Phenomenology of minimal theory of quasidilaton massive gravity

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The minimal theory of quasidilaton massive gravity with or without a Horndeski-type kinetic term for the quasidilaton field propagates only three physical modes: the two massive tensor polarizations and one scalar mode. This reduced number of degrees of freedom is realized by a Lorentz symmetry violation at cosmological scales and the presence of appropriate constraints that remove unwanted modes. Vacuum cosmological solutions have been considered in a previous work, and it has been shown that the late-time de Sitter attractor is stable under inhomogeneous perturbations. In this work, we explore the stability of cosmological solutions in the presence of matter fields. Assuming for simplicity that the quasidilaton scalar is on an attractor at the level of the background, we derive stability conditions in the subhorizon limit and find the scalar sound speeds, as well as the modification with respect to general relativity to the gravitational potential in the quasistatic approximation. We also find that the speed limit of gravitational waves coincides with the speed of light for any homogeneous and isotropic cosmological background, on or away from the attractor.

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I. INTRODUCTION

One of the most important questions in modern cosmology remains the elucidation of the origin of the current accelerated expansion of the Universe [1]. One of the studied avenues to address this so-called dark energy puzzle lies in large-distance, or infrared (IR), modifications to general relativity (GR). For example, gravity could be weaker at distances larger than the separation between galaxy clusters so that a large cosmological constant would not gravitate as strongly as in GR and would lead to a modest acceleration as observed. Alternatively, in the absence of a cosmological constant an IR modified behavior of gravity could dynamically source the acceleration of the expansion.

Among the IR modifications to Einstein gravity, adding a mass to the gravitational field is often presented as one of the simplest options, at least conceptually. In practice, it has however been difficult to construct a theory of massive gravity without compromising basic assumptions. In particular, the de Rham–Gabadadze–Tolley (dRGT) massive gravity [2], a unique healthy nonlinear theory of pure

massive gravity in four dimensions, does not accommodate Friedmann–Lemaître–Robertson–Walker (FLRW) solutions, meaning that either isotropy or homogeneity at large scales need to be considered as approximate [3]. Although this does not compromise the validity of the theory as long as the breaking of the approximate symmetry occurs at large enough distances [4], it effectively renders the analysis more tenuous. In order to evade the practical issues, several extensions of dRGT massive gravity have thus since then been proposed, for instance: massive bigravity [5], mass-varying massive gravity [6], modified matter couplings [7], scalar field extensions [8–12], and Lorentz-symmetry-violating extensions [13–17] (for the first linear study of a Lorentz-violating graviton mass see [18] and see also [19,20]). This has proven to be a fruitful path: several of these extensions are healthy and lead to interesting, non- Λ cold dark matter (non- Λ CDM) phenomenology.

The minimal theory of quasidilaton massive gravity [21,22], by means of a weak violation of Lorentz invariance and the addition of a scalar field called the quasidilaton, has been shown (i) to provide a stable cosmological vacuum de Sitter solution corresponding to a late-time de Sitter attractor and (ii) to accommodate the Vainshtein mechanism [23] to screen fifth force effects via a Horndeski-type kinetic term [24] for the quasidilaton field. On a similar basis as the minimal theory of massive gravity (MTMG) [13], the theory possesses constraints that remove the graviton scalar and vector modes. It thus propagates only

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3 degrees of freedom (d.o.f.), two tensors as in GR and the quasidilaton scalar, and can accommodate the Vainshtein screening for the quasidilaton. Therefore its phenomenology remains close to general relativity while giving a small mass (of cosmological value) to gravitational waves.

In this work, we study both background cosmology and linear perturbations in the presence of matter fields, thus extending the work done in [22]. We show that the dynamics of the quasidilaton scalar allow for an attractor that generalizes the attractor present in vacuum to the system with matter, analyze inhomogeneous perturbations around this attractor solution in the presence of matter, derive the stability conditions and find the expression for the effective gravitational constant in the quasistatic approximation.

The paper is organized as follows. In Sec. II we present the complete action for the minimal theory of quasidilaton massive gravity, in the unitary gauge. In Sec. III we present the cosmological background equations of motion and discuss the existence of the attractor solution. In Sec. IV we study the behavior of linear perturbations on the attractor. Finally we summarize and discuss our results in Sec. V.

II. ACTION IN THE UNITARY GAUGE

In order to construct a mass term for the physical metric field $g_{\mu\nu}$, one needs to introduce an additional, fiducial, metric $f_{\mu\nu}$. This nondynamical metric is taken as part of the definition of the theory. In the minimal theory of quasidilaton massive gravity, thanks to the quasidilation symmetry of the action [cf. (25)], one may choose a Minkowski fiducial metric and retain stable cosmological solutions [21,22]. For the remainder of the paper, we will thus keep $f_{\mu\nu} = \eta_{\mu\nu}$.

The graviton mass term, formed by contractions between the fiducial metric and the physical metric, will break the general covariance of the theory. While one may recover covariance by introducing four Stückelberg scalar fields, in what follows we remain in the “unitary gauge”: the gauge in which the Stückelberg fields do not appear. Note that once this gauge is chosen, a change of coordinates will correspond, in general, to a different physical solution.

Out of the two metrics, one can read off the Arnowitt-Deser-Misner lapse functions, shift vectors, three-dimensional metrics, and their inverses as

$$N = \frac{1}{\sqrt{-g^{00}}}, \quad N^i = \frac{1}{N^2} g^{i0},$$

$$\gamma^{ij} = g^{ij} + \frac{N^i N^j}{N^2}, \quad \gamma_{ik} \gamma^{kj} = \delta_i^j, \quad (1)$$

$$M = 1, \quad M^i = M_i = 0, \quad \tilde{\gamma}^{ij} = \tilde{\gamma}_{ij} = \delta_{ij}, \quad (2)$$

where $(N, N^i, \gamma^{ij}, \gamma_{ij})$ correspond to the physical sector; $(M, M^i, \tilde{\gamma}^{ij}, \tilde{\gamma}_{ij})$ correspond to the fiducial sector; and

$i, j, k \in \{1, 2, 3\}$. Using both three-dimensional metrics, one may further define $\mathfrak{K}^i_j$ and its inverse $\mathcal{K}^i_j$ such that

$$\mathfrak{K}^i_k \mathfrak{K}^k_j = \gamma^{ik} \tilde{\gamma}_{kj}, \quad \mathfrak{K}^i_k \mathcal{K}^k_j = \mathcal{K}^i_k \mathfrak{K}^k_j = \delta^i_j, \quad (3)$$

these two combinations being necessary to construct the graviton mass term.

In the present theory, the graviton mass terms break Lorentz invariance, in addition to being responsible for breaking general covariance. However, since matter remains Lorentz invariant, any experimental deviations from Lorentz symmetry are expected to originate only from graviton loops and to be further suppressed by the graviton mass, thus making them unobservable at present sensitivities. The graviton mass parameter is expected here to be of order of today’s Hubble rate, i.e., $m \sim H_0$, so as to become relevant at late times.

Building upon [22],¹ the action for the minimal quasidilaton, in the presence of matter, may be written using the objects defined above as

$$S = \tilde{S}_{\text{EH}} + \tilde{S}_\sigma + S_{\text{pot}} + S_C + S_m, \quad (4)$$

where $\tilde{S}_{\text{EH}}$ is a part that includes the Einstein-Hilbert action for the physical metric $g_{\mu\nu}$ (without cosmological constant, since this one is included in the graviton mass term), $\tilde{S}_\sigma$ is a part that includes the kinetic term of the quasidilaton scalar field σ , S_{pot} is the precursor graviton mass term, S_C is a part that includes the additional constraints which remove the scalar graviton mode, and S_m is the matter action.

For compactness, both kinetic terms for the graviton and the quasidilaton may be supplemented by a contribution from the quasidilaton cubic Horndeski sector, whose tertiary constraint mixes with the supplementary constraints used to minimize the number of physical d.o.f. in the theory [22]. We thus rewrite the action in a way that includes these contributions into the kinetic terms, i.e., the Einstein-Hilbert action as

$$\tilde{S}_{\text{EH}} = \int d^4x N \sqrt{\gamma} (R^{(3)} + \tilde{K}_{ij} \tilde{K}^{ij} - \tilde{K}^2), \quad (5)$$

and the quasidilaton kinetic term as

$$\tilde{S}_\sigma = \int d^4x \sqrt{-g} [P(X) - G(X) \square \sigma + \lambda_\chi (\tilde{\mathfrak{X}}_\sigma - X)], \quad (6)$$

where $R^{(3)}$ is the three-dimensional Ricci curvature; λ_χ is a Lagrange multiplier; $P(X)$ and $G(X)$ are free functions of the auxiliary field X ,

¹We refer the curious reader to both [21,22] for more details on the construction of the Lagrangian, and in particular the introduction of constraints and their Lagrange multipliers.

$$\begin{aligned}\tilde{K}_{ij} &= K_{ij} + \frac{1}{M_{\text{P}}^2} \frac{\lambda_T}{N} G_{,X} (X \gamma_{ij} + \sigma_{;i} \sigma_{;j}), \\ \tilde{\mathfrak{K}}_\sigma &= \frac{1}{2} [\partial_\perp \sigma^2 - \gamma^{ij} \partial_i \sigma \partial_j \sigma], \\ \widetilde{\partial_\perp \sigma} &= \partial_\perp \sigma + \frac{\lambda_T}{N} = \frac{1}{N} (\dot{\sigma} - N^i \sigma_{;i}) + \frac{\lambda_T}{N};\end{aligned}\quad (7)$$

and λ_T is the Lagrange multiplier for a tertiary constraint arising from the Horndeski sector. The comma generically indicates a derivative, as for example in $G_{,X} \equiv \partial G(X)/\partial X$. In the expressions (5)–(6), to simplify the form of the total action S we have replaced the usual extrinsic curvature K_{ij} (and its trace $K = \gamma^{ij} K_{ij}$) by $\tilde{K}_{ij}$ (and its trace $\tilde{K} = \gamma^{ij} \tilde{K}_{ij}$) and replaced the usual canonical kinetic term for the quasidilaton scalar field $\mathfrak{K}_\sigma$ by $\tilde{\mathfrak{K}}_\sigma$. This renders explicit the modification to the kinetic terms from the additional constraints and thus differentiates our theory from Lorentz-violating massive gravity theories which only modify the potential, e.g., as studied in [16,17].

From here on, we use a semicolon to indicate covariant derivatives with respect to γ_{ij} . The traces of $\mathcal{K}^i_j$ and $\mathfrak{K}^i_j$ are denoted as $\mathcal{K} \equiv \mathcal{K}^i_i$ and $\mathfrak{K} \equiv \mathfrak{K}^i_i$. Using these, the precursor graviton mass term is defined as

$$S_{\text{pot}} = \frac{M_{\text{P}}^2}{2} \int d^4x \sum_{i=0}^4 \mathcal{L}_i, \quad (8)$$

$$\mathcal{L}_0 = -m^2 c_0 e^{(4+\alpha)\sigma/M_{\text{P}}} \sqrt{\tilde{\gamma}} M, \quad (9)$$

$$\mathcal{L}_1 = -m^2 c_1 e^{3\sigma/M_{\text{P}}} \sqrt{\tilde{\gamma}} (N + M e^{\alpha\sigma/M_{\text{P}}} \mathcal{K}), \quad (10)$$

$$\mathcal{L}_2 = -m^2 c_2 e^{2\sigma/M_{\text{P}}} \sqrt{\tilde{\gamma}} \left[N \mathcal{K} + \frac{1}{2} M e^{\alpha\sigma/M_{\text{P}}} (\mathcal{K}^2 - \mathcal{K}^i_j \mathcal{K}^j_i) \right], \quad (11)$$

$$\mathcal{L}_3 = -m^2 c_3 e^{\sigma/M_{\text{P}}} \sqrt{\tilde{\gamma}} (N \mathfrak{K} + M e^{\alpha\sigma/M_{\text{P}}}), \quad (12)$$

$$\mathcal{L}_4 = -m^2 c_4 \sqrt{\tilde{\gamma}} N. \quad (13)$$

The bare cosmological constant is included in this potential term and is proportional to the parameter c_4 . However, as is discussed further in the text, c_4 is not necessary in order to accelerate the expansion of the Universe, as the rest of the graviton mass terms may also play this role. The remainder constraints part of the action is further divided as

$$S_C = S_{\lambda^i} + \tilde{S}_\lambda + S_{\lambda^2} + S_{\lambda_T}, \quad (14)$$

where

$$S_{\lambda^i} = \int d^4x \lambda^i \frac{m^2 M_{\text{P}}^2}{2} \left[\frac{1}{2} \sqrt{\tilde{\gamma}} \mathcal{D}_j (\Theta^{jk} \gamma_{ik}) - \mathcal{H} \partial_i \sigma \right], \quad (15)$$

$$\tilde{S}_\lambda = - \int d^4x \lambda \frac{m^2 M_{\text{P}}^2}{4} (2\mathcal{H} \widetilde{\partial_\perp \sigma} + \sqrt{\tilde{\gamma}} \tilde{K}_{ij} \Theta^{ij}), \quad (16)$$

$$S_{\lambda^2} = \int d^4x \lambda^2 \frac{m^4 M_{\text{P}}^2}{64N} \sqrt{\tilde{\gamma}} (2\Theta_{ij} \Theta^{ij} - \Theta^2), \quad (17)$$

$$S_{\lambda_T} = \int d^4x N \sqrt{\tilde{\gamma}} \frac{\lambda_T}{N} [G_{,X} \widetilde{\partial_\perp \sigma} \sigma^{;i}_{;i} - G_{,X} (\widetilde{\partial_\perp \sigma})^{;i} \sigma_{;i} - G_{,X} \partial_\perp X - P_{,X} \widetilde{\partial_\perp \sigma}], \quad (18)$$

with $i \in \{1, 2, 3\}$. Here we have defined

$$\begin{aligned}\Theta^{ij} &= e^{\alpha\sigma/M_{\text{P}}} \left\{ \frac{\sqrt{\tilde{\gamma}}}{\sqrt{\gamma}} [(c_1 e^{3\sigma/M_{\text{P}}} + c_2 e^{2\sigma/M_{\text{P}}} \mathcal{K}) \right. \\ &\quad \times (\mathcal{K}^i_k \gamma^{kj} + \gamma^{ik} \mathcal{K}^j_k) - 2c_2 e^{2\sigma/M_{\text{P}}} \tilde{\gamma}^{ij}] + 2c_3 e^{\sigma/M_{\text{P}}} \gamma^{ij} \Big\},\end{aligned}\quad (19)$$

$$\begin{aligned}\mathcal{H} &= \frac{e^{\alpha\sigma/M_{\text{P}}}}{M_{\text{P}}} \left\{ \sqrt{\tilde{\gamma}} \left[(4 + \alpha) c_0 e^{4\sigma/M_{\text{P}}} + (3 + \alpha) c_1 e^{3\sigma/M_{\text{P}}} \mathcal{K} \right. \right. \\ &\quad \left. \left. + \frac{2 + \alpha}{2} c_2 e^{2\sigma/M_{\text{P}}} (\mathcal{K}^2 - \mathcal{K}^i_j \mathcal{K}^j_i) \right] \right. \\ &\quad \left. + (1 + \alpha) c_3 \sqrt{\tilde{\gamma}} e^{\sigma/M_{\text{P}}} \right\}.\end{aligned}\quad (20)$$

Both λ_T and the corresponding constraint arise from the cubic Horndeski kinetic Lagrangian for the quasidilaton and from our specific choice of auxiliary variables, while the other Hamiltonian constraints still appear in the canonical case, albeit with different expressions.

Finally, for the matter action we assume that matter fields minimally couple to the four-dimensional physical metric,

$$g_{\mu\nu} dx^\mu dx^\nu = -N^2 dt^2 + g_{ij} (dx^i + N^i dt) (dx^j + N^j dt), \quad (21)$$

constructed from (N, N^i, g_{ij}) . Hereafter, for simplicity, we consider a perfect fluid. In particular one may choose an action *à la* Schutz and Sorkin [25], implemented as in [14]:

$$S_{\text{m}} = - \int d^4x (\sqrt{-g} \rho_{\text{m}}(n) + J^\mu \partial_\mu \varphi), \quad (22)$$

where $n = \sqrt{\frac{J_\mu J^\mu}{-N^2 \gamma}}$ and J^μ, φ are the (auxiliary) fields characterizing the fluid, or alternatively a k -essence scalar field

$$S'_{\text{m}} = \int d^4x \sqrt{-g} P_{\text{m}}(\mathfrak{X}_{\text{m}}), \quad (23)$$

with P_{m} a free function, $\mathfrak{X}_{\text{m}} = -\frac{1}{2} g^{\mu\nu} \nabla_\mu \phi_{\text{m}} \nabla_\nu \phi_{\text{m}}$ the canonical kinetic term for the matter scalar field ϕ_{m}

and for which the density and the sound speed can be expressed as

$$\rho_m = 2P'_m \mathfrak{X}_m - P_m, \quad c_{s,m}^2 = \frac{P'_m}{2P''_m \mathfrak{X}_m + P'_m}, \quad (24)$$

where we have used primes to denote derivatives of the free function. In the case of k -essence, when discussing perturbations, one needs to change variables from the scalar field perturbations $\delta\phi_m$ to density perturbations $\delta\rho$ in order to have a well-behaved dust limit ($P_m/\rho_m \rightarrow 0$, $c_{s,m}^2 \rightarrow 0$), for instance when computing the effective gravitational constant.

The total action is built to be invariant under the quasidilatation symmetry

$$\sigma \rightarrow \sigma + \sigma_0, \quad M \rightarrow M e^{-(1+\alpha)\sigma_0/M_P}, \quad \tilde{\gamma}_{ij} \rightarrow \tilde{\gamma}_{ij} e^{-2\sigma_0/M_P}, \quad (25)$$

with σ_0 a constant, and where nonzero values for the parameter α are also further admitted, since the gravitational action explicitly breaks Lorentz invariance. It has been shown in [21,22], that the quasidilatation symmetry allows the theory to contain stable self-accelerating background cosmologies without the need for a time dependence in the fiducial metric.

III. COSMOLOGICAL BACKGROUND

In the following we present the most important properties of the cosmological background. First, in Sec. III A we present our notation and give the background equations. Since the late-time dynamics is mainly characterized by an attractor solution, we then move on to describing this solution in Sec. III B and then comment on the cosmological evolution with respect to this attractor.

A. Background equations

Although this action is involved on arbitrary backgrounds, it has been shown in [22] that on a cosmological background, with a FLRW line element

$$ds^2 = -N^2(t)dt^2 + a^2(t)(\delta_{ij}dx^i dx^j), \quad (26)$$

and where all fields are taken to be time dependent only, the only solution is such that $\lambda = \lambda_T = \lambda^i = 0$, $i \in \{1, 2, 3\}$. For matter, we set $\rho_m = \rho_m^{(0)}(t) + \delta\rho_m$ but for the ease of notation drop the superscript (0). One can then obtain the background field equations from the minisuperspace action, which is greatly simplified compared to the general action.

To treat cosmological backgrounds with more ease, we define

$$\mathcal{X} = \frac{e^{\sigma/M_P}}{a}, \quad r = \frac{ae^{\alpha\sigma/M_P}}{N}, \quad \Sigma = \frac{\dot{\sigma}}{N}. \quad (27)$$

The cosmological background equations are then written as

$$E_1 \equiv 3M_P^2 H^2 - \rho_g - \rho_m = 0, \quad (28)$$

$$E_2 \equiv M_P^2 \frac{2\dot{H}}{N} + 3M_P^2 H^2 + P_g + P_m = 0, \quad (29)$$

$$E_\sigma \equiv \frac{1}{2} \mathcal{X} M_P^2 m^2 [(\alpha + 1)rJ + rJ_{,\mathcal{X}} \mathcal{X} + \Gamma_{,\mathcal{X}}] + P_{,\mathcal{X}} \Sigma \left(3H + \frac{\dot{\Sigma}}{\Sigma N} \right) + \Sigma^3 \frac{\dot{\Sigma}}{\Sigma N} (P_{,XX} + G_{,XX} \Sigma H) + 3G_{,\mathcal{X}} H \Sigma^2 \left(3H + \frac{\dot{H}}{HN} + 2 \frac{\dot{\Sigma}}{\Sigma N} \right) = 0, \quad (30)$$

$$E_X \equiv G_{,\mathcal{X}} \left(\frac{\dot{\Sigma}}{N} + 3H \Sigma \right) + P_{,\mathcal{X}} - \lambda_{\mathcal{X}} = 0, \quad (31)$$

$$E_{\lambda_T} \equiv E_X \Sigma = 0, \quad (32)$$

$$E_{\lambda_{\mathcal{X}}} \equiv X - \frac{1}{2} \Sigma^2 = 0, \quad (33)$$

$$E_{\lambda} \equiv \Gamma_{,\mathcal{X}} H M_P + \Sigma [(\alpha + 1)J + J_{,\mathcal{X}} \mathcal{X}] = 0, \quad (34)$$

whereas the equations for λ^i , E_{λ^i} , are trivially satisfied since they only include terms with spatial derivatives. We have defined the quantities

$$\rho_g \equiv 2XP_{,\mathcal{X}} - P + 6HG_{,\mathcal{X}} X \Sigma + \frac{M_P^2 m^2}{2} \Gamma, \quad (35)$$

$$P_g \equiv P - 2G_{,\mathcal{X}} X \frac{\dot{\Sigma}}{N} - \frac{M_P^2 m^2}{2} \left[\Gamma - \frac{\Gamma_{,\mathcal{X}}}{3} (r - 1) \right], \quad (36)$$

which can be interpreted as the contributions coming from the coupled sectors of the quasidilatation field and the graviton mass term to the Friedmann equations. In the previous expressions and in the following, for convenience, we also have grouped

$$\Gamma = \mathcal{X}^3 c_1 + 3\mathcal{X}^2 c_2 + 3\mathcal{X} c_3 + c_4, \quad J = \mathcal{X}^3 c_0 + 3\mathcal{X}^2 c_1 + 3\mathcal{X} c_2 + c_3. \quad (37)$$

Finally, we notice here that, in practice, one can generically solve the three background equations of motion (E_1 , E_2 , and E_σ) for P , r , and $\dot{H}$.

B. Cosmological attractor and evolution

Just as in [22], from the equation for λ , one can infer the existence of a late-time de Sitter attractor. In fact, Eq. (34) can be rewritten as

$$\frac{d}{dt}[a^{4+\alpha}\mathcal{X}^{1+\alpha}J(\mathcal{X})] = 0. \quad (38)$$

This result implies that

$$\mathcal{X}^{1+\alpha}J(\mathcal{X}) = \frac{cst}{a^{4+\alpha}}. \quad (39)$$

Notice that this result holds even in pure vacuum; i.e., this property is not changed by the presence of matter, since matter does not couple to λ . Notice also that we do not consider the space-time to be de Sitter, but will rather study the dynamics of the Universe on an attractor solution. Such attractor solutions are defined by

$$\begin{cases} \mathcal{X} = cst, & \text{for } \alpha = -4, \\ J = 0, \mathcal{X} = cst, & \text{for } \alpha \neq -4. \end{cases} \quad (40)$$

On the attractor, one has thus further

$$\Sigma|_{\text{attractor}} = HM_P, \quad X|_{\text{attractor}} = \frac{M_P^2 H^2}{2}. \quad (41)$$

Whenever $\mathcal{X} = cst$, the graviton potential induces an effective cosmological constant even without c_4 , which is the usual bare cosmological constant as found in GR. Therefore, in what follows, one may thus always safely assume $c_4 = 0$. This cosmological constant helps us however write the following condition on the background dynamics on the attractor, already given in [22]:

$$g(H) \equiv P_{,X} - 6 + 12G_{,X}H^2M_P + 3G_{,XX}H^4M_P^3 + H^2M_P^2P_{,XX} < 0, \quad \text{with } X = X|_{\text{attractor}}. \quad (42)$$

This consistency condition comes from requiring that an increase in the bare cosmological constant will always lead to an increase of the Hubble parameter. In fact, this quantity appears in several expressions for the background and for its perturbations.

We will not study the infinitely fine-tuned case $\alpha = -4$ in this work. Instead, we will focus on the interval $\alpha > -4$, for which the system tends to the attractor solution $J = 0$, having then neglected the strongly coupled solution $\mathcal{X} = 0$. Since, in general, the solution differs from the attractor solution by an inverse power law, the rate of change of such a deviation from the attractor (in a Hubble time) is $-(4 + \alpha)$, so that as long as α is not too close to -4 , then the solution is quickly approximated by the attractor solution. For example, when $\alpha \geq -1$, one may safely assume that the attractor has settled before or during the matter domination era.

In what follows, we characterize the background dynamics on the attractor with a few more details. The expression of r is

$$r - 1|_{\text{attractor}} = \frac{6H^2(3G_{,X}H^2M_P + P_{,X})}{m^2\Gamma_{,\mathcal{X}}\mathcal{X}} + \frac{6(P_m + \rho_m)g - 3g_1}{m^2M_P^2\Gamma_{,\mathcal{X}}\mathcal{X}} \frac{g - 3g_1}{g} \quad (43)$$

where g is given in (42) and $g_1 \equiv G_{,X}H^2M_P - 2$ are functions of H only since $X = X|_{\text{attractor}}$. The effective equation of state of dark energy $w_g = P_g/\rho_g$ is then

$$w_g|_{\text{attractor}} + 1 = -\frac{(P_m + \rho_m)(g + 6)}{\rho_g g}. \quad (44)$$

At late times, the Universe transitions to a de Sitter epoch guaranteed by the presence of the attractor, characterized by $w_g|_{\text{late}} = -1$. It is thus expected that any corrections close to this attractor appear in powers of H/H_{late} . We also have that if $-6 < g < 0$ the typical equation of state will be slightly $w_g > -1$ whereas in the other cases, the equation of state will be slightly $w_g < -1$. We invite the reader to see Fig. 2 for a depiction of these regions of interest in the parameter space.

To summarize, a conservative scenario for the cosmic evolution would be the following. First, the Universe undergoes the usual early-time phases, up to matter domination. During that time, the dynamics of the quasi-dilaton sector rapidly settles to the attractor $J = 0$, while all the deviations from GR remain negligible. Finally, as the contribution from dark energy becomes more important, one may measure an equation of state $w_g \neq -1$, but which will eventually settle to $w_g = -1$ at late times. During this last period modifications at the cosmological perturbation level may also arise.

Besides the attractor solution $J = 0$, other limits of the theory hold some interest. The limit $r \rightarrow 1$ corresponds to an asymptotic Minkowski limit of the theory, since it cancels the effective cosmological constant given by $\rho_g|_{\text{late}} = -P_g|_{\text{late}}$, as was shown in [22]. This can easily be seen from the expression of r on the attractor (43). Note also that the contributions from $P(X)$ and $G(X)$ cannot be neglected in general, since this would correspond to a case of strong coupling.

IV. LINEAR PERTURBATIONS

We present here an analysis of the action quadratic in perturbations for the tensor, vector, and scalar sectors. The different helicities do not mix at the linear level in the equations of motion, thus allowing us to study them separately also at the level of the action. In the case of scalar perturbations, the background is chosen to satisfy the attractor condition (40) (with $\alpha > -4$), whereas the analysis of vector and tensor perturbations is valid both on and away from the attractor.

Matter is taken to be a pressureless fluid, i.e., dust, in order to cover the matter domination and successive epochs. In fact, depending on the value of α , the attractor may be reached before or after the decay of matter fluids, be it radiation or matter. For such a reason, a study of early eras would in general necessitate a comprehension of the dynamics out of the attractor.

A. Tensor modes

One may include tensor perturbations by assuming

$$ds_3^2 = a^2(\delta_{ij} + h_{ij})dx^i dx^j, \quad (45)$$

$$N_i = 0, \quad (46)$$

$$N = N(t), \quad (47)$$

where $\delta^{ij}h_{ij} = 0 = \delta^{ik}\partial_k h_{ij}$. The Lagrangian quadratic in perturbations then reduces to

$$\mathcal{L} = \frac{M_{\text{p}}^2}{8} \sum_{\epsilon=+, \times} Na^3 \left[\frac{\dot{h}_\epsilon^2}{N^2} - \frac{1}{a^2} (\partial_i h_\epsilon)^2 - \mu_T^2 h_\epsilon^2 \right], \quad (48)$$

where $+$ and $\times$ denote the two different polarizations of tensor perturbations, and with

$$\mu_T^2 = \frac{1}{6} \mathcal{X} m^2 \left[\Gamma_{,\mathcal{X}} \mathcal{X} + (r-1) \frac{\Gamma_{,\mathcal{X}\mathcal{X}} \mathcal{X}^2}{2} \right]. \quad (49)$$

The Lagrangian (48) is valid both on and away from the attractor (40). The tensor mass, obtained by looking at the dispersion relation implied by the previous reduced action, is of order of m , which is supposed to be of order H_0 in order for this theory to explain the late-time acceleration of the Universe. We thus have $\mu_T \simeq H_0$. Furthermore, the tensor mass does not vanish in the Minkowski limit $r \rightarrow 1$. In the following dispersion relation, for the tensor modes,

$$\omega_T^2 = c_T^2 \frac{k^2}{a^2} + \mu_T^2, \quad (50)$$

we have $c_T^2 = 1$. This means that at all energy scales where m ($\simeq 10^{-33}$ eV) is negligible, e.g., for astrophysical phenomena, one recovers the same tensor mode phenomenology as in GR. In particular, this theory easily satisfies the most recent bounds on the graviton mass imposed by solar system observations, $|\mu_T| < \mathcal{O}(1) \times 10^{-24}$ eV [26], or by LIGO, $|\mu_T| < 7.7 \times 10^{-23}$ eV [27], and on the gravitational wave sound speed, i.e., $|1 - c_T/c| < \mathcal{O}(1) \times 10^{-15}$ [28]. It is expected that future space-based interferometers such as e.g., LISA [29] will provide a stronger bound $|\mu_T| < \mathcal{O}(1) \times 10^{-26}$ eV [30], and of course other model-dependent constraints can be investigated (for a short review see [31]).

B. Vector modes

The vector perturbations from the metric are given by

$$ds_3^2 = a^2(\delta_{ij} + \partial_{(i} E_{j)})dx^i dx^j, \quad (51)$$

$$N_i = B_i, \quad (52)$$

$$N = N(t), \quad (53)$$

while among the remaining fields only λ^i contributes by

$$\lambda^i = \delta \lambda^i, \quad (54)$$

since its background value is zero. All vector perturbations are taken to be transverse, i.e., $\partial^i E_i = \partial^i B_i = \partial^i \delta \lambda_i = 0$. Using the equations for $\delta \lambda_i$ one may integrate out all d.o.f., thus leaving no propagating vector mode. This was expected from the construction of the theory [22]. Again, this result is valid both on and away from the attractor (40).

C. Scalar modes

The scalar perturbations are given in the metric by

$$ds_3^2 = a^2(1 + 2\zeta)\delta_{ij}dx^i dx^j + 2a^2\partial_i\partial_j s dx^i dx^j, \quad (55)$$

$$N_i = N(t)\partial_i \chi, \quad (56)$$

$$N = N(t)(1 + \tilde{\alpha}), \quad (57)$$

and in the remaining fields by

$$X = X(t) + \delta X, \quad (58)$$

$$\sigma = \sigma(t) + \delta \sigma, \quad (59)$$

$$\lambda_\chi = \lambda_\chi(t) + \delta \lambda_\chi, \quad (60)$$

$$\lambda_T = \delta \lambda_T, \quad (61)$$

$$\lambda^i = \frac{1}{a^2} \partial_i \lambda_V, \quad (62)$$

$$\lambda = \delta \lambda. \quad (63)$$

Matter fields are also expanded about their background value. In particular, we set $\rho_m = \rho_m^{(0)}(t) + \delta \rho_m$, but drop the superscript (0) from the background quantity for the ease of notation. For the subsequent results, we will in fact rather use the gauge-invariant combination

$$\delta \equiv \frac{\delta \rho_m}{\rho_m} + 3\zeta, \quad (64)$$

instead of using directly $\delta \rho_m$.

Here in particular, the background is set to be the attractor solution (40) which is expected to be reached at early times, as discussed in Sec. III B. After expanding the action quadratically in the above linear perturbations, one may integrate out the nondynamical fields. Thus one obtains an action for two remaining propagating fields only, as well as equations setting the auxiliary variables. Details on the intermediary steps can be found in Appendix A.

With the field redefinition

$$\delta\sigma = \delta\sigma_2 - \frac{\beta_2}{\beta_3}\delta, \quad (65)$$

$$\delta = k\delta_2, \quad (66)$$

we find, once the other fields have been integrated out,

$$\begin{aligned} \mathcal{L} = & k^2 A_{11} \dot{\delta}_2^2 - \frac{\beta_3^2}{4\beta_1} \dot{\delta}\sigma_2^2 + \mathcal{B}(\dot{\delta}_2\delta\sigma_2 - \delta_2\dot{\delta}\sigma_2) - C_{11}\delta_2^2 \\ & - C_{22}\delta\sigma_2^2 - 2C_{12}\delta_2\delta\sigma_2, \end{aligned} \quad (67)$$

where the order of each coefficient in the high- k limit is

$$\begin{aligned} A_{11} &= \mathcal{O}(k^{-2}), & \frac{\beta_3^2}{4\beta_1} &= \mathcal{O}(k^0), & \mathcal{B} &= \mathcal{O}(k^{-1}), \\ C_{11} &= \mathcal{O}(k^0), & C_{12} &= \mathcal{O}(k^1), & C_{22} &= \mathcal{O}(k^2). \end{aligned} \quad (68)$$

We refer the reader to Appendix B for further details in the decomposition of the coefficients β_n and l_{mn} , as well as the terms in (68).

In what follows we successively study further the subhorizon limit (i.e., the extreme high- k limit), in which we find both no-ghost conditions and sound speeds for the two scalar modes, and the more specific quasistatic limit, in which we find both the effective gravitational constant characterizing matter perturbations and the η parameter. Aside from allowing us to compute unambiguously the aforementioned relevant quantities, both subhorizon limits are also justified for observational purposes. Indeed, in present studies of redshift-space distortion (RSD) (see e.g., [32–34]), the relevant range of wave numbers is $k \gg H_0$, but still in the regime of linear perturbation theory.

1. Subhorizon limit

In the high- k limit, we find the following approximated Lagrangian:

$$\begin{aligned} \mathcal{L} \approx & \frac{1}{2} N a^3 \left[Q_1 \frac{\dot{\delta}_2^2}{N^2} + Q_2 \frac{\dot{\delta}\sigma_2^2}{N^2} + \frac{a}{k} B \left(\frac{\dot{\delta}_2}{N} \delta\sigma_2 - \delta_2 \frac{\dot{\delta}\sigma_2}{N} \right) \right. \\ & \left. - L_{11} \delta_2^2 - 2L_{12} \frac{k}{a} \delta_2 \delta\sigma_2 - L_{22} \frac{k^2}{a^2} \delta\sigma_2^2 \right], \end{aligned} \quad (69)$$

where Q_1 , Q_2 , B , and L_{lm} ($l, m \in \{1, 2\}$), are all functions of time only. The friction coefficient B is subleading in the quasistatic approximation (see next section).

The kinetic coefficients read

$$Q_1 = a^2 \rho_m, \quad (70)$$

$$Q_2 = \frac{\Gamma_1^2}{\Gamma_2^2} \frac{q^2}{4Qd^2}, \quad (71)$$

where we have defined

$$\Gamma_1 = \mathcal{X}(c_1 \mathcal{X}^2 + 2c_2 \mathcal{X} + c_3) = \frac{1}{3} \Gamma_{,\mathcal{X}} \mathcal{X}, \quad (72)$$

$$\Gamma_2 = \mathcal{X}^2(c_1 \mathcal{X} + c_2) = \frac{1}{6} \Gamma_{,\mathcal{X}\mathcal{X}} \mathcal{X}^2, \quad (73)$$

$$\begin{aligned} q = & 2Q[2H^2(4 + \alpha)^2 + m^2\Gamma_2] - 3m^2[(2 - M_P H^2 G_{,\mathcal{X}})^2 \Gamma_2 \\ & + (4 + \alpha)(2 - M_P H^2 G_{,\mathcal{X}}) \Gamma_1], \end{aligned} \quad (74)$$

$$d = m^2 \Gamma_1 + 2H^2(4 + \alpha)(M_P H^2 G_{,\mathcal{X}} - 2), \quad (75)$$

$$\begin{aligned} Q = & 3M_P^3 G_{,\mathcal{X}\mathcal{X}} H^4 + \frac{3}{2} M_P^2 H^4 (G_{,\mathcal{X}})^2 + M_P^2 P_{,\mathcal{X}\mathcal{X}} H^2 \\ & + 6M_P G_{,\mathcal{X}} H^2 + P_{,\mathcal{X}} \equiv g + \frac{3}{2} (G_{,\mathcal{X}} H^2 M_P - 2)^2. \end{aligned} \quad (76)$$

From these, it is possible to read the no-ghost conditions in the high- k limit. The no-ghost condition for the field δ_2 reads

$$\rho_m > 0, \quad (77)$$

which is always trivially satisfied for canonical matter. The other nontrivial no-ghost condition leads to

$$Q > 0. \quad (78)$$

This latter condition is, in form, equal to the case of de Sitter, studied in [22].

For the ease of reading, we first decompose the mass coefficients in powers of m as

$$L_{11} = -\frac{a^2 \rho_m^2}{2q^2 M_P^3 \Gamma_1^2} (m^0 L_{11,0} + m^2 L_{11,2} + m^4 L_{11,4}), \quad (79)$$

$$L_{12} = -\frac{a \rho_m}{2dq M_P^3 \Gamma_1 \Gamma_2} (m^0 L_{12,0} + m^2 L_{12,2} + m^4 L_{12,4}), \quad (80)$$

$$L_{22} = \frac{1}{2d^2 M_P^3 \Gamma_2^2} (m^0 L_{22,0} + m^2 L_{22,2} + m^4 L_{22,4}). \quad (81)$$

The coefficients $L_{ij,k}$ are given in Appendix C for presentation purposes. From these mass coefficients, it is possible

to read the scalar sound speed in the high- k limit. In particular, for modes for which $\omega^2 = c_s^2 \frac{k^2}{a^2}$, we have

$$\left(\omega^2 Q_2 - L_{22} \frac{k^2}{a^2} \right) \delta\sigma_2 \approx 0, \quad (82)$$

so that

$$c_s^2 = \frac{L_{22}}{Q_2}, \quad (83)$$

whereas dust still has zero speed of propagation. The explicit subhorizon quantities such as the no-ghost conditions or the sound speed for scalar perturbations are one of the main results of this work.

2. Quasistatic approximation

Using the action (69), one obtains the following equations of motion:

$$\begin{aligned} -\frac{d}{dt} \left(a^3 Q_1 \frac{\dot{\delta}_2}{N} \right) - \frac{1}{2} \frac{d}{dt} \left(\frac{a^4}{k} B \delta\sigma_2 \right) - \frac{a^4}{2k} B \dot{\delta}\sigma_2 \\ - Na^3 L_{11} \delta_2 - Na^3 L_{12} \frac{k}{a} \delta\sigma_2 = 0, \end{aligned} \quad (84)$$

$$\begin{aligned} -\frac{d}{dt} \left(a^3 Q_2 \frac{\dot{\delta}\sigma_2}{N} \right) + \frac{a^4}{2k} B \dot{\delta}_2 + \frac{1}{2} \frac{d}{dt} \left(\frac{a^4}{k} B \delta_2 \right) \\ - Na^3 L_{12} \frac{k}{a} \delta_2 - Na^3 L_{22} \frac{k^2}{a^2} \delta\sigma_2 = 0. \end{aligned} \quad (85)$$

In the quasistatic approximation, where $\frac{\delta\sigma_2}{N^2} \simeq H \frac{\delta\sigma_2}{N} \simeq H^2 \delta\sigma_2 \ll \frac{k^2}{a^2} \delta\sigma_2$, these two equations become

$$L_{12} \delta_2 + L_{22} \frac{k}{a} \delta\sigma_2 \approx 0, \quad (86)$$

$$\frac{1}{Na^3} \frac{d}{dt} \left(a^3 Q_1 \frac{\dot{\delta}_2}{N} \right) + L_{11} \delta_2 + L_{12} \frac{k}{a} \delta\sigma_2 \approx 0. \quad (87)$$

By solving (86) for $\delta\sigma_2$ and substituting it to (87), one obtains the equation for δ_2 as

$$\frac{1}{N} \frac{d}{dt} \left(\frac{\dot{\delta}_2}{N} \right) + 2H \frac{\dot{\delta}_2}{N} + \frac{1}{\rho_m a^2} \left(L_{11} - \frac{L_{12}^2}{L_{22}} \right) \delta_2 \approx 0. \quad (88)$$

By comparing this with the definition of G_{eff} ,

$$\frac{1}{N} \frac{d}{dt} \left(\frac{\dot{\delta}}{N} \right) + 2H \frac{\dot{\delta}}{N} - 4\pi G_{\text{eff}} \rho_m \delta = 0, \quad (89)$$

one can obtain

$$\frac{G_{\text{eff}}}{G_N} = \frac{2M_{\text{P}}^2}{\rho_m a^2} \left(\frac{L_{12}^2}{L_{22}} - L_{11} \right). \quad (90)$$

This ratio is generally different from 1, aside from the GR limit discussed below.

In this same quasistatic approximation it is possible to show that the two gauge-invariant Bardeen potentials

$$\Psi = \tilde{\alpha} + \frac{\dot{\chi}}{N} - \frac{1}{N} \frac{d}{dt} \left(\frac{a^2 \dot{\chi}}{N} \right), \quad (91)$$

$$\Phi = -\zeta - H\chi + a^2 H \frac{\dot{\chi}}{N}, \quad (92)$$

both satisfy the Poisson equations

$$-\frac{k^2}{a^2} \Psi = \frac{1}{2M_{\text{P}}^2} \frac{G_{\text{eff}}}{G_N} \rho_m \delta, \quad (93)$$

$$-\frac{k^2}{a^2} \Phi = \frac{1}{2M_{\text{P}}^2} \frac{G_{\Phi}}{G_N} \rho_m \delta, \quad (94)$$

where the ratio G_{eff}/G_N is given in Eq. (90), whereas we can decompose

$$\frac{G_{\Phi}}{G_N} = \frac{g_{\Phi,n,4} m^4 + g_{\Phi,n,2} m^2 + g_{\Phi,n,0}}{g_{\Phi,d,4} m^4 + g_{\Phi,d,2} m^2 + g_{\Phi,d,0}} \quad (95)$$

with coefficients explicitly given in Appendix D for presentation purposes. By combining both equations, one then directly finds the slip parameter η , simply given by

$$\eta \equiv \frac{\Psi}{\Phi} = \frac{G_{\text{eff}}}{G_{\Phi}}, \quad (96)$$

which is also generally different from 1, aside from the GR limit discussed below.

Although we have given the general expressions for G_{eff} and η , and their values do depend on the choice of the model and the background, we can study, for example, their behavior at early times, i.e., when $\rho_m \rightarrow 3M_{\text{P}}^2 H^2$ and $m^2/H^2 \rightarrow 0$, in the case $|\Gamma_2| \ll 1$, $G_{,X} H^2 M_{\text{P}} \rightarrow 0$, and $G_{,XX} H^4 M_{\text{P}}^3 \rightarrow 0$. In fact, in this case we find

$$G_{\text{eff}} \rightarrow G_N, \quad (97)$$

$$\eta \rightarrow 1, \quad (98)$$

$$Q_2 \rightarrow \frac{(\alpha + 4)^2 (M_{\text{P}}^2 H^2 P_{,XX} + P_{,X}) \Gamma_1^2}{4\Gamma_2^2}, \quad (99)$$

$$c_s^2 \rightarrow \frac{3}{6 - M_{\text{P}}^2 H^2 P_{,XX} - P_{,X}}. \quad (100)$$

This result shows that there is parameter space for which it is possible to have at early times a GR limit in the behavior

FIG. 1. The effective gravitational constant for matter perturbations G_{eff}/G_N , evaluated in the present Universe, may become relevantly smaller than unity. Here its value is presented for a range of graviton masses μ_T^2/H_0^2 , using an illustrative choice of parameters (101).

of the growth of structure, together with a stable background, provided that $0 < M_{\text{p}}^2 H^2 P_{,XX} + P_{,X} < 6$. The simple limit $m \rightarrow 0$ corresponds to a limit towards GR plus a Horndeski sector.

As a proof of principle, we also present an example setup producing weak gravity in the present Universe. We choose

$$\alpha = 0, \quad \Gamma_1 = 1, \quad \Gamma_2 = 3, \quad \Omega_{m,0} \equiv \frac{\rho}{3M_{\text{p}}^2 H_0^2} = 0.3,$$

$$P_{,X} = 1, \quad G_{,X} = G_{,XX} = P_{,XX} = 0, \quad (101)$$

where all time-dependent values are assumed to be evaluated today. The resulting one-parameter family of models has a wide parameter space in which, at present, $G_{\text{eff}}/G_N|_{\text{today}} < 1$. We show this in Fig. 1, in which we plot G_{eff}/G_N versus μ_T^2/H_0^2 which is left as a free parameter by our previous choice. For example, a further choice of $\mu_T^2/H_0^2 = 5$ would yield $G_{\text{eff}}/G_N|_{\text{today}} \simeq 0.91$. It should be noted that in the analog Lorentz-invariant scalar-tensor theory case, a simple model of minimally coupled quintessence, it would not be possible to have an effective gravitational constant less than unity. Passing on the necessary specific choices in terms of parametrization, current data (Planck data combined with the CMASS galaxy catalog [35]) constrains deviations from $G_{\text{eff}}/G_N = 1$ up to about 10%. We therefore expect—although an accurate statement would necessitate an in-depth analysis—that while current data would only marginally constrain our theory, future surveys will be much more efficient in doing so.

The explicit expression for quantities derived in the quasistatic limit such as the effective gravitational constant or the η parameter are among the main results of this work.

V. SUMMARY AND DISCUSSION

In [22], it was shown that the minimal theory of quasidilaton theory could be extended with a Horndeski

structure to accommodate the Vainshtein screening, while retaining important features at the cosmological level. Among these are the existence of FLRW cosmologies and in particular the presence of a late-time de Sitter attractor, generally stable under perturbations. Here, we extend that work by showing that the dynamics of the quasidilaton scalar allow for an attractor that generalizes the attractor solution to the system with matter. This was expected since the attractor condition depends on the equation of motion for λ , which does not couple directly to the matter sector. The attractor thus obtained is shown to be stable in a wide range of parameters under inhomogeneous perturbations. It is sufficient to satisfy the no-ghost condition (78) and the positivity of the squared sound speed (83), in order to avoid ghost and gradient instabilities. We have then found that the speed limit of gravitational waves coincides with the speed of light for any homogeneous and isotropic cosmological background, on or away from the attractor. Furthermore, we have studied scalar perturbations of this theory in the quasistatic approximation and found a modified gravitational constant G_{eff} and slip parameter η for matter perturbations as shown in Eqs. (90) and (96).

This last result might be of particular interest as it is well known that it is in general hard to have a nontrivial modified-gravitational law (especially when we look for theories for which $G_{\text{eff}} < G_N$), which could be in principle able to explain weak gravity. It was shown in [15] that weak gravity can be easily achieved in the normal branch of MTMG. The minimal theory of quasidilaton massive gravity with a cubic Horndeski Lagrangian, while keeping the speed of the tensor modes equal to unity, might still then be able to achieve weak gravity. Although more work is needed in order to study this interesting possibility, the phenomenology of this theory looks promising in the light of future experiments which can shed light on the behavior of gravity, e.g., in RSD data.

In Fig. 2, we show how the free functions can be constrained already with healthiness requirements and measures of the equation of state of dark energy w_g . It is expected, from different models, that e.g., Euclid will constrain this equation of state to about 10% [36].

The construction presented in this work has concrete advantages with respect to other Lorentz breaking massive gravity theories with the same or a lesser number of gravitational d.o.f. In [37], it is pointed out that without modifying the kinetic term for gravity, as was instead done in this work, one may obtain theories with 3 or even 2 gravitational d.o.f. However, for the theory with 3 d.o.f., there exist signs of a strong coupling issue around Minkowski backgrounds. The theory with 2 d.o.f. has, to our knowledge, not yet been explored in the presence of matter fields and does not accommodate self-accelerating solutions, although perturbations may still be modified. As these theories are expressed as effective field theories, they come with a UV cutoff scale which can be in principle as

FIG. 2. Left: The unhealthy regions (shaded) in the $(g, G_X X/M_P)$ -parameter space, on the attractor (40). At a given time, g is a function of the Hubble rate H given by (42). “Background ghost” refers to a decrease in the Hubble rate when the bare cosmological constant is increased, which is avoided by the negativity of g . The scalar no-ghost condition (78) is the only nontrivial one. Right: The equation of state of dark energy w_g , on the attractor, is presented as a function of g [see Eq. (44)], for different values of the ratio of dust and dark energy densities ρ_d/ρ_g . At late times (full gray line), one tends towards an effective cosmological constant. The thick line corresponds to a ratio close to the current ratio. While the quantity $G_X X$ is most constrained by the no-ghost condition, any measure of the current or the past w_g will heavily constrain g . Bounds on w_g with e.g., Euclid may improve to about 10% for different models [36].

high as $\Lambda_2 \sim \sqrt{M_P m}$. This is high enough to accommodate solar-system probes of gravity. On the other hand, theories that modify the kinetic term can potentially have an even higher cutoff, and are expressed in a full nonlinear fashion. As was expressed before, finally, the minimal theory of quasidilaton massive gravity has the advantage that it can produce interesting cosmologies with new phenomenology with a fiducial structure that is purely Minkowskian, in the unitary gauge. One disadvantage is the presence of free functions, which we however expect will be well constrained both by phenomenology and the requirement of an efficient Vainshtein screening.

In future work, it will be interesting to make contact with observations at shorter scales. While finding the modified gravitational constant was a first step in this regard, several other parameters are relevant to observations. These should in principle be obtainable in the context of the minimal theory of quasidilaton massive gravity. Due to the presence of the quasidilaton field, one originally [21] expected the presence of a fifth force, thus potentially compromising the agreement of the theory at e.g., Solar System scales. Thanks to the extension by Horndeski-type terms [22] this can in principle be avoided by the Vainshtein effect. It remains important to study how this Vainshtein mechanism works to screen the potential fifth force in detail.

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APPENDIX A: INTEGRATION OF THE NONDYNAMICAL SCALAR PERTURBATIONS

We give here more details on some intermediary steps in our analysis of the scalar perturbations. Once the action is perturbed quadratically, one may then integrate out the nondynamical fields successively. For example, the equation of motion for the field λ_V sets the following constraint:

$$\zeta = \frac{1}{2} \frac{[c_1(\alpha + 6)\mathcal{X}^2 + 2c_2(5 + \alpha)\mathcal{X} + c_3(4 + \alpha)]\delta\sigma}{M_P \mathcal{X}(c_1 \mathcal{X} + c_2)}. \quad (\text{A1})$$

The equation of motion for $\delta\lambda_\chi$ sets another constraint, namely

$$\delta\lambda_T = M_P N H \tilde{\alpha} - \dot{\delta}\sigma + \frac{N\delta X}{M_P H}. \quad (\text{A2})$$

At this point we use the equation of motion for the field χ to integrate out the field $\tilde{\alpha}$. After replacing $\tilde{\alpha}$, we integrate out the field χ , via its own equation of motion. One can use the equation of motion for $\delta\lambda$ in order to set a constraint for δX in terms of the other fields.

After integrating out δX and all remaining Lagrange multipliers, the reduced Lagrangian is a function of $s, \delta\sigma$, and δ , which can be written as follows:

$$\mathcal{L} = A_{11}\dot{\delta}^2 - B_{12}(\dot{\delta}\delta\sigma - \delta\dot{\delta}\sigma) - l_{11}\delta^2 - 2l_{12}\delta\delta\sigma - l_{22}\delta\sigma^2 + \beta_1 s^2 + s(\beta_2\dot{\delta} + \beta_3\dot{\delta}\sigma + \beta_4\delta + \beta_5\delta\sigma). \quad (\text{A3})$$

One can solve the equation of motion for s as

$$s = -\frac{1}{2\beta_1}(\beta_2\dot{\delta} + \beta_3\dot{\delta}\sigma + \beta_4\delta + \beta_5\delta\sigma), \quad (\text{A4})$$

to integrate out s from the action.

APPENDIX B: COEFFICIENTS IN THE SCALAR PERTURBATIONS LAGRANGIAN

The coefficients in the Lagrangian (67) can be expressed in terms of the coefficients β_n and l_{mn} , defined in (A4), as

$$\mathcal{B} = \frac{[-4B_{12}\beta_1 + (-\beta_5 + \dot{\beta}_3)\beta_2 + \beta_3(\beta_4 - \dot{\beta}_2)]k}{4\beta_1}, \quad (\text{B1})$$

$$C_{22} = l_{22} - \frac{\beta_3\dot{\beta}_5}{4\beta_1} + \frac{\beta_5^2}{4\beta_1} - \frac{\dot{\beta}_3\beta_5}{4\beta_1} + \frac{\beta_3\beta_5\dot{\beta}_1}{4\beta_1^2}, \quad (\text{B2})$$

$$C_{11} = \frac{k^2\dot{\beta}_3\beta_2\beta_4}{2\beta_1\beta_3} + k^2l_{11} + \frac{k^2l_{22}\beta_2^2}{\beta_3^2} - \frac{k^2l_{12}\beta_2}{2\beta_3} + \frac{k^2\beta_2^2\beta_5^2}{4\beta_1\beta_3^2} - \frac{k^2\beta_2\beta_4\beta_5}{2\beta_1\beta_3} + \frac{k^2\beta_4^2}{4\beta_1} \\ + \frac{k^2\dot{\beta}_2^2}{4\beta_1} - \frac{k^2\dot{\beta}_2\dot{\beta}_3\beta_2}{2\beta_1\beta_3} + \frac{k^2\dot{\beta}_2B_{12}}{\beta_3} + \frac{k^2\dot{\beta}_2\beta_2\beta_5}{2\beta_1\beta_3} - \frac{k^2\dot{\beta}_2\beta_4}{2\beta_1} + \frac{k^2\dot{\beta}_3\beta_2^2}{4\beta_1\beta_3^2} - \frac{k^2\dot{\beta}_3B_{12}\beta_2}{\beta_3^2} - \frac{k^2\dot{\beta}_3\beta_2^2\beta_5}{2\beta_1\beta_3^2}, \quad (\text{B3})$$

$$C_{12} = kl_{12} - \frac{kl_{22}\beta_2}{\beta_3} - \frac{k\beta_2\dot{\beta}_3}{8\beta_1} - \frac{k\beta_2\beta_5^2}{4\beta_3\beta_1} + \frac{k\beta_3\dot{\beta}_2}{8\beta_1} + \frac{k\beta_4\beta_5}{4\beta_1} \\ - \frac{k\beta_3\dot{\beta}_1\dot{\beta}_2}{8\beta_1^2} + \frac{k\dot{\beta}_1\dot{\beta}_3\beta_2}{8\beta_1^2} - \frac{k\dot{\beta}_1\beta_2\beta_5}{8\beta_1^2} + \frac{k\beta_3\dot{\beta}_1\beta_4}{8\beta_1^2} - \frac{k\dot{\beta}_2\beta_5}{8\beta_1} - \frac{k\beta_3\dot{\beta}_4}{8\beta_1} + \frac{k\dot{\beta}_3\beta_2\beta_5}{4\beta_3\beta_1} - \frac{k\dot{\beta}_3\beta_4}{8\beta_1} + \frac{k\dot{\beta}_5\beta_2}{8\beta_1}. \quad (\text{B4})$$

One may further study the decomposition of β_n and l_{mn} in powers of k ,

$$\beta_1 = \frac{\beta_{11}k^6 + \beta_{12}k^4}{\beta_{13}k^2 + \beta_{14}}, \quad (\text{B5})$$

$$\beta_2 = \frac{\beta_{21}k^2}{\beta_{22}k^2 + \beta_{23}}, \quad (\text{B6})$$

$$\beta_3 = \frac{\beta_{31}k^2}{\beta_{32}}, \quad (\text{B7})$$

$$\beta_4 = \frac{\beta_{41}k^4 + \beta_{42}k^2}{\beta_{43}k^2 + \beta_{44}}, \quad (\text{B8})$$

$$\beta_5 = \frac{\beta_{51}k^6 + \beta_{52}k^4 + \beta_{53}k^2}{\beta_{54}k^2 + \beta_{55}}, \quad (\text{B9})$$

$$l_{11} = \frac{l_{111}k^4 + l_{112}k^2 + l_{113}}{l_{114}k^4 + l_{115}k^2 + l_{116}}, \quad (\text{B10})$$

$$l_{12} = \frac{l_{121}k^6 + l_{122}k^4 + l_{123}k^2 + l_{124}}{l_{125}k^4 + l_{126}k^2 + l_{127}}, \quad (\text{B11})$$

$$l_{22} = \frac{l_{221}k^6 + l_{222}k^4 + l_{223}k^2 + l_{224}}{l_{225}k^2 + l_{226}}, \quad (\text{B12})$$

$$B_{12} = \frac{B_{121}k^2 + B_{122}}{B_{123}k^2 + B_{124}}, \quad (\text{B13})$$

where the objects β_{mn} , l_{mno} , and B_{mno} are functions of time only.

Let us now calculate $\mathcal{B}$. We find that at highest order in k

$$\mathcal{B} = \frac{k(\beta_{13}(-\beta_{43}\beta_{51}\beta_{21}\beta_{32} + \beta_{41}\beta_{54}\beta_{22}\beta_{31})B_{123} - 4\beta_{32}\beta_{11}\beta_{22}\beta_{54}\beta_{43}B_{121})}{4\beta_{32}\beta_{22}\beta_{11}\beta_{54}\beta_{43}B_{123}} = 0 \cdot k + \mathcal{O}(k^{-1}),$$

so that we need to look for the next leading order, which is of order $\mathcal{O}(1/k)$; that is, $\mathcal{B} = a^4 B(t)/(2k) + \mathcal{O}(k^{-3})$.

Let us now calculate C_{22} . We find

$$C_{22} = \frac{k^4(4l_{221}\beta_{11}\beta_{54}^2 + l_{225}\beta_{13}\beta_{51}^2)}{4\beta_{11}l_{225}\beta_{54}^2} = 0 \cdot k^4 + \mathcal{O}(k^2),$$

on substituting the equations of motion. Then the next term is of order $\mathcal{O}(k^2)$, as in

$$\begin{aligned} \mathcal{C}_{22} = & \frac{k^2}{2\beta_{11}^2 l_{225}^2 \beta_{54}^3 \beta_{32}^2} \left[l_{225}^2 \left\{ \beta_{51} \left(\left[\left(\beta_{52} \beta_{13} + \frac{1}{2} \beta_{51} \beta_{14} \right) \beta_{11} - \frac{1}{2} \beta_{12} \beta_{13} \beta_{51} \right] \beta_{54} - \beta_{11} \beta_{13} \beta_{55} \beta_{51} \right) \beta_{32}^2 \right. \right. \\ & + \frac{1}{2} \{ [(-\dot{\beta}_{13} \beta_{31} - \dot{\beta}_{31} \beta_{13}) \beta_{51} - \dot{\beta}_{51} \beta_{13} \beta_{31}] \beta_{11} + \beta_{51} \dot{\beta}_{11} \beta_{13} \beta_{31} \} \beta_{54} + \beta_{51} \dot{\beta}_{54} \beta_{11} \beta_{13} \beta_{31} \} \beta_{54} \beta_{32} \\ & \left. \left. + \frac{1}{2} \beta_{31} \beta_{11} \beta_{13} \beta_{54} \beta_{51} \dot{\beta}_{32} \right\} + 2l_{225} l_{222} \beta_{32}^2 \beta_{11}^2 \beta_{54}^3 - 2l_{226} l_{221} \beta_{32}^2 \beta_{11}^2 \beta_{54}^3 \right]. \end{aligned} \quad (\text{B14})$$

Along the same lines we find that the leading order term for $\mathcal{C}_{11}$ which is of order k^2 vanishes, so that we need to evaluate the next leading order which is of order $\mathcal{O}(k^0)$. As for the term $\mathcal{C}_{12}$, we have that the leading order term, which is of order k^3 , vanishes and thus we need to evaluate the next leading order which is of order $\mathcal{O}(k)$.

APPENDIX C: COEFFICIENTS FOR THE MASS TERMS OF SCALAR PERTURBATIONS

The coefficients appearing in the mass terms for scalar perturbations, as defined in Eqs. (79)–(81), are given by

$$L_{11,0} = 16(\alpha + 4)^2 H^2 Q^2 \left[H^2 (\xi_2 - 6\Gamma_2 \iota_{-g_1}) + \frac{2\Gamma_2^2 (g - 3g_1) \rho_m}{M_{\text{P}}^2 g} \right], \quad (\text{C1})$$

$$L_{11,2} = 8(\alpha + 4) \Gamma_1 Q \left[H^2 (3g_1 \iota_0 \iota_2 + 2\Gamma_2 Q \epsilon) + \frac{6\Gamma_2^2 Q \rho_m}{M_{\text{P}}^2 g} \right], \quad (\text{C2})$$

$$L_{11,4} = 3\Gamma_1^2 g_1 \left[3g_1 (8\Gamma_2 \iota_{-g_1} + \xi_1) + 8\Gamma_2 Q \iota_{-(1+\alpha)} - \frac{18\Gamma_2^2 g_1 g_2 \rho_m}{H^2 M_{\text{P}}^2 g} \right], \quad (\text{C3})$$

$$L_{12,0} = 8(\alpha + 4)^2 H^2 Q \left\{ H^2 M_{\text{P}}^2 [(g_1 + 2)(\iota_0^2 - 6\Gamma_2 \iota_{-g_1}) - 2\Gamma_2 P_{,X} \iota_{-g_1}] + \frac{2\Gamma_2 \iota_{-g_1} (3g_1 - g) \rho_m}{g} \right\}, \quad (\text{C4})$$

$$\begin{aligned} L_{12,2} = & 2(\alpha + 4) \Gamma_1 M_{\text{P}}^2 \left(H^2 \{ 3g_1^2 \iota_{-6} \iota_0 + 2g_1 [3(\alpha + 4) \Gamma_1 \iota_{-(6+P_{,X})} + 2\Gamma_2 Q \epsilon] - 2Q \iota_0 \iota_{-2(3+\alpha)} \} \right. \\ & \left. - \frac{6\Gamma_2 \rho_m (3g_1 g_2 \iota_0 + 2Q \iota_{-g_1})}{M_{\text{P}}^2 g} \right), \end{aligned} \quad (\text{C5})$$

$$\begin{aligned} L_{12,4} = & \Gamma_1^2 M_{\text{P}}^2 \left\{ 12\Gamma_2 g_1^2 \iota_{-(3+\alpha)} - 3g_1 (\iota_{-(3+\alpha)}^2 + \Gamma_2^2 (-\alpha^2 - 6\alpha + 2P_{,X} + 3)) + 4\Gamma_2 g \iota_{-(1+\alpha)} \right. \\ & \left. - \frac{18\Gamma_2^2 g_1 g_2 \rho_m}{H^2 M_{\text{P}}^2 g} \right\}, \end{aligned} \quad (\text{C6})$$

$$\begin{aligned} L_{22,0} = & 4(\alpha + 4)^2 H^2 \left(g_1 H^2 M_{\text{P}}^2 \{ 2\Gamma_2 P_{,X} (\iota_{-g_1} + \iota_0) - (g_1 + 2) [\iota_0^2 - 6\Gamma_2 (\iota_{-g_1} + \iota_0)] \} \right. \\ & \left. + 2\rho_m \left[\frac{3}{g} (g_1 \iota_{-g_1}^2 + g_2 \iota_0^2) - \iota_{-g_1}^2 \right] \right), \end{aligned} \quad (\text{C7})$$

$$\begin{aligned} L_{22,2} = & 4(\alpha + 4) \Gamma_1 \left(H^2 M_{\text{P}}^2 \{ g_1 (\iota_{2-2\alpha} \iota_{-g_1} - \Gamma_2 g_1 \iota_{-5}) - (\alpha + 4) \Gamma_1 [\iota_0 - 2\Gamma_2 (P_{,X} + 6)] \} \right. \\ & \left. + \frac{\rho_m}{g} [6(\alpha + 4) \Gamma_1 \Gamma_2 g_2 - 3\iota_{-g_1}^2] \right), \end{aligned} \quad (\text{C8})$$

$$L_{22,4} = \frac{\Gamma_1^2}{H^2} \left(H^2 M_{\text{P}}^2 \xi_3 + 6\Gamma_2^2 g_2 \frac{\rho_m}{g} \right), \quad (\text{C9})$$

where we have defined $\iota_n \equiv (4 + \alpha)\Gamma_1 + n\Gamma_2$, for $n \in \mathbb{R}$, and

$$g_1 = G_{,X}H^2M_P - 2, \quad (C10)$$

$$g_2 = 2G_{,X}H^2M_P + G_{,XX}H^4M_P^3, \quad (C11)$$

$$\epsilon = 2(\alpha + 4)\Gamma_1 - (3 + 2\alpha)\Gamma_2 \quad (C12)$$

$$\xi_1 = (\alpha + 4)^2\Gamma_1^2 - 4(\alpha + 4)\Gamma_1\Gamma_2 - 2(P_{,X} + 6)\Gamma_2^2 \quad (C13)$$

$$\xi_2 = (\alpha + 4)^2\Gamma_1^2 + 6(\alpha + 4)\Gamma_1\Gamma_2 + 2(P_{,X} + 6)\Gamma_2^2 \quad (C14)$$

$$\xi_3 = 3(\alpha + 4)^2\Gamma_1^2 - 4(\alpha + 4)(\alpha + g_1 + 2)\Gamma_1\Gamma_2 + 2(2(\alpha + 3)g_1 + P_{,X} + 6)\Gamma_2^2 \quad (C15)$$

$$g = P_{,X} - 6 + 12G_{,X}H^2M_P + 3G_{,XX}H^4M_P^3 + H^2M_P^2P_{,XX} = Q - \frac{3}{2}g_1^2, \quad (C16)$$

and Γ_1 , Γ_2 , q , d , and Q are given in Eqs. (72)–(76).

APPENDIX D: COEFFICIENTS FOR G_Φ/G_N

The coefficients appearing in Eq. (95), which describes the effective gravitational constant for the Bardeen perturbation Φ , are explicitly given by

$$g_{\Phi,n,4} = -2\Gamma_1^2H^2\iota_2M_P^2g_{1-(1+\alpha)}, \quad (D1)$$

$$g_{\Phi,n,2} = -4(\alpha + 4)\Gamma_1H^2(3\rho_m[\Gamma_2g_1\iota_2 + \Gamma_2(g_2 - 2)\iota_0 - \iota_0^2] + H^2M_P^2g\{g_1[\iota_0\iota_{6-\alpha} - 2(2\alpha + 3)\Gamma_2^2] + \iota_0[\Gamma_2(2\alpha + P_{,X} + 12) - \iota_0]\}), \quad (D2)$$

$$g_{\Phi,n,0} = -8(\alpha + 4)^2H^4\{\rho_m[\iota_2\iota_{-g_1}(3g_1 - g) + 3g_2\iota_0^2] + H^2M_P^2g[\Gamma_2g_1\iota_2(3g_1 + P_{,X} + 6) + \iota_0(g_1\iota_{-6} + 2\iota_{-(6+P_{,X})})]\}, \quad (D3)$$

$$g_{\Phi,d,4} = -\Gamma_1^2[H^2M_P^2g(4\iota_{-(3+\alpha)}\iota_{-g_1} - \xi_1) + 6\Gamma_2^2g_2\rho_m], \quad (D4)$$

$$g_{\Phi,d,2} = 12(\alpha + 4)\Gamma_1H^2\rho_m[\iota_{-g_1}^2 - 2(\alpha + 4)\Gamma_1\Gamma_2g_2] - 4(\alpha + 4)\Gamma_1H^4M_P^2g\{(g_1 + 2)[-2\alpha\Gamma_2\iota_{-g_1} + 3\Gamma_2(\iota_{g_1} + \iota_2) + \iota_0\iota_{-2(2+g_1)}] - \xi_3 + 2\Gamma_2\iota_1P_{,X}\}, \quad (D5)$$

$$g_{\Phi,d,0} = -24(\alpha + 4)^2H^4\rho_m[g_1\iota_{-g_1}^2 + (\alpha + 4)^2\Gamma_1^2g_2] + 8(\alpha + 4)^2H^4\rho_mg\iota_{-g_1}^2 + 4(\alpha + 4)^2g_1H^6M_P^2g\{(g_1 + 2)[\iota_0^2 - 6\Gamma_2(\iota_{-g_1} + \iota_0)] - 2\Gamma_2P_{,X}(\iota_{-g_1} + \iota_0)\}, \quad (D6)$$

where we have used definitions (C10)–(C16) and (72)–(76).

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