Scale dependence of the small x transverse momentum dependent gluon distribution

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We investigate the scale dependence of transverse momentum dependent(TMD) gluon distribution by taking into account the multiple rescattering effect. We find that in the Collins-2011 scheme, the scale dependence of the small- x gluon TMD is governed by the same renormalization group(RG) equation that holds at moderate or large x . Following the standard procedure, one then can resum both the double leading logarithm and the single leading logarithm by jointly solving the Collins-Soper equation and the RG equation in a way that is compatible with the small- x formalism.

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I. INTRODUCTION

One of the central scientific goals to be achieved at the current and future facilities, including the JLab 12 GeV upgrade, RHIC, and the planned electron-ion collider(EIC) is to reveal the three-dimensional structure of a nucleon or nuclei by measuring the final state produced particle transverse momentum spectrum in high-energy scatterings. The extraction of parton transverse momentum dependent (TMD) distributions that encode information on the internal structure of the nucleon or nuclei from physical observables relies on the QCD factorization theorem. As a leading power approximation, TMD factorization in the moderate- or large- x region has been well established during the past few decades [1–4]. However, since high-twist contributions arising from multiple gluon rescattering are no longer negligible at small x , it is nontrivial to justify TMD factorization in a saturation regime. Some recent attempts to address this issue have been made in Refs. [5–11].

The purpose of the current work is to further extend and refine the previous analysis presented in Refs. [10,11]. The key to unifying small- x formalism and the TMD approach is a two-step evolution procedure, which can be best demonstrated using a color neutral scalar particle production through gluon fusion ($gg \rightarrow H$) process as an example. Below the produced particle mass and transverse momentum are denoted as Q and $k_\perp$, respectively. For a comparison, we first review the conventional treatment in moderate x region, where the calculation of the differential

cross section can be formulated in collinear factorization. At high order, the collinear divergence and various large logarithm terms show up. After absorbing the collinear divergence and the associated logarithm $\ln \frac{Q^2}{\mu^2}$ into the renormalized gluon PDF, we are still left with large double logarithm term $\alpha_s^n \ln^{2n} \frac{Q^2}{k_\perp^2}$. To facilitate resumming these large logarithms to all orders, one can introduce gluon TMD distribution. The large $k_\perp$ logarithms then can be resummed by solving the Collins-Soper evolution equation [1,4] that governs $\ln \zeta_c$ dependence of gluon TMD. Here ζ_c is a parameter introduced in the Collins 2011 scheme [4] for regulating the light cone divergence. It plays a role of varying hard scale which allows one to smoothly evolve from the scale Q^2 down to $k_\perp^2$.

When the center mass of energy S is much larger than Q^2 , the large logarithm $\ln \frac{S}{Q^2} \sim \ln \frac{1}{x}$ appearing in the high-order calculation could be more important than the logarithm $\ln \frac{Q^2}{\mu^2}$. One, thus, should formulate the calculation in color glass condensate(CGC) effective theory [12,13] to first take care of the logarithm $\ln \frac{S}{Q^2}$. They can be summed by solving the Balitsky-Kovchegov(BK) [14,15] equation that describes $\ln \frac{1}{x}$ dependence of multiple point function—the basic nonperturbative ingredient in CGC calculation. In the kinematic region where $Q^2 \gg k_\perp^2$, logarithm $\alpha_s^n \ln^{2n} \frac{Q^2}{k_\perp^2}$ terms also arise in high-order calculations. When these logarithms are much larger than leading order high-twist contributions suppressed by the power of $\frac{k_\perp^2}{Q^2}$ [7], TMD factorization should be employed where all subleading power contributions are systematically ignored. The equivalence between the leading power part of the CGC result and TMD factorization calculation at tree level can

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be verified by utilizing the operator relation between the derivative of multiple point function and gluon TMD matrix element [5,6]. What we gain by making the leading power approximation is that large $\alpha_s^n \ln^{2n} \frac{Q^2}{k_\perp^2}$ logarithm terms can be resummed to all orders in the context of TMD factorization.

To achieve such a resummation, it is necessary to show that the properly defined gluon TMD accommodates the similar large logarithm and satisfies the Collins-Soper equation in the small- x limit. It has indeed been verified in a recent work [10] that gluon TMD computed at the next-to-leading order in a quark target model satisfies both the Balitsky-Fadin-Kuraev-Lipatov(BFKL) equation [16,17] and the Collins-Soper equation in the small x limit. The similar analysis was later extended to saturation regime by calculating the small- x gluon TMD in terms of multiple point functions using the CGC approach [11]. Schematically, the derived Weizsäcker-Williams (WW)-type gluon TMD takes the following form,

$$xG(x, k_\perp, \zeta_c^2 = \mu_F^2 = Q^2) = -\frac{2}{\alpha_s} \int \frac{d^2 x_\perp d^2 y_\perp}{(2\pi)^2} e^{ik_\perp \cdot b_\perp} \times \mathcal{H}(\alpha_s(Q)) \text{Exp} \left\{ - \int_{\mu_b}^Q \frac{d\mu}{\mu} \left(A \ln \frac{Q^2}{\mu^2} + B \right) \right\} \times \mathcal{F}(x_\perp, y_\perp), \quad (1)$$

where $\mu_b = \frac{2e^{-\gamma_E}}{|b_\perp|}$ with $b_\perp = x_\perp - y_\perp$, and μ_F is the factorization scale. $\mathcal{F}$ is the Fourier transform of the WW gluon distribution,

$$\mathcal{F}(x_\perp, y_\perp) = \langle \text{Tr}[\partial_\perp^i U(x_\perp) U^\dagger(y_\perp) \partial_\perp^i U(y_\perp) U^\dagger(x_\perp)] \rangle, \quad (2)$$

where $U(x_\perp)$ is the Wilson line in the fundamental representation. It absorbs all large logarithm $\ln \frac{1}{x}$ terms from the hard part with the help of the BK equation. The remaining logarithms are resummed into the exponentiation known as the Sudakov factor by solving the Collins-Soper equation. We are eventually left with a hard coefficient $\mathcal{H}(\alpha_s(Q))$ that only has finite contributions. The Similar result holds for the dipole-type gluon distribution.

The hard coefficients A and B can be calculated perturbatively. In the previous work [11], we only took into account the double leading logarithm contribution in CGC framework, and determined the coefficient A to be $A = \frac{\alpha_s C_A}{\pi}$ at leading order, which is the same as the one in the standard Collins-Soper-Sterman(CSS) formalism [3]. The purpose of the current work is to sort out the single leading logarithm contribution in the saturation regime, i.e., to fix the coefficient B . To this end, one has to study not only the $\ln \zeta_c$ dependence but also the factorization scale μ_F dependence of the small- x gluon TMD. In other words, we aim at deriving a renormalization group(RG) equation for the gluon TMD in saturation regime. By jointly solving the RG equation and

the Collins-Soper equation, one is able to resum both the double leading logarithm and single leading logarithm contributions.

From a theoretical point of view, completing the previous analysis on $k_\perp$ resummation in the saturation regime is interesting in its own right. On the other hand, the present work is further motivated by the fact that very rich polarization dependent phenomenology in saturation regime has been discovered in recent years [18–26]. It is time to lay down a solid theoretical ground for performing phenomenological studies of the relevant physical observables which can be measured at RHIC, LHC, and the planned EIC.

The rest of this paper is organized as follows. In the next section, we compute the anomalous dimension of the small- x gluon TMD by isolating the ultraviolet(UV) divergent part. The most nontrivial part of our analysis is to investigate how the UV divergence is affected in the presence of multiple gluon rescattering. The detailed derivation will be presented. The paper is summarized in Sec. III.

II. DERIVATION

There are two widely used $k_\perp$ -dependent unpolarized gluon distributions with different gauge link structures: (1) the WW-type distribution [27] with a staple like gauge link, and (2) the dipole-type distribution [28] with a close loop gauge link. These two types of gluon distributions can be directly probed through two-particle correlation in different high-energy scattering reactions [5,6,27–32]. In Ref. [11], we demonstrated that both gluon TMDs computed in CGC formalism satisfy the Collins-Soper equation after matching them onto the renormalized quadrupole and dipole amplitudes, respectively. At leading order, the Collins-Soper equation reads [4]

$$\frac{\partial \ln G(x, b_\perp, \mu^2, \zeta_c^2)}{\partial \ln \zeta_c} = K(b_\perp, \mu) = -\frac{\alpha_s C_A}{\pi} \ln \frac{\mu^2 b_\perp^2}{4e^{-2\gamma_E}}, \quad (3)$$

where $G(x, b_\perp, \mu^2, \zeta_c^2)$ is the Fourier transform of gluon TMD, which can be related to the derivative of the quadrupole amplitude for the WW case. The logarithm $\ln \frac{1}{x}$ dependence of the operator $G(x, b_\perp, \mu^2, \zeta_c^2)$ is described by the BK equation. On the other hand, the factorization scale dependence of gluon TMD is governed by the renormalization group equation which takes form,

$$\frac{d \ln G(x, b_\perp, \mu^2, \zeta_c^2)}{d \ln \mu} = \gamma_G(g(\mu), \zeta_c^2/\mu^2) \quad (4)$$

The anomalous dimension γ_G is also the function of ζ_c . Its ζ_c dependence can be explicitly separated out as the following [4],

$$\gamma_G(g(\mu), \zeta_c^2/\mu^2) = \gamma_G(g(\mu), 1) - \frac{1}{2} \gamma_K(g(\mu)) \ln \frac{\zeta_c^2}{\mu^2}, \quad (5)$$

with $\gamma_K(g(\mu) = -\frac{dK(b_\perp, \mu)}{d \ln \mu} = \frac{2\alpha_s C_A}{\pi}$ at one-loop order. Following the standard procedure, it can be readily deduced from the evolution equations,¹

$$G(x, b_\perp, Q^2, Q^2) = \text{Exp} \left\{ \int_{\mu_b}^Q \frac{d\mu}{\mu} \left(\gamma_G(g(\mu), 1) - \frac{\alpha_s C_A}{\pi} \ln \frac{Q^2}{\mu^2} \right) \right\} \times G(x, b_\perp, \mu_b^2, \mu_b^2), \quad (6)$$

where the large logarithm is resummed into the Sudakov factor.

In the dilute limit, it is shown [10] that both the double leading and single leading logarithms can be resummed. In the saturation regime, only the double leading logarithm contribution $\text{Exp} \left\{ - \int_{\mu_b}^Q \frac{d\mu}{\mu} \frac{\alpha_s C_A}{\pi} \ln \frac{Q^2}{\mu^2} \right\}$ was taken into account in the previous analysis [11]. The value of $\gamma_G(g(\mu), 1)$ is not yet fixed for the saturation case. The purpose of the present work is to compute the anomalous dimension of the small- x gluon TMD in the saturation regime. Once the anomalous dimension is worked out, the single leading logarithm can be resummed to all orders by solving the CS and RG equations as shown above.

As an example, we focus on the WW case in this paper. The calculation can be straightforwardly extended to the dipole case. Our starting point is the matrix element definition of the WW gluon distribution,

$$xG(x, k_\perp) = \int \frac{d\xi^- d^2 \xi_\perp}{(2\pi)^3 P^+} e^{ik_\perp \cdot \xi_\perp - ixP^+ \xi^-} \times \langle P | F_a^{+i}(\xi^-, \xi_\perp) \mathcal{L}_{ab}^\dagger(\xi^-, \xi_\perp) \times \mathcal{L}_{bc}(0, 0_\perp) F_c^{+i}(0) | P \rangle, \quad (7)$$

where $F_a^{+i}(\xi^-, \xi_\perp)$ is the gauge field strength tensor and the gauge link is further fixed to be the past pointing one in the adjoint representation,

$$\mathcal{L}_{ab}^\dagger(\xi^-, \xi_\perp) = \mathcal{P} \exp \left[-g_0 \int_{-\infty}^{\xi^-} dz^- f^{dab} A_d^+(z^-, \xi_\perp) \right] \quad (8)$$

$$\mathcal{L}_{bc}(0, 0_\perp) = \mathcal{P} \exp \left[-g_0 \int_0^{-\infty} dz^- f^{dbc} A_d^+(z^-, \xi_\perp) \right]. \quad (9)$$

The WW-type gluon TMD also can be defined in the fundamental representation,

$$xG(x, k_\perp) = 2 \int \frac{d\xi^- d^2 \xi_\perp}{(2\pi)^3 P^+} e^{ik_\perp \cdot \xi_\perp - ixP^+ \xi^-} \times \langle P | \text{Tr} [F_a^{+i}(\xi^-, \xi_\perp) U^{[+]\dagger} F_c^{+i}(0) U^{[+]}] | P \rangle, \quad (10)$$

¹It has been found [33] that the final implementation of the TMD evolution depends on the particular choice of integration path in the (μ, ζ_c) plane. This deserves further investigation.

FIG. 1. Tree diagrams contributing to the WW-type gluon TMD. The double lines represent gauge link in the adjoint representation.

where U denotes the gauge link in the fundamental representation.

One can readily determine the anomalous dimension by isolating the ultraviolet (UV) divergence of the small- x gluon TMD. To do so, one has to go beyond the conventional treatment of small- x formalism in which the Eikonal approximation is applied everywhere. This is because the UV divergent part does not have $1/x$ enhancement and could be missed in the leading power small- x approximation. We, thus, have to carry out the calculation in the full QCD. The extra care should be taken when performing the Eikonal approximation to simplify calculation.

In order to fix conventions and do a warm-up exercise, we start with the tree-level calculation, though the UV divergence is absent at the tree level. Diagrams illustrated in Fig. 1 give rise to the leading-order contributions. In the small- x limit, the dominant contribution is from the A^+ component in the Feynman gauge or $A^- = 0$ light cone gauge. Hereafter, we perform the calculation in the light cone gauge with $A^- = 0$. It is trivial to compute the graph Fig. 1(a) and its conjugate part, which leads to

$$xG_0(x, k_\perp) = 4\pi \int \frac{dx^- d^2 x_\perp}{(2\pi)^2} \frac{dy^- d^2 y_\perp}{(2\pi)^2} e^{ik_\perp \cdot (x_\perp - y_\perp)} \times \langle \partial_i A_a^+(x^-, x_\perp) \partial_i A_a^+(y^-, y_\perp) \rangle_x. \quad (11)$$

It is well known that the gauge link is built through gluon rescattering. The diagram Fig. 1(b) gives rise to the first nontrivial term of the Taylor expansion of the gauge link,

$$xG_1(x, k_\perp) = 4\pi \int \frac{dx^- d^2 x_\perp}{(2\pi)^2} \frac{dy^- d^2 y_\perp}{(2\pi)^2} e^{ik_\perp \cdot (x_\perp - y_\perp)} \times \left\langle \partial_i A_a^+(x^-, x_\perp) \left(-g_0 \int_{y^-}^{-\infty} dz^- f^{bac} A_b^+(z^-, y_\perp) \right) \times \partial_i A_c^+(y^-, y_\perp) \right\rangle_x, \quad (12)$$

where g_0 associated with the gauge potential $A_b^+(z^-, y_\perp)$ is the bare strong coupling constant, which will be renormalized after including one-loop correction. Note that in

FIG. 2. One-loop virtual correction to the gluon TMD without taking into account gauge link. In the small- x limit, the A^+ component of the incoming gluon gives rise to the dominant contribution. The diagram d has vanishing contribution. The ghost contribution is absent in the gauge $A^- = 0$.

the small- x limit, one usually only takes into account A^+ component contribution. It is possible to restore $\partial^+ A_\perp$ part of the gluon field strength $F_{+\perp}$ is missed in our calculation. However, due to the gauge invariance, one anticipates that the scale dependence associated with operator $\partial_\perp A^+$ and $\partial^+ A_\perp$ should be the same. It is sufficient to only compute $\partial_\perp A^+$ contribution for the current purpose. Similarly, the graph Fig. 1(c) results in

$$\begin{aligned}
 xG_2(x, k_\perp) &= 4\pi \int \frac{dx^- d^2 x_\perp}{(2\pi)^2} \frac{dy^- d^2 y_\perp}{(2\pi)^2} e^{ik_\perp \cdot (x_\perp - y_\perp)} \langle \partial_i A_d^+(x^-, x_\perp) \\
 &\times \left(g_0^2 \int_{y^-}^{-\infty} dz^- f^{dab} A_d^+(z^-, y_\perp) \right. \\
 &\times \left. \int_{y^-}^{z^-} dz_1^- f^{bdc} A_b^+(z_1^-, y_\perp) \right) \partial_i A_c^+(y^-, y_\perp) \rangle_x \quad (13)
 \end{aligned}$$

It is straightforward to resum gluon rescattering to all orders. The WW-type gluon TMD in the small- x limit eventually can be cast into the following form in the fundamental representation,

$$\begin{aligned}
 xG(x, k_\perp) &= -\frac{8\pi}{g_0^2} \int \frac{d^2 x_\perp}{(2\pi)^2} \frac{d^2 y_\perp}{(2\pi)^2} e^{ik_\perp \cdot (x_\perp - y_\perp)} \\
 &\times \langle \text{Tr}[\partial_i U(x_\perp)] U^\dagger(y_\perp) [\partial_i U(y_\perp)] U^\dagger(x_\perp) \rangle_x, \quad (14)
 \end{aligned}$$

where the strong coupling constant appear in the Wilson lines is the bare one. The above matrix element obtained through tree diagram calculation is consistent with the

matrix element definition given in Eq. (2), which only captures the leading contribution in the power of $1/x$. In contrast, the gluon TMD definition Eq. (7) [or Eq. (10)] is valid at arbitrary x and keeps not only the leading $\ln \frac{1}{x}$ terms but also the leading contributions in the power of μ^2/ζ_c^2 and $k_\perp^2/\zeta_c^2$. Therefore, the correct UV behavior and the $\ln \zeta_c$ dependence of the gluon TMD only can be obtained by computing the expectation value of the matrix element in Eq. (7) [or Eq. (10)], rather than the matrix element given in Eq. (14).

UV divergence only arises in virtual corrections. There are four virtual graphs without gluon rescattering as shown in Fig. 2. It is easy to verify that the contribution from the graph Fig. 2(d) vanishes. We now start with computing the vertex correction shown in Fig. 2(a). To avoid the interaction between the radiated gluon and color source inside target, our calculation is performed in the light cone gauge ($A^- = 0$), in which the gluon propagator reads

$$\left(-g^{\mu\nu} + \frac{p^\mu l^\nu + p^\nu l^\mu}{l \cdot p - i\epsilon} \right) \frac{i}{l^2 + i\epsilon}, \quad (15)$$

where the prescription $\frac{1}{l \cdot p - i\epsilon}$ for regulating the light cone divergence is proven to be the most convenient choice for our calculation. The contribution from the graph Fig. 2(a) is expressed as the product of the corresponding hard part and the gluon TMD matrix element without the gauge link being included:

$$\text{Fig.2(a)} \propto \mathcal{H}_{2a}(k) xG_0(x, k_\perp). \quad (16)$$

The hard part is given by

$$\mathcal{H}_{2a}(k) = \frac{\alpha_s C_A}{4\pi^3} \int dl^+ d^2 l_\perp dl^- \frac{-i[(k_\perp^2 - k_\perp \cdot l_\perp)(l_\perp^2 - k_\perp \cdot l_\perp) - l^- k^+ k_\perp \cdot l_\perp]}{[2l^+ l^- - l_\perp^2 + i\epsilon][2l^- (l^+ - k^+) - (l_\perp - k_\perp)^2 + i\epsilon][l^- - i\epsilon][l^+ + i\epsilon]k_\perp^2} \quad (17)$$

where $k_\perp^2$ in the denominator arises when we make the following conversion by partial integration,

$$k_\perp^2 \langle A_d^+(x^-, x_\perp) A_d^+(y^-, y_\perp) \rangle \Rightarrow \langle \partial_i A_d^+(x^-, x_\perp) \partial_i A_d^+(y^-, y_\perp) \rangle. \quad (18)$$

We proceed by performing contour integration on l^- ,

$$\begin{aligned} \mathcal{H}_{2a}(k) = & -\frac{\alpha_s C_A}{2\pi^2} \int d^2 l_\perp \int_0^{k^+} dl^+ \frac{(k_\perp^2 - k_\perp \cdot l_\perp)(l_\perp^2 - k_\perp \cdot l_\perp) - l_\perp^2 k_\perp \cdot l_\perp k^+ / (2l^+)}{[(l^+ - k^+)l_\perp^2 - (l_\perp - k_\perp)^2 l^+ + i\epsilon]l_\perp^2 k_\perp^2} \\ & + \frac{\alpha_s C_A}{2\pi^2} \int d^2 l_\perp \int_{k^+}^\infty dl^+ \frac{(k_\perp^2 - k_\perp \cdot l_\perp)(l_\perp^2 - k_\perp \cdot l_\perp)}{(l_\perp - k_\perp)^2 l_\perp^2 l^+ k_\perp^2}. \end{aligned} \quad (19)$$

As we are only interested in the UV behavior of the small- x gluon TMD, the external transverse momentum $k_\perp$ can be neglected in the integral in the first line. It is then trivial to carry out the elementary integration for l^+ . With the help of the identity

$$\int d^2 l_\perp \frac{(k_\perp^2 - k_\perp \cdot l_\perp)(l_\perp^2 - k_\perp \cdot l_\perp)}{(l_\perp - k_\perp)^2 l_\perp^2} = \int d^2 l_\perp \frac{k_\perp \cdot l_\perp (l_\perp^2 - k_\perp \cdot l_\perp)}{(l_\perp - k_\perp)^2 l_\perp^2}, \quad (20)$$

one arrives at

$$\begin{aligned} \mathcal{H}_{2a}(k) \approx & -\frac{\alpha_s C_A}{2\pi^2} \int d^2 l_\perp \left\{ \int_0^{k^+} dl^+ \frac{l^+ - k^+}{l_\perp^2} - \int_{k^+}^\infty \frac{dl^+}{l^+} \frac{l_\perp \cdot (l_\perp - k_\perp)}{2(l_\perp - k_\perp)^2 l_\perp^2} \right\} \\ = & \frac{\alpha_s C_A}{2\pi^2} \left\{ \frac{1}{2} \int \frac{d^2 l_\perp}{l_\perp^2} + \int_{k^+}^\infty \frac{dl^+}{l^+} \int d^2 l_\perp \frac{l_\perp \cdot (l_\perp - k_\perp)}{2(l_\perp - k_\perp)^2 l_\perp^2} \right\}. \end{aligned} \quad (21)$$

In contrast to a covariant gauge calculation, the vertex correction we obtained is free from the Collins-Soper-type light cone divergence. But the second integration $\int_{k^+}^\infty \frac{dl^+}{l^+}$ has the BFKL/BK-type light cone divergence when l^+ goes to infinity and leads to the small- x evolution of gluon TMD that is beyond the scope of the current work.

We turn to discuss the Wilson line self-energy correction,

$$\text{Fig.2(b)} \propto \mathcal{H}_{2b}(k) xG_0(x, k_\perp), \quad (22)$$

with the hard part

$$\mathcal{H}_{2b}(k) = \frac{\alpha_s C_A}{4\pi^3} \int dl^+ d^2 l_\perp dl^- \frac{i}{[2l^+ l^- - l_\perp^2 + i\epsilon][l^- - i\epsilon][l^+ + i\epsilon]} = -\frac{\alpha_s C_A}{2\pi^2} \int \frac{d^2 l_\perp}{l_\perp^2} \int_0^\infty \frac{dl^+}{l^+}, \quad (23)$$

which contains the light cone divergence when l^+ goes to zero. Such end point singularity can be cured by introducing a soft factor in the Collins-2011 scheme.

The gluon self-energy graph in Fig. 2(c) gives

$$\text{Fig. 2(c)} \propto \frac{1}{2} \mathcal{H}_{2c}(k) xG_0(x, k_\perp). \quad (24)$$

According to the LSZ reduction formula, half the one-loop correction of the gluon propagator contributes to the anomalous dimension of the gluon TMD, while another half contributes to the renormalization of gauge field. That is why we include a factor $\frac{1}{2}$ in the above equation. Once again, we use the residue theorem to perform the l^- integration in the hard part. This gives

$$\begin{aligned} \mathcal{H}_{2c}(k) = & \frac{1}{2} \frac{\alpha_s C_A}{4\pi^3} \int dl^+ d^2 l_\perp dl^- \frac{4i[(2l^+ - k^+)l^- - k_\perp^2]}{[2l^+ l^- - l_\perp^2 + i\epsilon][2l^- (l^+ - k^+) - (l_\perp - k_\perp)^2 + i\epsilon]k_\perp^2} \\ = & \frac{1}{2} \frac{\alpha_s C_A}{2\pi^2} \int d^2 l_\perp \int_0^{k^+} dl^+ \frac{2(2l^+ - k^+)l_\perp^2 / l^+ - 2k_\perp^2}{2[(l^+ - k^+)l_\perp^2 - l^+(l_\perp - k_\perp)^2]k_\perp^2}. \end{aligned} \quad (25)$$

As before, to explicitly isolate the UV pole contribution, the external transverse momentum $k_\perp$ is set to be zero. The l^+ integration can be done by very elementary methods,

$$\begin{aligned}\mathcal{H}_{2c}(k) &\approx \frac{1}{2} \frac{\alpha_s C_A}{2\pi^2} \int d^2 l_\perp \int_0^{k^+} dl^+ \frac{[(k^+)^2 + 4k^+ l^+ - 4(l^+)^2]}{(k^+)^3 l_\perp^2} \\ &= \frac{\alpha_s C_A}{2\pi^2} \int \frac{d^2 l_\perp}{l_\perp^2} \frac{5}{6}.\end{aligned}\quad (26)$$

Put all contributions from Fig. 2 together,

$$\begin{aligned}\mathcal{H}_{2a}(k) + \mathcal{H}_{2b}(k) + \frac{1}{2} \mathcal{H}_{2c}(k) \\ \approx \frac{\alpha_s C_A}{2\pi^2} (2\pi\mu)^{2\epsilon} \int \frac{d^{2-2\epsilon} l_\perp}{l_\perp^2} \left\{ \frac{11}{12} - \int_0^{k^+} \frac{dl^+}{l^+} \right\} \\ - \frac{\alpha_s C_A}{4\pi^2} \int_{k^+}^\infty \frac{dl^+}{l^+} \int d^2 l_\perp \frac{k_\perp^2}{l_\perp^2 (k_\perp - l_\perp)^2},\end{aligned}\quad (27)$$

where dimensional regularization is introduced to regulate the UV divergence in the first term. Some of finite terms might be missed at intermediate steps. However, such treatment is sufficient as we only need to compute UV pole terms for the current purpose. The second term combining with the Hermitian conjugate contributions gives rise to the virtual part of the evolution kernel of the BFKL equation. As observed in the earlier work, the large logarithms $\ln \frac{1}{x}$ and $\ln \frac{\zeta_c^2}{\mu^2}$ receive contributions from the well separated phase regions. It is easy to see that the UV divergence cancels out in the phase space region $k^+ \leq l^+ \leq \infty$. This is also the precise reason why one has to formulate the calculation in full QCD rather than in small- x formalism where many subleading terms in the power of $1/x$ are missed, including UV pole terms.

The end point singularity in the first term in Eq. (27) is canceled by the soft factor in the Collins-2011 scheme. Combining with contributions from the Hermitian conjugate diagrams, the subtracted gluon TMD then takes form

$$\begin{aligned}xG(x, k_\perp, \mu^2, \zeta_c^2) &\approx \frac{\alpha_s C_A}{2\pi^2} (2\pi\mu)^{2\epsilon} \int \frac{d^{2-2\epsilon} l_\perp}{l_\perp^2} \left\{ \frac{11}{6} - \ln \frac{\zeta_c^2}{l_\perp^2} \right\} \\ &\times xG_0(x, k_\perp) + \text{UV c.t.}\end{aligned}\quad (28)$$

One finds that both the factorization scale μ and the parameter ζ_c dependence of the gluon TMD show up at the next-to-leading order. The UV counterterm is added in the above equation to give a finite result at $\epsilon = 0$. Note that the collinear divergence in our calculation is absent once the incoming gluon transverse momentum $k_\perp$ is restored. In a conventional collinear factorization

calculation, the remaining collinear divergence can be removed after matching TMD onto gluon PDF.

The UV counterterm in the $\overline{\text{MS}}$ scheme is determined as [4],

$$\text{UV c.t.} = -\frac{\alpha_s C_A}{2\pi} S_\epsilon \left[\frac{1}{\epsilon^2} + \frac{1}{\epsilon} \left(\frac{11}{6} - \ln \frac{\zeta_c^2}{\mu^2} \right) \right], \quad (29)$$

where $S_\epsilon = \frac{(4\pi)^\epsilon}{\Gamma(1-\epsilon)}$. The anomalous dimension of the gluon TMD can be computed accordingly,

$$\begin{aligned}\gamma_G(g(\mu), \zeta_c^2/\mu^2) &= \frac{d(-\frac{\alpha_s C_A}{2\pi} S_\epsilon [\frac{1}{\epsilon^2} + \frac{1}{\epsilon} (\frac{11}{6} - \ln \frac{\zeta_c^2}{\mu^2})])}{d \ln \mu} \\ &= \frac{\alpha_s C_A}{\pi} \left(\frac{11}{6} - \ln \frac{\zeta_c^2}{\mu^2} \right),\end{aligned}\quad (30)$$

which is the same as the standard one. With this anomalous dimension, one reproduces the common Sudakov factor including both double leading logarithm and single leading logarithm contributions in the dilute limit.

We now calculate virtual correction in the presence of gluon rescattering. As argued before, it is not appropriate to first resum all-order gluon rescattering into the Wilson lines by applying the Eikonal approximation because the UV divergent part is the subleading contribution without $1/x$ enhancement. Instead, one should work out the UV part before resumming gluon rescattering. Here we start with one gluon rescattering case.

First of all, it is easy to check that diagrams with four gluon vertex have vanishing contribution in the gauge we specified above. We start evaluating the vertex correction from Figs. 3(a)–3(c). It is convenient to calculate the following combination,

$$\begin{aligned}&\frac{1}{2} \text{Fig.3(a)} + \text{Fig.3(c)} \\ &\propto \mathcal{H}_{\frac{q}{2}+c}(k, k_1) \int \frac{dx^- d^2 x_\perp}{(2\pi)^2} \frac{dy^- d^2 y_\perp}{(2\pi)^2} \frac{dz^- d^2 z_\perp}{(2\pi)^2} \\ &\quad \times e^{ik_\perp \cdot x_\perp} e^{-i(k_\perp - k_{1\perp}) \cdot y_\perp} e^{-ik_{1\perp} \cdot z_\perp} e^{i(k^+ - k_1^+) y^-} \\ &\quad \times e^{ik_1^+ z^-} \langle A_a^+(x^-, x_\perp) g_0 f^{bac} A_b^+(z^-, z_\perp) A_c^+(y^-, y_\perp) \rangle_x,\end{aligned}\quad (31)$$

with the hard part being given by

$$\begin{aligned}\mathcal{H}_{\frac{q}{2}+c}(k, k_1) &\propto \frac{C_A}{2} \frac{\alpha_s}{4\pi^3} \int d^2 k_{1\perp} dk_1^+ \int dl^+ d^2 l_\perp dl^- \frac{-i}{k_1^+ + i\epsilon} \\ &\quad \times \frac{[k_\perp \cdot (k_\perp - k_{1\perp} - l_\perp)][l_\perp \cdot (l_\perp - k_\perp + k_{1\perp})] - l^+ k_\perp \cdot l_\perp}{[2l^+ l^- - l_\perp^2 + i\epsilon][2l^-(l^+ - k^+ + k_1^+) - (l_\perp - k_\perp + k_{1\perp})^2 + i\epsilon][l^- - i\epsilon][l^+ + i\epsilon]}.\end{aligned}\quad (32)$$

FIG. 3. Virtual correction with one gluon rescattering. Collinear reducible diagrams [4] are not shown here.

As stated previously, we do not aim at getting the complete result. To clearly extract the UV divergent part associated with the leading power contribution, we set $k_{1\perp}$ to be zero and make the Taylor expansion in terms of the power $k_{\perp}/l_{\perp}$,

$$\begin{aligned} \mathcal{H}_{2+c}(k, k_1) &\propto \frac{C_A}{2} \frac{\alpha_s}{2\pi^2} \int d^2 k_{1\perp} dk_1^+ \int d^2 l_{\perp} \frac{-1}{k_1^+ + i\epsilon} \\ &\times \left\{ \int_0^{k^+ - k_1^+} dl^+ \frac{l^+ - k^+ + \frac{3}{2} k_1^+}{(k^+ - k_1^+)^2} \frac{k_{\perp}^2}{l_{\perp}^2} \right. \\ &\left. - \int_{k^+ - k_1^+}^{\infty} \frac{dl^+}{l^+} \frac{k_{\perp}^2}{l_{\perp}^2} \right\}. \end{aligned} \quad (33)$$

We rearrange the kinematic factor $k_{\perp}^2$ into the soft part and combine it with the gluon TMD matrix element by partial integration,

$$\begin{aligned} &k_{\perp}^2 \langle A_a^+(x^-, x_{\perp}) g_0 f^{bac} A_b^+(z^-, z_{\perp}) A_c^+(y^-, y_{\perp}) \rangle \\ \Rightarrow &\langle \partial_i A_a^+(x^-, x_{\perp}) g_0 f^{bac} A_b^+(z^-, z_{\perp}) \partial_i A_c^+(y^-, y_{\perp}) \rangle. \end{aligned} \quad (34)$$

Since the hard part is no longer dependent on $k_{1\perp}$, one can carry out the integration over $k_{1\perp}$. This produces a delta function $\delta^2(z_{\perp} - y_{\perp})$. The integration for $z_{\perp}$ then can be

trivially done. After performing the integral over k_1^+ by the residue theorem, one obtains

$$\begin{aligned} \frac{1}{2} \text{Fig.3(a)} + \text{Fig.3(c)} &\approx \frac{\alpha_s C_A}{4\pi^2} \int \frac{d^2 l_{\perp}}{l_{\perp}^2} \left\{ \int_{k^+}^{\infty} \frac{dl^+}{l^+} + \frac{1}{2} \right\} \\ &\times x G_1(x, k_{\perp}), \end{aligned} \quad (35)$$

which differs from the Fig. 2(a) by a factor 1/2. We will show that another half contribution comes from the combination $\frac{1}{2} \text{Fig. 3(a)} + \text{Fig. 3(b)}$. To simplify the calculation of $\frac{1}{2} \text{Fig. 3(a)} + \text{Fig. 3(b)}$, we play the following trick. One can treat the k_1 gluon as the collinear one (the error is power suppressed), and thus apply the Ward identity to the internal gluon line in Fig. 3(b). The hard part from Fig. 3(b) can be subsequently separated into two parts,

$$\begin{aligned} \text{Fig.3(b)} &\propto \mathcal{H}_b(k, k_1) \int \frac{dx^- d^2 x_{\perp}}{(2\pi)^2} \frac{dy^- d^2 y_{\perp}}{(2\pi)^2} \frac{dz^- d^2 z_{\perp}}{(2\pi)^2} \\ &\times e^{ik_{\perp} \cdot x_{\perp}} e^{-i(k_{\perp} - k_{1\perp}) \cdot y_{\perp}} e^{-ik_{1\perp} \cdot z_{\perp}} e^{i(k^+ - k_1^+) y^-} \\ &\times e^{ik_1^+ z^-} \langle A_a^+(x^-, x_{\perp}) g_0 f^{bac} A_b^+(z^-, z_{\perp}) \\ &\times A_c^+(y^-, y_{\perp}) \rangle_x \end{aligned} \quad (36)$$

where $\mathcal{H}_b(k, k_1)$ reads,

$$\begin{aligned} \mathcal{H}_b(k, k_1) &\propto \frac{C_A}{2} \frac{\alpha_s}{4\pi^3} \int d^2 k_{1\perp} dk_1^+ \int dl^+ d^2 l_{\perp} dl^- \frac{-i}{k_1^+ + i\epsilon} \\ &\times \left\{ \frac{[k_{\perp} \cdot (k_{\perp} - k_{1\perp} - l_{\perp})][l_{\perp} \cdot (l_{\perp} - k_{\perp} + k_{1\perp})] - l^- k^+ k_{\perp} \cdot l_{\perp}}{[2(l^+ + k_1^+)l^- - (l_{\perp} + k_{1\perp})^2 + i\epsilon][2l^-(l^+ - k^+ + k_1^+) - (l_{\perp} - k_{\perp} + k_{1\perp})^2 + i\epsilon][l^- - i\epsilon][l^+ + k_1^+ + i\epsilon]} \right. \\ &\left. - \frac{[k_{\perp} \cdot (k_{\perp} - k_{1\perp} - l_{\perp})][l_{\perp} \cdot (l_{\perp} - k_{\perp} + k_{1\perp})] - l^- k^+ k_{\perp} \cdot l_{\perp}}{[2l^+ l^- - l_{\perp}^2 + i\epsilon][2l^-(l^+ - k^+ + k_1^+) - (l_{\perp} - k_{\perp} + k_{1\perp})^2 + i\epsilon][l^- - i\epsilon][l^+ + k_1^+ + i\epsilon]} \right\}. \end{aligned} \quad (37)$$

Note that we relabeled the gluon momentum flow in the above equation. The internal gluon line sandwiched by the two incoming gluon lines carries momentum l . It is easy to check that the second term in the above equation is canceled out by the half of the contribution from Fig. 3(a). We are left with the first term once combining Fig. 3(b) with the half of Fig. 3(a),

$$\begin{aligned} \mathcal{H}_{b+\frac{g}{2}}(k, k_1) &\propto \frac{C_A}{2} \frac{\alpha_s}{4\pi^3} \int d^2 k_{1\perp} dk_1^+ \int dl^+ d^2 l_\perp dl^- \frac{-i}{k_1^+ + i\epsilon} \frac{1}{[2(l^+ + k_1^+)l^- - (l_\perp + k_{1\perp})^2 + i\epsilon]} \\ &\times \frac{[k_\perp \cdot (k_\perp - k_{1\perp} - l_\perp)][l_\perp \cdot (l_\perp - k_\perp + k_{1\perp})] - l^- k^+ k_\perp \cdot l_\perp}{[2l^- (l^+ - k^+ + k_1^+) - (l_\perp - k_\perp + k_{1\perp})^2 + i\epsilon][l^- - i\epsilon][l^+ + k_1^+ + i\epsilon]}, \end{aligned} \quad (38)$$

which can be further simplified by changing integration variable $l \rightarrow l + k_1$ and neglecting $k_{1\perp}$ in the numerator,

$$\begin{aligned} \mathcal{H}_{b+\frac{g}{2}}(k, k_1) &\propto \frac{C_A}{2} \frac{\alpha_s}{4\pi^3} \int d^2 k_{1\perp} dk_1^+ \int dl^+ d^2 l_\perp dl^- \frac{-i}{k_1^+ + i\epsilon} \frac{1}{[2l^+ l^- - l_\perp^2 + i\epsilon]} \\ &\times \frac{[k_\perp \cdot (k_\perp - l_\perp)][l_\perp \cdot (l_\perp - k_\perp)] - l^- k^+ k_\perp \cdot l_\perp}{[2l^- (l^+ - k^+) - (l_\perp - k_\perp)^2 + i\epsilon][l^- - i\epsilon][l^+ + i\epsilon]}. \end{aligned} \quad (39)$$

This turns out to be the same as the hard part of Fig. 2(a) except for the color factor. Following the procedure outlined above, the UV pole term extracted from Figs. 3(a)–3(c) is given by

$$\begin{aligned} \text{Fig.3(a)} + \text{Fig.3(b)} + \text{Fig.3(c)} &\approx \frac{\alpha_s C_A}{2\pi^2} \int \frac{d^2 l_\perp}{l_\perp^2} \left\{ \int_{k^+}^\infty \frac{dl^+}{l^+} - \int_0^{k^+} dl^+ \frac{l^+ - k^+}{(k^+)^2} \right\} xG_1(x, k_\perp) \\ &= \frac{\alpha_s C_A}{2\pi^2} \int \frac{d^2 l_\perp}{l_\perp^2} \left\{ \frac{1}{2} + \int_{k^+}^\infty \frac{dl^+}{l^+} \right\} xG_1(x, k_\perp). \end{aligned} \quad (40)$$

The vertex correction now is correctly reproduced with one gluon rescattering being taken into account.

Diagrams Figs. 3(f) and 3(g) also represent vertex correction, which however, do not contribute to the scale evolution of gluon TMD. Instead, they are responsible for the running of the strong coupling constant in the gauge link together with gluon self-energy diagram in Fig. 3(i) and the Wilson line self-energy diagram in Fig. 3(h). The similar calculation for the diagram in Fig. 3(f) leads to

$$\begin{aligned} \mathcal{H}_f(k_1, k_2) &\propto \frac{C_A}{2} \frac{\alpha_s}{4\pi^3} \int d^2 k_{1\perp} dk_1^+ \int dl^+ d^2 l_\perp dl^- \frac{1}{k_1^+ + i\epsilon} \\ &\times \frac{-i[l_\perp^2 - k_{1\perp} \cdot l_\perp]}{[2l^+ l^- - l_\perp^2 + i\epsilon][2l^- (l^+ - k_1^+) - (l_\perp - k_{1\perp})^2 + i\epsilon][l^- - i\epsilon][l^+ + i\epsilon]} \\ &= \frac{C_A}{2} \frac{\alpha_s}{2\pi^2} \int d^2 k_{1\perp} d^2 l_\perp \int dk_1^+ \frac{-1}{k_1^+ + i\epsilon} \\ &\times \left\{ \int_0^{k_1^+} dl^+ \frac{l_\perp^2 - k_{1\perp} \cdot l_\perp}{[(l^+ - k_1^+)l_\perp^2 - (l_\perp - k_{1\perp})^2 l^+ + i\epsilon]l_\perp^2} - \int_{k_1^+}^\infty dl^+ \frac{l_\perp^2 - k_{1\perp} \cdot l_\perp}{(l_\perp - k_{1\perp})^2 l_\perp^2 l^+} \right\}. \end{aligned} \quad (41)$$

The external transverse momentum $k_{1\perp}$ is set to be zero, one has

$$\mathcal{H}_f(k_1, k_2) \approx \frac{C_A}{2} \frac{\alpha_s}{2\pi^2} \int d^2 k_{1\perp} \int \frac{d^2 l_\perp}{l_\perp^2} \int \frac{dk_1^+}{k_1^+ + i\epsilon} \left\{ \int_0^{k_1^+} \frac{dl^+}{k_1^+} + \int_{k_1^+}^\infty \frac{dl^+}{l^+} \right\}. \quad (42)$$

By carrying out the contour integration for k_1^+ , the lower limit of the second integration is constrained to be zero. We arrive at

$$\text{Fig.3(f)} \approx \frac{C_A}{2} \frac{\alpha_s}{2\pi^2} \int \frac{d^2 l_\perp}{l_\perp^2} \left[1 + \int_0^\infty \frac{dl^+}{l^+} \right] xG_1(x, k_\perp). \quad (43)$$

The hard part of Fig. 3(g) is written as

$$\begin{aligned} \mathcal{H}_g(k_1, k_2) &\propto \frac{C_A \alpha_s}{2} \frac{1}{4\pi^3} \int d^2 k_{1\perp} dk_1^+ \int dl^+ d^2 l_\perp dl^- \frac{1}{k_1^+ + i\epsilon} \\ &\quad \times \frac{-i}{[2l^+ l^- - l_\perp^2 + i\epsilon][l^- - i\epsilon][k_1^+ + l^+ + i\epsilon]} \\ &= \frac{C_A \alpha_s}{2} \frac{1}{2\pi^2} \int d^2 k_{1\perp} \frac{dk_1^+}{k_1^+ + i\epsilon} \int \frac{d^2 l_\perp}{l_\perp^2} \int_{k_1^+}^{\infty} \frac{dl^+}{l^+ + i\epsilon}. \end{aligned} \quad (44)$$

After integrating over k_1^+ , one obtains

$$\text{Fig.3(g)} \approx \frac{C_A \alpha_s}{2} \frac{1}{2\pi^2} \int \frac{d^2 l_\perp}{l_\perp^2} \int_0^{\infty} \frac{dl^+}{l^+} x G_1(x, k_\perp). \quad (45)$$

The contributions from the Wilson line self-energy are listed as follows,

$$\text{Fig.3(d)} = \text{Fig.3(h)} = -\frac{\alpha_s C_A}{2\pi^2} \int \frac{d^2 l_\perp}{l_\perp^2} \int_0^{\infty} \frac{dl^+}{l^+} x G_1(x, k_\perp), \quad (46)$$

and the gluon self-energy diagrams,

$$\text{Fig.3(e)} \approx \text{Fig.3(i)} \approx \frac{\alpha_s C_A}{2\pi^2} \int \frac{d^2 l_\perp}{l_\perp^2} \frac{5}{12} x G_1(x, k_\perp). \quad (47)$$

We are now ready to assemble all pieces together. First, one notices that the light cone divergence is canceled out among Figs. 3(f)–3(h). Including the gluon self-energy diagram, we have

$$\text{Fig.3(f + g + h + i)} \approx \frac{\alpha_s C_A}{2\pi^2} \int \frac{d^2 l_\perp}{l_\perp^2} \frac{11}{12} x G_1(x, k_\perp), \quad (48)$$

from which one can reproduce the one-loop beta function that describes the scale dependence of the strong coupling constant. The summation of the rest of the diagrams in Fig. 3 gives

$$\text{Fig.3(a + b + c + d + e)} \approx \frac{\alpha_s C_A}{2\pi^2} \int \frac{d^2 l_\perp}{l_\perp^2} \left\{ \frac{11}{12} - \int_0^{k^+} \frac{dl^+}{l^+} \right\}. \quad (49)$$

Adding up Hermitian conjugate contributions and the soft factor, the final result reads

Fig.3(a + b + c + d + e + f + g + h + i)

$$\begin{aligned} &\approx \frac{\alpha_s C_A}{2\pi^2} (2\pi\mu)^{2\epsilon} \int \frac{d^{2-2\epsilon} l_\perp}{l_\perp^2} \left\{ \frac{11}{12} + \frac{11}{6} - \ln \frac{\zeta_c^2}{l_\perp^2} \right\} \\ &\quad \times x G_1(x, k_\perp) + \text{UV c.t.} \end{aligned} \quad (50)$$

The extra UV divergence can be removed by replacing the bare strong coupling constant g_0 with a renormalized one g in the gauge link,

$$\begin{aligned} &-g_0 \int_{y^-}^{-\infty} dz^- f^{bac} A_b^+(z^-, y_\perp) \\ &\Rightarrow -g \int_{y^-}^{-\infty} dz^- f^{bac} A_b^+(z^-, y_\perp), \end{aligned} \quad (51)$$

with

$$g_0 = g\mu^\epsilon \left[1 - \frac{\alpha_s C_A}{2\pi\epsilon} S_\epsilon \frac{11}{12} \right]. \quad (52)$$

Here, the quark-loop contribution is not included. The one-loop virtual correction to the gluon TMD now takes the form

$$\begin{aligned} x G(x, k_\perp, \mu^2, \zeta_c^2) &\approx \frac{\alpha_s C_A}{2\pi^2} (2\pi\mu)^{2\epsilon} \int \frac{d^{2-2\epsilon} l_\perp}{l_\perp^2} \left\{ \frac{11}{6} - \ln \frac{\zeta_c^2}{l_\perp^2} \right\} \\ &\quad \times \{x G_0(x, k_\perp) + x G_1(x, k_\perp)\} + \text{UV c.t.} \end{aligned} \quad (53)$$

where $x G_1$ denotes the gluon TMD with the gauge link $-g \int_{y^-}^{-\infty} dz^- f^{bac} A_b^+(z^-, y_\perp)$. It is easy to see that the anomalous dimension is not affected by gluon rescattering effect. This is more or less expected because the short distance physics (UV divergence) can not be altered by physics happens in long distance (gluon rescattering).

Before proceeding to compute virtual correction for the two gluon rescattering case, we now carry out a consistency check. To facilitate extracting the UV divergence, we systematically ignored the external transverse momenta. If one keeps transverse momenta $k_\perp$ and $k_{1\perp}$ finite, one is able to recover the corresponding small- x evolution equation. To this end, let us sum all terms containing the rapidity divergence $\int_{k^+}^{\infty} \frac{dl^+}{l^+}$ from diagrams Figs. 3(a)–3(c) and 3(h),

$$\begin{aligned} &\int_{k^+}^{\infty} \frac{dl^+}{l^+} \int d^2 k_{1\perp} d^2 l_\perp \left\{ -\frac{[k_\perp \cdot (k_\perp - k_{1\perp} - l_\perp)][l_\perp \cdot (l_\perp - k_\perp + k_{1\perp})]}{l_\perp^2 (l_\perp - k_\perp + k_{1\perp})^2} - \frac{[k_\perp \cdot (k_\perp - l_\perp)][l_\perp \cdot (l_\perp - k_\perp)]}{l_\perp^2 (l_\perp - k_\perp)^2} \right. \\ &\quad \left. + \frac{k_\perp^2}{l_\perp^2} e^{ik_\perp \cdot x_\perp} e^{-i(k_\perp - k_{1\perp}) \cdot y_\perp} e^{-ik_{1\perp} \cdot z_\perp} \langle \partial_\perp^i A_a^+(x^-, x_\perp) g_0 f^{bac} \int_{y^-}^{\infty} dz^- A_b^+(z^-, z_\perp) \partial_\perp^i A_c^+(y^-, y_\perp) \rangle \right\}. \end{aligned} \quad (54)$$

FIG. 4. One-loop virtual correction with two gluon rescattering. Collinear reducible diagrams [4] are not shown here.

After a few steps of algebraic manipulations, the above formula can be cast into the form

$$\begin{aligned}
 & \int_{k^+}^{\infty} \frac{dl^+}{l^+} \int d^2 k_{1\perp} d^2 l_{\perp} k_{\perp}^2 \left\{ \frac{1}{l_{\perp}^2} - \frac{l_{\perp} \cdot (l_{\perp} - k_{\perp} + k_{1\perp})}{2l_{\perp}^2 (l_{\perp} - k_{\perp} + k_{1\perp})^2} \right. \\
 & - \frac{l_{\perp} \cdot (l_{\perp} - k_{\perp})}{2l_{\perp}^2 (l_{\perp} - k_{\perp})^2} e^{ik_{\perp} \cdot x_{\perp}} e^{-i(k_{\perp} - k_{1\perp}) \cdot y_{\perp}} e^{-ik_{1\perp} \cdot z_{\perp}} \\
 & \times \langle \partial_{\perp}^i A_a^+(x^-, x_{\perp}) g_0 f^{bac} \int_{y^-}^{\infty} dz^- A_b^+(z^-, z_{\perp}) \\
 & \left. \times \partial_{\perp}^i A_c^+(y^-, y_{\perp}) \rangle \right\}. \quad (55)
 \end{aligned}$$

To make the expression more symmetrical, one can further split it into two identical parts and then exchange the integration variables $(z^-, z_{\perp}) \Leftrightarrow (y^-, y_{\perp})$ in the second part. After having done so, one immediately recognizes this as the virtual contribution of the evolution kernel of the DMMX equation [34], which is the BK-type equation governing the rapidity evolution of the WW gluon TMD. As long as the small- x evolution of the properly defined gluon TMD is described by the BK-type equation, the saturation phenomenology will automatically emerge. All saturation dynamic is inherently encoded in the matrix element definition of the gluon TMD given by Eq. (7). Our main point is that the UV divergence is localized in the phase space region $\int_0^{k^+} dl^+$ separately while the rapidity evolution is driven by gluons from the phase space region $\int_{k^+}^{\infty} dl^+$. It appears not to be easy task to extend the analysis to the higher-order gluon rescattering case through brutal force calculation as shown above. However, more complete treatment including

the all-order gluon rescattering effect, is feasible by making systematic small- x approximation [11].

To generalize the calculation to the two gluon rescattering case, let us reexamine the evaluations of Figs. 3(a)–3(c) from a different aspect of view. We start with investigating the k_1^+ pole structure of these three diagrams. Figures 3(b) and 3(c) generate the pole

$$\frac{1}{k_1^+ + l^+ + i\epsilon}, \quad (56)$$

while a double pole emerges in Fig. 3(a),

$$\frac{1}{k_1^+ + l^+ + i\epsilon} \frac{1}{k_1^+ + i\epsilon}. \quad (57)$$

If one picks up $\frac{1}{k_1^+ + l^+ + i\epsilon}$ pole contributions and carries out the l^- integration by closing the contour circles around the pole $\frac{1}{l^2 + i\epsilon}$, the Wilson line and the internal gluon line are effectively put on shell. Moreover, the external transverse momenta can be neglected when computing the UV counterterm. All exchanged gluons except for the left-most incoming gluon can be treated as the collinear ones. The Ward identity argument then can be applied in a physical gauge calculation. It is easy to see that the pole $\frac{1}{k_1^+ + l^+ + i\epsilon}$ contributions from three diagrams are canceled out due to the Ward identity. We are left with the $\frac{1}{k_1^+ + i\epsilon}$ pole contribution from Fig. 3(a). At this step, one can directly isolate the $\frac{1}{k_1^+ + i\epsilon}$ pole contribution using the residue theorem. The

rest calculation is exactly same as that for the standard vertex correction represented by Fig. 2(a).

In an analogous way, the calculation of Fig. 4 can be simplified by playing the same trick. For instance, the $\frac{1}{k_2^+ + l^+ + i\epsilon}$ pole contributions from Figs. 4(j)–4(l) are canceled out. Such cancelation also occurs among Figs. 4(m)–4(o). We are left with the $\frac{1}{k_2^+ + i\epsilon}$ pole contributions from Figs. 4(j) and 4(m). After carrying out integration over k_2^+ using the residue theorem, it becomes obvious that the calculations of Figs. 4(j) and 4(m) are the same as that for Figs. 3(f) and 3(g). We end up with

$$\begin{aligned} & \text{Fig.4(j + k + l + m + n + o)} \\ & \approx \frac{\alpha_s C_A}{2\pi^2} \int \frac{d^2 l_\perp}{l_\perp^2} \left[\frac{1}{2} + \int_0^\infty \frac{dl^+}{l^+} \right] x G_2(x, k_\perp). \end{aligned} \quad (58)$$

Other vertex diagrams can be grouped and lead to

$$\text{Fig.4(r + s)} \approx \frac{\alpha_s C_A}{2\pi^2} \int \frac{d^2 l_\perp}{l_\perp^2} \left[\frac{1}{2} + \int_0^\infty \frac{dl^+}{l^+} \right] x G_2(x, k_\perp). \quad (59)$$

Combining with the Wilson line self-energy diagrams Figs. 4(p) and 4(t), and gluon self-energy diagrams Figs. 4(q) and 4(u), we obtain the UV pole contributions as

$$\begin{aligned} & \text{Fig.4(i + j + k + l + m + n + o + p)} \\ & \approx \text{Fig.4(r + s + t + u)} \\ & \approx \frac{\alpha_s C_A}{2\pi^2} \int \frac{d^2 l_\perp}{l_\perp^2} \frac{11}{12} x G_2(x, k_\perp). \end{aligned} \quad (60)$$

The UV divergence from these diagrams can be absorbed into the gauge link by making the following replacement,

$$\begin{aligned} & \mathbf{g}_0^2 \int_{y^-}^{-\infty} dz^- f^{dab} A_d^+(z^-, y_\perp) \int_{y^-}^{z^-} dz_1^- f^{bdc} A_b^+(z_1^-, y_\perp) \\ & \Rightarrow \mathbf{g}^2 \int_{y^-}^{-\infty} dz^- f^{dab} A_d^+(z^-, y_\perp) \int_{y^-}^{z^-} dz_1^- f^{bdc} A_b^+(z_1^-, y_\perp). \end{aligned} \quad (61)$$

The rest of the diagrams contribute to the anomalous dimension of the gluon TMD. Here, we adopt the same strategy to simplify the calculation of the vertex correction from Figs. 4(a)–4(g). One notices that Figs. 4(b)–4(d) can be grouped together, while the cancelation of the $\frac{1}{k_2^+ + l^+ + i\epsilon}$ pole contributions occurs among Figs. 4(e)–4(g). After integrating out k_2^+ , we are ready to take care of k_1^+ pole contributions in a similar manner. Eventually, the same vertex correction is reproduced with graphs Figs. 4(a)–4(g). Combining with the Wilson line self-energy diagram in

Fig. 4(h) and gluon self-energy diagram in Fig. 4(i), one obtains

$$\begin{aligned} & \text{Fig.4(a + b + c + d + e + f + g + h + i)} \\ & \approx \frac{\alpha_s C_A}{2\pi^2} \int \frac{d^2 l_\perp}{l_\perp^2} \left[\frac{11}{12} - \int_0^{k^+} \frac{dl^+}{l^+} \right] x G_2(x, k_\perp). \end{aligned} \quad (62)$$

Now it is easy to see that the identical UV pole structure of the gluon TMD is recovered for the two-gluon rescattering case.

The method introduced above can be recursively applied to multiple gluon rescattering case starting from the right-most gluon attachment. The evaluation of diagrams with n gluon rescattering always can be reduced to the calculation of diagrams with $n - 1$ gluon attachment. The UV structure is not affected no matter how many soft gluons are exchanged. This is expected because as a quite general principle, long distance physics does not cause any impact on short distance physics. We verified this statement by explicit calculations for this specific case.

Now let us summarize our calculation. We computed the gluon TMD defined in Eq. (7) in terms of the operator given in Eq. (14). The calculation is compatible with the conventional treatment of the small- x formalism. To study the scale dependence of the gluon TMD, the UV pole terms from virtual corrections are explicitly worked out. Apart from leading to the scale evolution of gluon TMD, virtual correction also results in other effects: (1) the running of strong coupling constant, all the bare strong coupling constant in Eq. (14) should be replaced with the renormalized ones; (2) the bare gauge fields in Eq. (14) are replaced with the renormalized fields. The derived anomalous dimension of gluon TMD is $\gamma_G(g(\mu), 1) = \frac{\alpha_s C_A}{\pi} \frac{11}{6}$ (it is straightforward to include the quark loop contribution), which is the same as the one calculated without taking into account multiple gluon rescattering. Therefore, we conclude that both the double leading and single leading logarithms can be resummed to all orders in saturation regime by solving the CS evolution equation and the RG equation.

III. SUMMARY

This work is devoted to the study of the resummation of the single leading logarithms in a way consistent with the small- x formalism. In the Collins-2011 scheme, the double leading logarithm and single leading logarithm can be resummed into an exponentiation i.e., the Sudakov factor, by solving the Collins-Soper equation and the RG equation. In a previous publication, we showed that small- x gluon TMDs do satisfy the Collins-Soper equation. To derive the RG equation, we compute the one-loop virtual corrections to the WW-type gluon TMD in the presence of multiple gluon rescattering. It has been found that the UV divergence only arises in the region $0 < l^+ < k^+$ while the small- x evolution is driven by gluons carrying longitudinal momentum $l^+ > k^+$. One can first extract the scale

dependence of the gluon TMD, and then make small- x approximation to derive the BK like equation. As long as the rapidity dependence of the gluon TMD is shown to be governed by the DMMX equation, saturation phenomena will automatically emerge. In this sense, our calculation is valid for arbitrary small- x down to saturation regime. It is not surprising that the anomalous dimension of small- x gluon TMD determined through UV pole terms is found to be the same as the one calculated in the conventional way at one-loop order.

Our analysis can be straightforwardly applied to other cases, for instance, the WW-type gluon distribution with a future pointing gauge link and the dipole-type gluon distribution. We reached the same conclusion that the perturbative part of the resulting Sudakov factor takes

the same form in dense medium. However, it is not yet clear if the nonperturbative part of the Sudakov factor is affected by saturation effect. We leave this for future study. Nevertheless, it is now clear that the full k_t resummation machinery can be employed to perform the relevant phenomenological studies of physical observables involving two well-separated scales in the saturation regime.

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