.14\text{--}0.37$, $\Gamma(\rho\eta)/\Gamma(\pi\omega) = 0.42\text{--}0.45$, $\Gamma(KK^*)/\Gamma(\rho\eta) = 0.47\text{--}0.57$, and $\Gamma(\rho\eta')/\Gamma(\rho\eta) = 0.17\text{--}0.37$. This information is valuable for the further experimental study and confirmation of $\rho_2(1940)$.

Another 2^{--} resonance reported in Ref. [21], the $\rho_2(2225)$ as the 3^3D_2 state, is studied. Its partial decay width is presented in the left side of Fig. 8. Since $\rho_2(2225)$ is assumed as the second excited state, its width is strongly dependent on the R value, and thus it possesses a broad width range with $R = 4.5\text{--}5.1 \text{ GeV}^{-1}$. Unlike the other three observed resonances, our calculated width is somewhat smaller than the experimental value 335^{+100}_{-50} . But notice that the Crystal Barrel Collaboration announced this resonance and its experimental width error is evidently larger than the other three resonances, so more experimental information is needed to test our result. The main decay channels of $\rho_2(2225)$ include $\pi\omega(1420)$, $\pi h_1(1170)$, $\pi\omega$, $\pi a_0(1450)$, and $\pi a_2(1320)$ with branching ratios equal to 13.4%–18.8%, 11.0%–14.1%, 12.1%–13.2%, 8.9%–11.0%, and 3.9%–14.4%, respectively. On the other hand, $\rho\rho$, $\rho\eta$, $\rho(1450)\eta$, $\rho b_1(1235)$, and $\pi\pi_2(1670)$ are subordinate decay channels. Also, sequential two-body decays of the $\pi\omega(1420)$, $\omega a_2(1320)$, $\eta b_1(1235)$, and $\pi\omega(1650)$ channels become the $\omega\eta\pi^0$ final state, so it explains the experimental process $p\bar{p} \rightarrow \rho_2(2225) \rightarrow \omega\eta\pi^0$. Here, detailed decay width ratios are also presented: $\Gamma(\pi\omega(1420))/\Gamma(\pi h_1(1170)) = 0.96\text{--}1.70$, $\Gamma(\pi\omega)/\Gamma(\pi\omega(1420)) = 1.10\text{--}1.51$, $\Gamma(\pi a_0(1450))/\Gamma(\pi\omega) = 0.74\text{--}0.84$, $\Gamma(\rho\eta)/\Gamma(\pi\omega) = 0.38\text{--}0.40$,

$$\begin{aligned} \Gamma(\rho(1450\eta))/\Gamma(\rho\eta) &= 1.08\text{--}1.91, & \Gamma(KK^*(1410))/\Gamma(KK^*(1430)) &= 1.34\text{--}1.93, \\ \Gamma(KK^*)/\Gamma(\rho\eta') &= 1.00\text{--}1.07, & \Gamma(\rho\eta')/\Gamma(\rho\eta) &= 0.45\text{--}0.52, \\ \Gamma(\pi\omega(1650))/\Gamma(KK^*) &= 0.45\text{--}0.63. \end{aligned}$$

The mass of $\rho_2(4D)$, which is absent from experiments, is predicted as 2424 MeV, and its calculated total width is plotted in the lower right of Fig. 6, which presents a broad width range relying on R . Our suggested total decaying width is 97 MeV when choosing suitable R and γ values. The branching ratios of $\rho_2(4D)$ are illustrated in the right column of Fig. 8. In addition, because of higher excitation, both the total width and the branching ratios are sharply dependent on the R value. The main channels consist of $\pi h_1(1170)$, $\pi\omega$, $\pi\omega(1420)$, and $\pi a_0(1450)$. Except for the $\pi h_1(1170)$ channel, which has a stable ratio of 15%, the other three main channels, $\pi\omega$, $\pi\omega(1420)$, and $\pi a_0(1450)$, show wider ratio ranges 11.6%–12.9%, 9.8%–14.4%, and 10.7%–13.35%, respectively. Also, we list several relative ratios weakly dependent on R : $\Gamma(\pi h_1(1170))/\Gamma(\pi\omega) = 1.21\text{--}1.25$, $\Gamma(\pi\omega)/\Gamma(\pi\omega(1420)) = 0.90\text{--}1.19$, $\Gamma(\rho b_1(1235))/\Gamma(\rho\eta) = 0.62\text{--}1.13$, $\Gamma(\rho(1450)\eta)/\Gamma(\rho\eta) = 0.91\text{--}0.95$, $\Gamma(b_1(1235)\eta)/\Gamma(\rho(1450)\eta) = 0.67\text{--}0.71$, and $\Gamma(\rho\eta')/\Gamma(\rho\eta) = 0.42\text{--}0.45$. This information is helpful for future experimental investigations.

FIG. 9. The total widths of the ϕ_2 family ($n = 1$ to 4) with dependence on R and γ . Different colors correspond to different width values and the contour curves are marked with the width values. In particular, the red longitudinal solid lines depict the R dependence when $\gamma = 7.1$.

D. ϕ_2 family

Information about the ϕ_2 family has remained absent from experiments, so theoretical prediction of the absent states can assist researchers in searching for them and discovering more properties. By combining the analysis of the GI model and Regge trajectory, masses of $\phi_2(1D) - \phi_2(4D)$ are obtained in Table I, and their total widths are illustrated in Fig. 9. Considering our previous work regarding the K_2^* family [16], complete $J^{PC} = 2^{--}$ light meson nonets are established, and systematic analysis can be performed. The analysis of ϕ_2 states proceeds in more detail as follows.

The mass of $\phi_2(1D)$ is suggested as 1904 MeV, near its partner $\phi_3(1850)$. If taking a typical value of $\gamma = 7.1$, its total width is predicted about 255 MeV with mild R dependence shown in the top left of Fig. 9, and it is comparable with the calculation in Ref. [19]. Two-body decay properties of $\phi_2(1D)$ are presented at the top of Fig. 10, where the KK^* decay mode is dominant with the fraction up to 58.5%–66.2%, and K^*K^* , $KK_1(1270)$, and $\eta\phi$ channels also hold considerable proportions. Similar branching ratio results appear in Ref. [19], except that of $\phi\eta$ is slightly larger than our prediction. Besides, all channel ratios exhibit linear dependence with changing R , which probably results from the smooth wave function of the ground state. Thus, studying partial width ratios in experiments can appropriately test our prediction. These are

FIG. 10. The branching ratios of $\phi_2(1D)$ and $\phi_2(2D)$ with R dependence. Decay channels with branching ratios less than 1% are neglected.

$$\Gamma(K^*K^*)/\Gamma(KK^*)=0.28-0.40, \Gamma(KK_1(1270))/\Gamma(K^*K^*)=0.41-0.47, \text{ and } \Gamma(KK_1(1270))/\Gamma(\eta\phi) = 0.96-1.59.$$

Under the assignment of 2^3D_2 , the mass with 2151 MeV is employed, and corresponding total width and branching ratios are plotted in the top right part of Fig. 9 and at the bottom of Fig. 10, respectively. The total width of $\phi_2(2D)$ is 206 MeV when taking the proposed R value 3.7 GeV^{-1} and $\gamma = 7.1$. The two-body strong decay properties, collected at the bottom of Fig. 10, indicate that the decay channels devoted to main contributions include $KK^*(1410)$, KK^* , K^*K^* , $KK_2^*(1430)$, and $KK_1(1270)$, taking proportions 34.1%–41.0%, 17.4%–23.0%, 5.4%–22.1%, 7.3%–19.2%, and 0.1%–11.2%, respectively. Since BESIII declared numerous states around 2.2 GeV, $\Gamma(\eta'\phi)/\Gamma(\eta\phi) = 0.19-0.38$ should be an ideal ratio to distinguish $\phi_2(2D)$.

As for $\phi_2(3D)$, its mass is advised as 2372 MeV by the extrapolation from the Regge trajectory with the mass of the ground state calculated from the GI model. Its recommended total width value is 124 MeV from the bottom left of Fig. 9. We list the branching ratios of $\phi_2(3D)$ on the left side of Fig. 11. $KK^*(1410)$, KK^* , $K^*K^*(1410)$, $KK_2^*(1430)$, $KK_1(1400)$, and $K^*K_1(1270)$ are the main channels with branching ratios 23.5%–31.8%, 12.3%–22.0%, 23.6%–4.7%, 5.4%–13.1%, 4.3%–10.4%, and

FIG. 11. The branching ratios of $\phi_2(3D)$ and $\phi_2(4D)$ with R dependence. Decay channels with branching ratios less than 1% are neglected.

1.1%–9.3%, respectively. In particular, decay modes $K^*K^*(1410)$, $K^*K_1(1270)$, and $KK_3^*(1780)$ are significantly dependent on R mainly because of the node effects. By comparison, $KK^*(1410)$ is the dominant decay channel and it is more stable than other modes. A similar phenomenon appears for $\phi_2(2D)$, too. A characteristic partial width ratio helpful for experimental exploration is $\Gamma(\eta'\phi)/\Gamma(\eta\phi) = 0.39$ – 0.66 .

Finally, we analyze the decay behaviors of the $\phi_2(4D)$ state with its mass proposed as 2574 MeV. As the highest excitation considered in the ϕ_2 family, it provides complicated properties of strong decays due to the R value sensitivity and node effects. Its total width variation tendency is presented in the lower right corner of Fig. 9 with an advisable value of 75 MeV when $\gamma = 7.1$ and $R = 4.2 \text{ GeV}^{-1}$. Relevant branching ratios are shown in the right of Fig. 11, where $KK^*(1410)$ is the dominant mode holding a 25.5%–36.0% fraction, and KK^* , $KK_1(1400)$, and $KK_2^*(1430)$ are also main channels. Some typical relative branching fractions are presented here, $\Gamma(KK_1(1400))/\Gamma(KK^*) = 0.66$ – 0.73 , $\Gamma(\eta\phi)/\Gamma(K^*K^*) = 0.58$ – 0.97 , and $\Gamma(\eta'\phi)/\Gamma(\eta\phi) = 0.58$ – 0.74 . This predicted information about the missing ϕ_2 family may inspire more experimental efforts to hunt them.

IV. SUMMARY AND DISCUSSION

Since the quark model was proposed, it has achieved great success in explaining and predicting numerous hadrons. However, there also remain some unexplained exotic states beyond the quark model, such as glueballs and hybrids. To distinguish between conventional and exotic mesons, it is crucial that we comprehend conventional-meson spectroscopy very well. Currently, the experimental information of $J^{PC} = 2^{--}$ unflavored light mesons is still scarce except for the results of the Crystal Barrel detector in 2002, and thus they are not well established. Therefore, a systematical and complete study of 2^{--} unflavored light mesons is necessary to stimulate mounting experimental efforts to this sector.

In this work, an investigation of the mass spectrum was first performed by combining the analysis of the Regge trajectory and GI model in Sec. II, which indicates that the four experimental resonances are assigned as the first and second excitation of the ω_2 and ρ_2 families, respectively. In addition, this categorization is also supported by the corresponding features of two-body decays allowed by OZI rule. More importantly, with predicted masses $\sim 1.7 \text{ GeV}$ for ω_2 and ρ_2 ground states, total decay widths are suggested as 220 and 390 MeV. Furthermore, since $\omega_2(1D)$ is masked by $\pi_2(1670)$ and $\rho_2(1D)$ is difficult to reconstruct with a broad width and a 4π final state, we can explain why they are still not found in experiments. Detailed decay properties of the missing ground resonances

TABLE II. The partial decay width of ground states in units of MeV. Corresponding R value ranges are 3.7–4.3 for the ω_2 and ρ_2 ground states and 3.1–3.7 for the ϕ_2 ground state.

Channels	$\omega_2(1D)$	Channels	$\rho_2(1D)$	Channels	$\phi_2(1D)$
$\pi\rho$	142–165	$\pi a_2(1320)$	206–213	KK^*	149–167
$\pi b_1(1235)$	25–26	$\rho\rho$	47–56	K^*K^*	46–59
$\omega\eta$	24–27	$\pi\omega$	47–55	$KK_1(1270)$	19–28
KK^*	11–12	$\rho\eta$	24–27	$\eta\phi$	18–20
		$\pi a_1(1260)$	16–22		

are given by QPC model in Sec. III. In addition, for the absolutely missing ϕ_2 family, their partial decay behaviors are also discussed by adopting advisable masses from $n = 1$ to 4. The branching ratios of the main and subordinate channels are illustrated in Sec. III. A certain partial decay width is obtained by multiplying the corresponding branching ratio with the total width. The main decay widths of ground states are listed in Table II.

Because of the merely theoretical inputs of the masses for most states, it is necessary to discuss the mass dependence of decay widths. For example, when adding 50 MeV to the masses of the ω_2 family, the main decay widths of $\omega_2(4D)$ increase by a maximum of 24%. For the lower state, their main decay widths are relatively less sensitive to mass variation. Similarly, the mass dependence of the ρ_2 and ϕ_2 families can be estimated.

Strong decay information is vital to search for missing ground states and higher excitations. We hope our study can inspire more experimental interest to find them and establish the 2^{--} light meson family in experiments eventually. BESIII, CMD-3, SND, KEDR, COMPASS, and E852 are suitable platforms to explore these issues. In particular, BESIII is a promising facility with the highest luminosity running at 2.0–4.6 GeV, and its main focus still is exploring light hadrons in the following years. Also, it is hopeful that many states can be experimentally better understood in the relatively near future at the Jefferson Lab. Potential channels, $\omega\eta$ for ω_2 , $\rho\eta$ for ρ_2 , and $\eta\phi$ for ϕ_2 , are suggested to explore these ground states. KK^* is an ideal channel which contributes to all 2^{--} light unflavored states and to sizable branching ratios.

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