Exploring dark matter, neutrino mass, and $R_{K^{(*)},\phi}$ anomalies in $L_\mu - L_\tau$ model

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We investigate Majorana dark matter in a new variant of $U(1)_{L_\mu-L_\tau}$ gauge extension of the Standard Model, where the scalar sector is enriched with an inert doublet and a $(\bar{3}, 1, 1/3)$ scalar leptoquark. We compute the weakly interacting massive particle–nucleon cross section in the leptoquark portal and the relic density mediated by inert doublet components, the leptoquark, and the new Z' boson. We constrain the parameter space consistent with the Planck limit on the relic density and the PICO-60 and LUX bounds on the spin-dependent direct detection cross section. Furthermore, we constrain the new couplings from the present experimental data on $\text{Br}(\tau \rightarrow \mu\nu_\tau\bar{\nu}_\mu)$, $\text{Br}(B \rightarrow X_s\gamma)$, $\text{Br}(\bar{B}^0 \rightarrow \bar{K}^0\mu^+\mu^-)$, $\text{Br}(B^+ \rightarrow K^+\tau^+\tau^-)$, and $B_s - \bar{B}_s$ mixing, which occur at the one-loop level in the presence of the Z' and leptoquark. Using the allowed parameter space, we estimate the form-factor-independent $P'_{4,5}$ observables and the lepton nonuniversality parameters R_K , R_{K^*} , and R_ϕ . We also briefly discuss neutrino mass generation at the one-loop level and the viable parameter region to explain current neutrino oscillation data.

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I. INTRODUCTION

Though the experimentally measured values of various physical observables are in excellent agreement with the Standard Model (SM) predictions, there are many open unsolved problems—such as the matter-antimatter asymmetry, the hierarchy problem, the dark matter (DM) content of the Universe, etc.—which make us believe that there is something beyond the SM. In this regard, the study of rare semileptonic B decay processes provide an ideal testing ground to critically test the SM and to look for a possible extension of it. Although so far we have not observed any clear indication of new physics (NP) in the B sector, there are several physical observables associated with flavor-changing neutral current $b \rightarrow s l^+ l^-$ processes which have $(2-4)\sigma$ [1–6] discrepancies. In particular, the observation of the 3σ anomaly in the P'_5 angular observables [4] and the decay rate [5] of $B^0 \rightarrow K^{*0}\mu^+\mu^-$ processes have recently attracted a lot of attention. The decay rate of $B_s \rightarrow \phi\mu^+\mu^-$ also has a 3σ deviation compared to its SM prediction [3]. Furthermore, the LHCb Collaboration has observed the

violation of lepton universality in the $B^+ \rightarrow K^+ l^+ l^-$ process in the low $q^2 \in [1, 6]$ GeV^2 region [2],

$$R_K^{\text{Expt}} = \frac{\text{Br}(B^+ \rightarrow K^+\mu^+\mu^-)}{\text{Br}(B^+ \rightarrow K^+e^+e^-)} = 0.745^{+0.090}_{-0.074} \pm 0.036, \quad (1)$$

which has a 2.6σ deviation from the corresponding SM result [7]

$$R_K^{\text{SM}} = 1.0003 \pm 0.0001. \quad (2)$$

In addition, an analogous lepton nonuniversality (LNU) parameter (R_{K^*}) has also been observed in $B^0 \rightarrow K^{*0} l^+ l^-$ processes [1],

$$\begin{aligned} R_{K^*}^{\text{Expt}} &= \frac{\text{Br}(B^0 \rightarrow K^{*0}\mu^+\mu^-)}{\text{Br}(B^0 \rightarrow K^{*0}e^+e^-)} \\ &= 0.66^{+0.11}_{-0.07} \pm 0.03, \quad q^2 \in [0.045, 1.1] \text{ GeV}^2, \\ &= 0.69^{+0.11}_{-0.07} \pm 0.05, \quad q^2 \in [1.1, 6] \text{ GeV}^2, \end{aligned} \quad (3)$$

which correspond to deviations of 2.2σ and 2.4σ from their SM predictions [8],

$$\begin{aligned} R_{K^*}^{\text{SM}}|_{q^2 \in [0.045, 1.1] \text{ GeV}^2} &= 0.92 \pm 0.02, \\ R_{K^*}^{\text{SM}}|_{q^2 \in [1.1, 6] \text{ GeV}^2} &= 1.00 \pm 0.01. \end{aligned} \quad (4)$$

To resolve the above $b \rightarrow s l l$ anomalies, we extend the SM gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$ with a local

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$U(1)_{L_\mu-L_\tau}$ symmetry. The anomaly-free $L_\mu - L_\tau$ gauge extensions [9,10] are attractive, as they have a minimal number of new particles and parameters, making them phenomenologically valuable. The model is quite simple in structure, and is suitable for studying the phenomenology of DM, neutrinos, and the flavor anomalies [11–15]. It has been well explored in the dark matter context in the literature [12–15], in both the gauge and scalar portals. In the literature [16–19], DM, neutrino, and flavor phenomenology have also been investigated in several $U(1)$ -extended models. The approach of adding color triplet particles to shed light on the flavor sector (thereby connecting with the dark sector) is interesting. Leptoquarks (LQs) are not only advantageous in addressing the flavor anomalies, but they also act as a mediator between the visible and dark sectors. A few works have already studied this idea [20–23].

Leptoquarks are hypothetical color triplet gauge particles, with either spin 0 (scalar) or spin 1 (vector), which connect the quark and lepton sectors and thus carry both baryon and lepton numbers simultaneously. They can arise from various extended SM scenarios [24–35] that treat quarks and leptons on equal footing, such as the grand unified theories [24–27], the color $SU(4)$ Pati-Salam model [28–32], the extended technicolor model [33,34] and the composite models of quarks and leptons [35]. In this article, we study a new version of the $U(1)_{L_\mu-L_\tau}$ gauge extension of the SM with a $(\bar{3}, 1, 1/3)$ scalar LQ (SLQ) and an inert doublet, to study the phenomenology of dark matter and neutrino mass generation and to compute the flavor observables within a single framework. The SLQ mediates the annihilation channels contributing to the relic density and also plays a crucial role in direct searches as well, providing a spin-dependent weakly interacting massive particle (WIMP)-nucleon cross section which is quite

sensitive to the recent and ongoing direct-detection experiments such as PICO-60 and LUX. The Z' gauge boson of the extended $U(1)$ symmetry and the SLQ also play an important role in settling the known issues of the flavor sector. In this regard, we would like to investigate whether the observed anomalies in the rare leptonic/semileptonic decay processes mediated by $b \rightarrow sl^+l^-$ transitions can be explained in the present framework. We analyze the implications of the model on both the DM and flavor sectors, in particular on $B \rightarrow Vl^+l^-$ ($V = K^*, \phi$) decay modes. In the literature [36–59] there have been many attempts to explain the observed anomalies of rare B decays in the scalar leptoquark model.

The paper is structured as follows. We describe the particle content, relevant Lagrangian and interaction terms, and the pattern of symmetry breaking in Sec. II. We derive the mass eigenstates of the new fermions and the scalar spectrum in Sec. III. We then provide a detailed study of DM phenomenology regarding the relic density and direct-detection observables in Sec. IV. The mechanism of generating light neutrino masses at the one-loop level consistent with the current oscillation data is illustrated in Sec. V. Section VI contains the additional constraint on the new parameters obtained from the existing anomalies of the flavor sector, like $\text{Br}(\tau \rightarrow \mu \nu_\tau \bar{\nu}_\mu)$, $\text{Br}(B \rightarrow X_s \gamma)$, $\text{Br}(\bar{B}^0 \rightarrow \bar{K}^0 \mu^+ \mu^-)$, $\text{Br}(B^+ \rightarrow K^+ \tau \tau)$, and $B_s - \bar{B}_s$ mixing. We then investigate the impact of the additional $U(1)_{L_\mu-L_\tau}$ gauge symmetry on the $R_{K^{(*)}}$ and R_ϕ LNU parameters and the optimized $P'_{4,5}$ observables in Sec. VII. We summarize our findings in Sec. VIII.

II. NEW $L_\mu - L_\tau$ MODEL WITH A SCALAR LEPTOQUARK

We study the well-known anomaly-free $U(1)_{L_\mu-L_\tau}$ extension of the SM containing three additional neutral fermions

TABLE I. Fields of the proposed $U(1)_{L_\mu-L_\tau}$ model and their charges.

	Field	$SU(3)_C \times SU(2)_L \times U(1)_Y$	$U(1)_{L_\mu-L_\tau}$	Z_2
Fermions	$Q_L \equiv (u, d)_L^T$	$(\mathbf{3}, \mathbf{2}, 1/6)$	0	+
	u_R	$(\mathbf{3}, \mathbf{1}, 2/3)$	0	+
	d_R	$(\mathbf{3}, \mathbf{1}, -1/3)$	0	+
	$e_L \equiv (\nu_e, e)_L^T$	$(\mathbf{1}, \mathbf{2}, -1/2)$	0	+
	e_R	$(\mathbf{1}, \mathbf{1}, -1)$	0	+
	$\mu_L \equiv (\nu_\mu, \mu)_L^T$	$(\mathbf{1}, \mathbf{2}, -1/2)$	1	+
	μ_R	$(\mathbf{1}, \mathbf{1}, -1)$	1	+
	$\tau_L \equiv (\nu_\tau, \tau)_L^T$	$(\mathbf{1}, \mathbf{2}, -1/2)$	-1	+
	τ_R	$(\mathbf{1}, \mathbf{1}, -1)$	-1	+
	N_e	$(\mathbf{1}, \mathbf{1}, 0)$	0	-
	N_μ	$(\mathbf{1}, \mathbf{1}, 0)$	1	-
	N_τ	$(\mathbf{1}, \mathbf{1}, 0)$	-1	-
Scalars	H	$(\mathbf{1}, \mathbf{2}, 1/2)$	0	+
	η	$(\mathbf{1}, \mathbf{2}, 1/2)$	0	-
	ϕ_2	$(\mathbf{1}, \mathbf{1}, 0)$	2	+
	S_1	$(\bar{\mathbf{3}}, \mathbf{1}, 1/3)$	-1	-

N_e, N_μ, N_τ , with $L_\mu - L_\tau$ charges 0, 1, and -1 , respectively. A scalar singlet ϕ_2 with charge $+2$ under the new $U(1)$ is added to spontaneously break the local $U(1)_{L_\mu - L_\tau}$ gauge symmetry. We also introduce an inert doublet η and a scalar leptoquark $S_1(\bar{3}, 1, 1/3)$ with $L_\mu - L_\tau$ charges 0 and -1 to

the scalar content of the model. We impose an additional Z_2 symmetry under which all the new fermions, η , and the leptoquark are odd and the rest are even. The particle content and their corresponding charges are displayed in Table I.

The Lagrangian of the present model can be written as

$$\begin{aligned} \mathcal{L} = & \mathcal{L}_{\text{SM}} - \frac{1}{4} Z'_{\mu\nu} Z'^{\mu\nu} - g_{\mu\tau} \bar{\mu}_L \gamma^\mu \mu_L Z'_\mu - g_{\mu\tau} \bar{\mu}_R \gamma^\mu \mu_R Z'_\mu + g_{\mu\tau} \bar{\tau}_L \gamma^\mu \tau_L Z'_\mu + g_{\mu\tau} \bar{\tau}_R \gamma^\mu \tau_R Z'_\mu \\ & + \bar{N}_e i \not{\partial} N_e + \bar{N}_\mu (i \not{\partial} - g_{\mu\tau} Z'_\mu \gamma^\mu) N_\mu + \bar{N}_\tau (i \not{\partial} + g_{\mu\tau} Z'_\mu \gamma^\mu) N_\tau - \frac{f_\mu}{2} (\bar{N}_\mu^c N_\mu \phi_2^\dagger + \text{H.c.}) \\ & - \frac{f_\tau}{2} (\bar{N}_\tau^c N_\tau \phi_2 + \text{H.c.}) - \frac{1}{2} M_{ee} \bar{N}_e^c N_e - \frac{1}{2} M_{\mu\tau} (\bar{N}_\mu^c N_\tau + \bar{N}_\tau^c N_\mu) - \sum_{q=d,s,b} (y_{qR} \bar{d}_{qR}^c S_1 N_\mu + \text{H.c.}) \\ & - \sum_{l=e,\mu,\tau} (Y_{\beta l} (\bar{\ell}_L)_\beta \tilde{\eta} N_{lR} + \text{H.c.}) + \left| \left(i \partial_\mu - \frac{g}{2} \boldsymbol{\tau}^a \cdot \mathbf{W}_\mu^a - \frac{g'}{2} B_\mu \right) \eta \right|^2 + \left| \left(i \partial_\mu - \frac{g'}{3} B_\mu + g_{\mu\tau} Z'_\mu \right) S_1 \right|^2 \\ & + |(i \partial_\mu - 2g_{\mu\tau} Z'_\mu) \phi_2|^2 - V(H, \eta, \phi_2, S_1), \end{aligned} \quad (5)$$

where the scalar potential V is

$$\begin{aligned} V(H, \eta, \phi_2, S_1) = & \mu_H^2 H^\dagger H + \lambda_H (H^\dagger H)^2 + \mu_\eta (\eta^\dagger \eta) + \lambda_{H\eta} (H^\dagger H) (\eta^\dagger \eta) + \lambda_\eta (\eta^\dagger \eta)^2 + \lambda'_{H\eta} (H^\dagger \eta) (\eta^\dagger H) \\ & + \frac{\lambda''_{H\eta}}{2} [(H^\dagger \eta)^2 + \text{H.c.}] + \mu_2^2 (\phi_2^\dagger \phi_2) + \lambda_2 (\phi_2^\dagger \phi_2)^2 + \mu_S^2 (S_1^\dagger S_1) + \lambda_S (S_1^\dagger S_1)^2 \\ & + [\lambda_{H2} (\phi_2^\dagger \phi_2) + \lambda_{HS} (S_1^\dagger S_1)] (H^\dagger H) + \lambda_{S2} (\phi_2^\dagger \phi_2) (S_1^\dagger S_1) + \lambda_{\eta 2} (\phi_2^\dagger \phi_2) (\eta^\dagger \eta) + \lambda_{S\eta} (S_1^\dagger S_1) (\eta^\dagger \eta). \end{aligned} \quad (6)$$

The gauge symmetry $SU(2)_L \times U(1)_Y \times U(1)_{L_\mu - L_\tau}$ is spontaneously broken to $SU(2)_L \times U(1)_Y$ by assigning a vacuum expectation value (VEV) v_2 to the complex singlet ϕ_2 . Then, the SM Higgs doublet breaks the SM gauge group to a low-energy theory by obtaining a VEV v . The new neutral gauge boson Z' associated with the $U(1)$ extension absorbs the massless pseudoscalar in ϕ_2 to become massive. The neutral components of the fields H and ϕ_2 can be written in terms of real scalars and pseudoscalars as

$$\begin{aligned} H^0 &= \frac{1}{\sqrt{2}} (v + h) + \frac{i}{\sqrt{2}} A^0, \\ \phi_2 &= \frac{1}{\sqrt{2}} (v_2 + h_2) + \frac{i}{\sqrt{2}} A_2. \end{aligned} \quad (7)$$

The inert doublet is denoted by $\eta = (\eta^+_{\frac{1}{2}}, \eta^0_{\frac{1}{2}})$, with $\eta^0 = \frac{\eta_e + i\eta_o}{\sqrt{2}}$. The masses of its charged and neutral components are given by

$$\begin{aligned} M_{\eta^+}^2 &= \mu_\eta^2 + \frac{\lambda_{H\eta}}{2} v^2 + \frac{\lambda_{\eta 2}}{2} v_2^2, \\ M_{\eta_e}^2 &= \mu_\eta^2 + \frac{\lambda_{\eta 2}}{2} v_2^2 + (\lambda_{H\eta} + \lambda'_{H\eta} + \lambda''_{H\eta}) \frac{v^2}{2}, \\ M_{\eta_o}^2 &= \mu_\eta^2 + \frac{\lambda_{\eta 2}}{2} v_2^2 + (\lambda_{H\eta} + \lambda'_{H\eta} - \lambda''_{H\eta}) \frac{v^2}{2}. \end{aligned} \quad (8)$$

The masses obtained by the colored scalar and the gauge boson Z' are

$$\begin{aligned} M_{S_1}^2 &= 2\mu_S^2 + \lambda_{HS} v^2 + \lambda_{S2} v_2^2, \\ M_{Z'} &= 2v_2 g_{\mu\tau}. \end{aligned} \quad (9)$$

When discussing the results, we consider the following benchmark values for the masses of the scalar spectrum: $(M_{S_1}, M_{\eta^+}, M_{\eta_{e,o}}) = (1.2, 2, 1.5)$ TeV.

III. MIXING IN THE FERMION AND SCALAR SECTORS

The fermion and scalar mass matrices take the forms

$$M_N = \begin{pmatrix} \frac{1}{\sqrt{2}} f_\mu v_2 & M_{\mu\tau} \\ M_{\mu\tau} & \frac{1}{\sqrt{2}} f_\tau v_2 \end{pmatrix}, \quad M_S = \begin{pmatrix} 2\lambda_H v^2 & \lambda_{H2} v v_2 \\ \lambda_{H2} v v_2 & 2\lambda_2 v_2^2 \end{pmatrix}. \quad (10)$$

One can diagonalize the above mass matrices as $U_{\alpha(\zeta)}^T M_{N(S)} U_{\alpha(\zeta)} = \text{diag}[M_{N_-(H_1)}, M_{N_+(H_2)}]$, where

$$U_\theta = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}, \quad (11)$$

with $\zeta = \frac{1}{2} \tan^{-1} \left(\frac{\lambda_{H_2^2} v v_2}{\lambda_2 v_2^2 - \lambda_H v^2} \right)$ and $\alpha = \frac{1}{2} \tan^{-1} \left(\frac{2M_{\mu\tau}}{(f_\tau - f_\mu)(v_2/\sqrt{2})} \right)$. We denote the scalar mass eigenstates as H_1 and H_2 , where H_1 is assumed to be the observed Higgs at the LHC with $M_{H_1} = 125.09$ GeV and $v = 246$ GeV. We use the minimum value of the mixing parameter ζ to stay within the LHC limits on the Higgs decay width. We denote the fermion mass eigenstates as N_- and N_+ , where the lightest one (N_-) is the probable dark matter candidate in the present work.

IV. DARK MATTER PHENOMENOLOGY

A. Relic abundance

The model allows the dark matter (N_-) to have gauge- and scalar-mediated annihilation channels. The possible contributing diagrams are provided in Fig. 1, which are mediated by $(H_1, H_2, \eta^+, \eta^0, S_1, Z')$. Majorana DM in the $H_{1,2}$ portal (upper row in Fig. 1) has already been well explored in the literature [60,61]. Here, we focus on (Z', S_1, η) -mediated channels (middle and bottom rows in Fig. 1) contributing to DM observables, which we later connect with radiative neutrino mass as well as flavor observables. The relic abundance of dark matter is given by

$$\Omega h^2 = \frac{1.07 \times 10^9 \text{ GeV}^{-1}}{M_{\text{Pl}} g_*^{1/2}} \frac{1}{J(x_f)}. \quad (12)$$

Here, the Planck mass $M_{\text{Pl}} = 1.22 \times 10^{19}$ GeV and $g_* = 106.75$ denotes the total number of effective relativistic degrees of freedom. The function $J(x_f)$ reads as

$$J(x_f) = \int_{x_f}^{\infty} \frac{\langle \sigma v \rangle(x)}{x^2} dx. \quad (13)$$

The thermally averaged annihilation cross section $\langle \sigma v \rangle$ is given by the expression

$$\langle \sigma v \rangle(x) = \frac{x}{8M_-^5 K_2^2(x)} \int_{4M_-^2}^{\infty} \hat{\sigma} \times (s - 4M_-^2) \sqrt{s} K_1 \left(\frac{x\sqrt{s}}{M_-} \right) ds, \quad (14)$$

where K_1 and K_2 denote the modified Bessel functions, $x = M_-/T$, where T is the temperature, and $\hat{\sigma}$ is the dark matter annihilation cross section. The analytical expression for the freeze-out parameter x_f is

$$x_f = \ln \left(\frac{0.038 g M_{\text{Pl}} M_- \langle \sigma v \rangle(x_f)}{(g_* x_f)^{1/2}} \right). \quad (15)$$

Here g represents the number of degrees of freedom of the dark matter particle N_- .

As seen in the left panel of Fig. 2, the relic density with the s -channel contribution meets the Planck limit [62] near the resonance in the propagator (H_1, H_2, Z'), i.e., near $M_- = \frac{M_{\text{prop}}}{2}$. We restrict our discussion to the mass regions (in GeV) $100 \leq M_{Z'} \leq 1000$ and $80 \leq M_- \leq 1000$, and H_2

FIG. 1. Feynman diagrams contributing to the relic density.

FIG. 2. Behavior of the relic density plotted against DM mass with $M_{H_2} = 2.2$ TeV, for varying $M_{Z'}$ and $g_{\mu\tau}$ (left panel) and y_{qR} (right panel). Black horizontal dotted lines denote the 3σ range of the Planck limit [62].

is considered to be sufficiently large such that its resonance does not meet the Planck limit below the 1 TeV DM mass region. Now, in this DM mass range the channels mediated by (Z', η, S_1) drive the relic density observable, where the gauge coupling $g_{\mu\tau}$ controls the s -channel contribution, while $Y_{\beta l}$ and y_{qR} are relevant in the t -channel contributions. Hence, the relevant parameters in our investigation are $(M_-, g_{\mu\tau}, M_{Z'}, Y_{\beta l}, y_{qR})$. The effect of these parameters on the relic abundance is made transparent in Fig. 2, where we consider $Y_{\beta l} \sim 10^{-2}$ in order to explain neutrino masses at the one-loop level. The left panel shows the variation of the relic density with the gauge parameters $g_{\mu\tau}$ and $M_{Z'}$, while the right panel depicts the variation with the parameter y_{qR} . No significant constraints on $M_{Z'}$ and $g_{\mu\tau}$ are observed; however, the relic density has an appreciable effect on the $M_- - (y_{qR})^2$ parameter space, which will be discussed in the next section.

B. Direct searches

Moving to direct searches, the Z' -mediated WIMP-nucleon interaction is not possible at tree level as the Z' boson does not couple to quarks. The t -channel scalar (H_1, H_2) exchange can give a spin-independent (SI)

contribution, but this is not relevant for our purposes. In the scalar portal, one can obtain a contribution from the spin-dependent (SD) interaction mediated by the SLQ of the form

$$\mathcal{L}_{\text{eff}}^{\text{SD}} \simeq \frac{y_{qR}^2 \cos^2 \alpha}{4(M_{S_1}^2 - M_-^2)} \bar{N} \gamma^\mu \gamma^5 N_- \bar{q} \gamma_\mu \gamma^5 q. \quad (16)$$

The s -channel process is depicted in the left diagram of Fig. 3, and the corresponding cross section is given by [63]

$$\sigma_{\text{SD}} = \frac{\mu_r^2}{\pi} \frac{\cos^4 \alpha}{(M_{S_1}^2 - M_-^2)^2} [y_{dR}^2 \Delta_d + y_{sR}^2 \Delta_s]^2 J_n(J_n + 1), \quad (17)$$

where the angular momentum $J_n = \frac{1}{2}$ and the reduced mass $\mu_r = \frac{M_- M_n}{M_- + M_n}$, where $M_n \simeq 1$ GeV for a nucleon. The values of the quark spin functions Δ_q can be found in Ref. [63]. Now, it is obvious that it can constrain the parameters M_- and $(y_{qR})^2$. The left panel of Fig. 4 shows the $M_- - (y_{qR})^2$ parameter space (green and red regions) that remains after imposing the Planck 3σ limit [62] on the current relic density. Here, the region shown in green is excluded by the

FIG. 3. Diagrams involving the SLQ relevant for direct-detection studies.

FIG. 4. Left: The $M_- - (y_{qR})^2$ parameter space consistent up to the 3σ level of the Planck limit [62] on the relic density. Right: The SD WIMP-proton cross section as a function of DM mass. Dashed lines represent the recent bounds obtained from PICO-60 [64] and LUX [67]. Green (red) dots in both panels represent Planck-allowed and PICO-excluded (Planck- and PICO-allowed) data points.

most stringent PICO-60 limit [64] on the SD WIMP-proton cross section, as seen in the right panel. Apart from the tree-level interaction, one-loop penguin diagrams (middle and right panels of Fig. 3) involving the SLQ mediated by the SM neutral gauge boson (Z) and the neutral scalars (H_1, H_2) can provide SD and SI contributions, respectively. The effective interaction Lagrangian relevant for the SD cross section is given by [65,66]

$$\mathcal{L}_{\text{eff}}^{\text{SD-loop}} \simeq \xi_{q'} \bar{N}_- \gamma^\mu \gamma^5 N_- \bar{q}' \gamma_\mu \gamma^5 q', \quad (18)$$

where

$$\xi_{q'} = \sum_{q=d,s,b} \frac{y_{qR}^2 \cos^2 \alpha a_{q'}}{32\pi^2 M_Z^2} \left[(v_q + a_q) G\left(\frac{M_-^2}{M_{S_1}^2}\right) \right]. \quad (19)$$

The loop function takes the form

$$G(x) = -1 + \frac{2(x + (1-x) \ln(1-x))}{x^2}. \quad (20)$$

The corresponding WIMP-nucleon cross section is given by

$$\sigma_{\text{SD}}^{\text{loop}} = \frac{16\mu_r^2}{\pi} J_n(J_n + 1) (\xi_u \Delta_u + \xi_d \Delta_d + \xi_s \Delta_s)^2. \quad (21)$$

In the above expression, $v_q = \frac{g}{2\cos\theta_w} (-\frac{1}{2} + \frac{2}{3}\sin^2\theta_w)$, $a_q = -\frac{g}{4\cos\theta_w}$, and $a_{q'} = \frac{g}{4\cos\theta_w} (-\frac{g}{4\cos\theta_w})$ for $q' = u(d, s)$. For the SI contribution, the effective interaction term is

$$\mathcal{L}_{\text{eff}}^{\text{SI-loop}} \simeq \Lambda_{q'} \bar{N}_- N_- \bar{q}' q', \quad (22)$$

and the corresponding cross section is given by

$$\sigma_{\text{SI}}^{\text{loop}} = \frac{4\mu_r^2}{\pi} M_n^2 \left(\frac{\Lambda_{q'}}{m_{q'}} \right)^2 f_n^2, \quad (23)$$

where

FIG. 5. One-loop SD (left) and SI (right) contributions. The stringent upper limits of PICO-60 [64] and LUX [67] are given in the left panel, while PandaX-II [68], XENON1T [69], and LUX [70] bounds are given in the right panel.

FIG. 6. Radiative generation of neutrino mass.

$$\Lambda_{q'} = - \sum_{q=d,s,b} \frac{y_{qR}^2 \cos^2 \alpha}{16\pi^2} \left[\frac{\lambda_{HS} v \cos \zeta - \lambda_{S2} v_2 \sin \zeta}{M_{H_1}^2} + \frac{\lambda_{HS} v \sin \zeta + \lambda_{S2} v_2 \cos \zeta}{M_{H_2}^2} \right] G_1 \left(\frac{M_-^2}{M_{S_1}^2} \right) \frac{m_{q'}}{v M_-}.$$

Here $G_1(x) = \frac{x+(1-x)\ln(1-x)}{x}$. For the proton, the typical value of the scalar form factor f_n is ~ 0.3 . The left and right panels of Fig. 5 show the one-loop SD and SI contributions, respectively, for the parameter space consistent with Planck data. We see that these contributions are well below the current upper limits set by direct-detection collaborations, such as PICO-60 [64] and LUX [67] for SD cases and PandaX-II [68], XENON1T [69], and LUX [70] for SI cases. Thus, they have no impact on the range of model parameters.

FIG. 7. Allowed parameter space consistent with the current data from the neutrino sector in the Y_{II} - M_- plane.

V. RADIATIVE NEUTRINO MASS

The light neutrino mass can be generated at the one-loop level from the Yukawa interaction with the inert doublet in Eq. (5), and the corresponding diagram is shown in Fig. 6. Assuming that $m_0^2 = (M_{\eta_e}^2 + M_{\eta_o}^2)/2$ is much greater than $M_{\eta_e}^2 - M_{\eta_o}^2 = \lambda_{H\eta}'' v^2$, the expression for the radiatively generated neutrino mass [71] is given by

$$(\mathcal{M}_\nu)_{\beta\gamma} = \frac{\lambda_{H\eta}'' v^2}{16\pi^2} \sum_{l=e,\mu,\tau} \frac{Y_{\beta l} Y_{\gamma l} M_{Dl}}{m_0^2 - M_{Dl}^2} \left[1 + \frac{M_{Dl}^2}{m_0^2 - M_{Dl}^2} \ln \frac{M_{Dl}^2}{m_0^2} \right]. \quad (24)$$

Here, $M_{Dl} = (U^T M_N U)_l = \text{diag}(M_{ee}, M_-, M_+)$ and the fermion mass eigenstates $N_{Dl} = U_{lm}^\dagger N_m$. The light neutrino mass matrix (24) can be expressed as

$$(\mathcal{M}_\nu)_{\beta\gamma} \equiv (Y^T \Lambda Y)_{\beta\gamma}, \quad (25)$$

where the matrix Λ is defined as $\Lambda = \text{diag}(\Lambda_1, \Lambda_2, \Lambda_3)$, with

$$\Lambda_l = \frac{\lambda_{H\eta}'' v^2}{16\pi^2} \frac{M_{Dl}}{m_0^2 - M_{Dl}^2} \left[1 + \frac{M_{Dl}^2}{m_0^2 - M_{Dl}^2} \ln \frac{M_{Dl}^2}{m_0^2} \right]. \quad (26)$$

Generation-wise, the $L_\mu - L_\tau$ charges of SM leptons match the charges of the new fermions N_l (0, +1, and -1, respectively). The Yukawa interaction term is written using

an inert doublet η with vanishing $L_\mu - L_\tau$ charge, and thus the Yukawa matrix $Y_{\beta l}$ is diagonal. Hence, the neutrino mass matrix (25) is diagonal by model construction, i.e., $\mathcal{M}_\nu = \text{diag}(M_1, M_2, M_3)$. In order to find the viable region for model parameters consistent with the current neutrino oscillation data (i.e., $6.93 \leq \Delta M_{\text{sol}}^2 [10^{-5} \text{ eV}^2] \leq 7.97$, $2.37 \leq \Delta M_{\text{atm}}^2 [10^{-3} \text{ eV}^2] \leq 2.63$ [72]) and the cosmological bound on the sum of the light neutrino mass ($\sum_i M_i < 0.23 \text{ eV}$ [73]), we perform a scan by varying the parameters in the following ranges (where the masses are in GeV):

$$\begin{aligned} 1000 &\leq M_{ee} \leq 3000, & 100 &\leq M_- \leq 1000, \\ 2000 &\leq M_+ \leq 5000, & 1000 &\leq m_0 \leq 2000, \\ 0.0001 &\leq Y_{ee}, Y_{\mu\mu}, Y_{\tau\tau} \leq 0.05. \end{aligned} \quad (27)$$

The allowed parameter space in the $Y_{ll}-M_-$ plane that satisfies the above constraints from the neutrino sector is shown in Fig. 7. Thus, the proposed model gives a viable region of parameter space consistent with recent oscillation data in the context of the radiative light neutrino mass matrix.

VI. FLAVOR PHENOMENOLOGY

The general effective Hamiltonian responsible for the quark-level transition $b \rightarrow sl^+l^-$ is given by [74,75]

$$\begin{aligned} \mathcal{H}_{\text{eff}} = & -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \left[\sum_{i=1}^6 C_i(\mu) O_i \right. \\ & \left. + \sum_{i=7,9,10} (C_i(\mu) O_i + C'_i(\mu) O'_i) \right], \end{aligned} \quad (28)$$

where G_F is the Fermi constant and $V_{qq'}$ denote the Cabibbo-Kobayashi-Maskawa matrix elements. The C_i 's stand for the Wilson coefficients evaluated at the renormalized scale $\mu = m_b$ [76], where the sum over i includes the current-current operators ($i = 1, 2$) and the QCD-penguin operators ($i = 3, 4, 5, 6$). The quark-level operators mediating leptonic/semileptonic processes are given as

$$\begin{aligned} O_7^{(\prime)} &= \frac{e}{16\pi^2} [\bar{s}\sigma_{\mu\nu}(m_s P_{L(R)} + m_b P_{R(L)})b] F^{\mu\nu}, \\ O_9^{(\prime)} &= \frac{\alpha_{\text{em}}}{4\pi} (\bar{s}\gamma^\mu P_{L(R)}b)(\bar{l}\gamma_\mu l), \\ O_{10}^{(\prime)} &= \frac{\alpha_{\text{em}}}{4\pi} (\bar{s}\gamma^\mu P_{L(R)}b)(\bar{l}\gamma_\mu \gamma_5 l), \end{aligned} \quad (29)$$

where α_{em} denotes the fine-structure constant and $P_{L,R} = (1 \mp \gamma_5)/2$ are the chiral operators. The primed operators are absent in the SM, but can exist in the proposed $L_\mu - L_\tau$ model.

In the previous section we discussed the available new parameter space consistent with the DM observables which are within their respective experimental limits. However, these parameters can be further constrained from the quark and lepton sectors, as we show in the following subsections.

A. $B_s - \bar{B}_s$ mixing

In this subsection, we discuss the constraint on the new parameters from the mass difference between the B_s meson mass eigenstates (ΔM_s), which characterizes the $B_s - \bar{B}_s$ mixing phenomena. In the SM, $B_s - \bar{B}_s$ mixing proceeds to an excellent approximation through the box diagram with an internal top quark and W -boson exchange. The effective Hamiltonian describing the $\Delta B = 2$ transition is given by [77]

$$\mathcal{H}_{\text{eff}} = \frac{G_F^2}{16\pi^2} \lambda_t^2 M_W^2 S_0(x_t) \eta_B (\bar{s}b)_{V-A} (\bar{s}b)_{V-A}, \quad (30)$$

where $\lambda_t = V_{tb} V_{ts}^*$, η_B is the QCD correction factor, and the loop function $S_0(x_t)$ is given by [77]

$$S_0(x_t) = \frac{4x_t - 11x_t^2 + x_t^3}{4(1-x_t)^2} - \frac{3}{2} \frac{\log x_t x_t^3}{(1-x_t)^3}, \quad (31)$$

with $x_t = m_t^2/M_W^2$. Using Eq. (30), the $B_s - \bar{B}_s$ mass difference in the SM is given as

$$\begin{aligned} \Delta M_s^{\text{SM}} &= 2|M_{12}^{\text{SM}}| = \frac{\langle \bar{B}_s | \mathcal{H}_{\text{eff}} | B_s \rangle}{M_{B_s}} \\ &= \frac{G_F^2}{6\pi^2} M_W^2 \lambda_t^2 \eta_B \hat{B}_s f_{B_s}^2 M_{B_s} S_0(x_t). \end{aligned} \quad (32)$$

Using the input parameters from Refs. [78,79], the SM predicted value of ΔM_s is

$$\Delta M_s^{\text{SM}} = (17.426 \pm 1.057) \text{ ps}^{-1}, \quad (33)$$

and the corresponding experimental value is [78]

$$\Delta M_s^{\text{Expt}} = 17.761 \pm 0.022 \text{ ps}^{-1}. \quad (34)$$

Even though the theoretical prediction is in good agreement with the experimental $B_s - \bar{B}_s$ oscillation data, it does not completely rule out the possibility of new physics.

The box diagrams for $B_s - \bar{B}_s$ mixing in the presence of a singlet SLQ and $N_\pm$ are shown in Fig. 8. The effective Hamiltonian in the presence of NP is given by

$$\mathcal{H}_{\text{eff}} = \frac{(y_{sR} y_{bR})^2}{128\pi^2 M_{S_1}^2} \cos^2 \alpha \sin^2 \alpha C_{B_s}^{\text{NP}} (\bar{s}b)_{V+A} (\bar{s}b)_{V+A}, \quad (35)$$

where

FIG. 8. Box diagrams of $B_s - \bar{B}_s$ mixing with a leptoquark in the loop.

$$C_{B_s}^{\text{NP}} = 2k(\chi_-, \chi_-, 1) + 4k(\chi_-, \chi_+, 1) + 2k(\chi_+, \chi_+, 1) \\ + \chi_- j(\chi_-, \chi_-, 1) \\ + 2\sqrt{\chi_- \chi_+} j(\chi_-, \chi_+, 1) + \chi_+ j(\chi_+, \chi_-, 1), \quad (36)$$

with $\chi_{\mp} = M_{\mp}^2/M_{S_1}^2$ and the loop functions $k(\chi_{\pm}, \chi_{\mp}, 1)$ and $j(\chi_{\pm}, \chi_{\mp}, 1)$ are given in Appendix A. Using Eq. (35), the mass difference of $B_s - \bar{B}_s$ mixing due to the exchange of S_1 and $N_{\pm}$ is found to be

$$\Delta M_s^{\text{NP}} = \frac{(y_{sR} y_{bR})^2}{48\pi^2 M_{S_1}^2} \cos^2 \alpha \sin^2 \alpha C_{B_s}^{\text{NP}} \eta_B \hat{B}_{B_s} f_{B_s}^2 M_{B_s}. \quad (37)$$

Including the NP contribution arising due to the SLQ exchange, the total mass difference can be written as

$$\Delta M_s = \Delta M_s^{\text{SM}} \left[1 + \frac{C_{B_s}^{\text{NP}} \cos^2 \alpha \sin^2 \alpha}{8G_F^2 V_{tb}^2 V_{ts}^{*2} M_W^2 S_0(x_t)} \left(\frac{(y_{sR} y_{bR})^2}{M_{S_1}^2} \right) \right]. \quad (38)$$

Using Eqs. (33) and (34) in Eq. (38), one can put bounds on $(y_{qR})^2$ and M_- .

B. $B \rightarrow Kl^+l^-$ process

The rare semileptonic $B \rightarrow Kl^+l^-$ process is mediated via $b \rightarrow sl^+l^-$ quark-level transitions. In the current framework, the $b \rightarrow sl^+l^-$ transitions can occur via the Z' -exchanging one-loop penguin diagrams shown in Fig. 9. The penguin diagram in the left panel is suppressed by the factor $m_b/M_{\pm}$, thus providing a negligible contribution. Furthermore, due to the zero hypercharge of the dark matter fermion, the diagrams with SM neutral bosons (Z, γ) replacing Z' in the right panel of Fig. 9 are not possible in the present framework. The matrix elements of the various hadronic currents between the initial B meson and K meson in the final state are related to the form factors $f_{+,0}$ as follows [7,80]:

FIG. 9. Penguin diagrams of $b \rightarrow sll$ processes, where $l = \mu, \tau$ with a leptoquark in the loop.

$$\langle K(p_K) | \bar{s} \gamma^\mu b | B(p_B) \rangle = f_+(q^2) (p_B + p_K)^\mu \\ + [f_0(q^2) - f_+(q^2)] \frac{M_B^2 - M_K^2}{q^2} q^\mu, \quad (39)$$

where $p_B(p_K)$ and $M_B(M_K)$ denote the four-momenta and mass of the $B(K)$ meson and q^2 is the momentum transfer. By using Eq. (39), the transition amplitude of the $B \rightarrow K\mu^+\mu^-$ process is given by

$$\mathcal{M} = \frac{1}{2^5 \pi^2} \frac{y_{bR} y_{sR} g_{\mu\tau}^2}{M_{Z'}^2} \mathcal{V}_{sb}(\chi_-, \chi_+) \\ \times [\bar{u}(p_B) \gamma^\mu (1 + \gamma_5) u(p_K)] [\bar{v}(p_2) \gamma_\mu u(p_1)], \quad (40)$$

where p_1 and p_2 are the four-momenta of charged leptons and the loop function $\mathcal{V}_{sb}(\chi_-, \chi_+)$ is given in Appendix A [20,81]. Now, comparing the amplitude (40) with the amplitude obtained from the effective Hamiltonian (28), we obtain a new Wilson coefficient associated with the right-handed semileptonic electroweak penguin operator $\mathcal{O}'_9$ as

$$C_9^{\text{NP}} = \frac{\sqrt{2}}{2^4 \pi G_F \alpha_{\text{em}} V_{tb} V_{ts}^*} \frac{y_{bR} y_{sR} g_{\mu\tau}^2}{M_{Z'}^2} \mathcal{V}_{sb}(\chi_-, \chi_+). \quad (41)$$

The differential branching ratio of the $B \rightarrow Kll$ process with respect to q^2 is given by

$$\frac{d\text{Br}}{dq^2} = \tau_B \frac{G_F^2 \alpha_{\text{em}}^2 |V_{tb} V_{ts}^*|^2}{2^8 \pi^5 M_B^3} \sqrt{\lambda(M_B^2, M_K^2, q^2)} \beta_l f_+^2 \\ \times \left(a_l(q^2) + \frac{c_l(q^2)}{3} \right), \quad (42)$$

where

$$\begin{aligned}
a_l(q^2) &= q^2 |F_P|^2 + \frac{\lambda(M_B^2, M_K^2, q^2)}{4} (|F_A|^2 + |F_V|^2) \\
&\quad + 2m_l(M_B^2 - M_K^2 + q^2) \text{Re}(F_P F_A^*) + 4m_l^2 M_B^2 |F_A|^2, \\
c_l(q^2) &= -\frac{\lambda(M_B^2, M_K^2, q^2)}{4} \beta_l^2 (|F_A|^2 + |F_V|^2), \quad (43)
\end{aligned}$$

with

$$\begin{aligned}
F_V &= \frac{2m_b}{M_B} C_7^{\text{eff}} + C_9^{\text{eff}} + C_9^{\text{NP}}, \quad F_A = C_{10}, \\
F_P &= m_l C_{10} \left[\frac{M_B^2 - M_K^2}{q^2} \left(\frac{f_0(q^2)}{f_+(q^2)} - 1 \right) - 1 \right], \quad (44)
\end{aligned}$$

and

$$\begin{aligned}
\lambda(a, b, c) &= a^2 + b^2 + c^2 - 2(ab + bc + ca), \\
\beta_l &= \sqrt{1 - 4m_l^2/q^2}. \quad (45)
\end{aligned}$$

For numerical estimation, we have used the lifetime and masses of particles from Ref. [78], and the form factors are taken from Ref. [82]. The experimental limits on the branching ratios of $\bar{B}^0 \rightarrow \bar{K}^0 \mu^+ \mu^-$ and $B^+ \rightarrow K^+ \tau^+ \tau^-$ processes are [78]

$$\text{Br}(\bar{B}^0 \rightarrow \bar{K}^0 \mu^+ \mu^-)|^{\text{Expt}} = (3.39 \pm 0.34) \times 10^{-7}, \quad (46)$$

$$\text{Br}(B^+ \rightarrow K^+ \tau^+ \tau^-)|^{\text{Expt}} < 2.25 \times 10^{-3}, \quad (47)$$

while their predicted values in the SM are

$$\text{Br}(\bar{B}^0 \rightarrow \bar{K}^0 \mu^+ \mu^-)|^{\text{SM}} = (1.82 \pm 0.15) \times 10^{-7}, \quad (48)$$

$$\text{Br}(B^+ \rightarrow K^+ \tau^+ \tau^-)|^{\text{SM}} = (1.56 \pm 0.125) \times 10^{-7}. \quad (49)$$

Since Z' does not couple to the electron, the branching ratio of the $B \rightarrow Ke^+e^-$ process is considered to be SM-like. The decay modes $\bar{B}^0 \rightarrow \bar{K}^0 \mu^+ \mu^-$ and $B^+ \rightarrow K^+ \tau^+ \tau^-$ can constrain all four parameters, i.e., $(y_{qR})^2$, $g_{\mu\tau}$, $M_{Z'}$, and M_- .

C. $B \rightarrow X_s \gamma$ process

The $B \rightarrow X_s \gamma$ process involves a $b \rightarrow s \gamma$ quark-level transition, and the experimental limit on the corresponding branching ratio is given by [83]

$$\text{Br}(B \rightarrow X_s \gamma)|_{E_\gamma > 1.6 \text{ GeV}}^{\text{Expt}} = (3.32 \pm 0.16) \times 10^{-4}. \quad (50)$$

Figure 10 shows the one-loop penguin diagram of the $b \rightarrow s \gamma$ process mediated by the SLQ and $N_\pm$.

Including the NP contribution, the total branching ratio of $B \rightarrow X_s \gamma$ is given by

FIG. 10. Feynman diagram of $b \rightarrow s \gamma$ processes in the presence of a scalar leptoquark.

$$\text{Br}(B \rightarrow X_s \gamma) = \text{Br}(B \rightarrow X_s \gamma)|^{\text{SM}} \left(1 + \frac{C_7^{\gamma \text{NP}}}{C_7^{\gamma \text{SM}}} \right)^2, \quad (51)$$

where the predicted SM branching ratio is [84]

$$\text{Br}(B \rightarrow X_s \gamma)|_{E_\gamma > 1.6 \text{ GeV}}^{\text{SM}} = (3.36 \pm 0.23) \times 10^{-4}. \quad (52)$$

The new $C_7^{\gamma \text{NP}}$ Wilson coefficient obtained from Fig. 10 is given by

$$C_7^{\gamma \text{NP}} = -\frac{\sqrt{2}/3}{8G_F V_{tb} V_{ts}^*} \frac{y_{bR} y_{sR}}{M_{S_1}^2} (J_1(\chi_-) \cos^2 \alpha + J_1(\chi_+) \sin^2 \alpha), \quad (53)$$

where the loop functions $J_1(\chi_\pm)$ are given as [20]

$$J_1(\chi_\pm) = \frac{1 - 6\chi_\pm + 3\chi_\pm^2 + 2\chi_\pm^3 - 6\chi_\pm^2 \log \chi_\pm}{12(1 - \chi_\pm)^4}. \quad (54)$$

Using Eqs. (50), (52), and (53) in Eq. (51), the parameters $(y_{qR})^2$ and M_- can be constrained.

D. $\tau \rightarrow \mu \nu_\tau \bar{\nu}_\mu$ process

In the presence of a Z' boson, the $\tau \rightarrow \mu \nu_\tau \bar{\nu}_\mu$ process can occur via a box diagram, as shown in Fig. 11. There are four possible one-loop box diagrams with the Z' connected to the lepton legs. The total branching ratio of this process is given by [85]

FIG. 11. One-loop box diagram of $\tau \rightarrow \mu \nu_\tau \bar{\nu}_\mu$ processes.

FIG. 12. Left: The constraint on $g_{\mu\tau}$ and $M_{Z'}$ obtained from $\text{Br}(B^+ \rightarrow K^+ \tau^+ \tau^-)$, $\text{Br}(\bar{B}^0 \rightarrow \bar{K}^0 \mu^+ \mu^-)$, and $\text{Br}(\tau \rightarrow \mu \nu_\tau \bar{\nu}_\mu)$ observables. The dashed orange line denotes the neutrino trident bound [86,87]. Right: Blue data points denote the allowed parameter space obtained from $\text{Br}(\bar{B}^0 \rightarrow \bar{K}^0 \mu^+ \mu^-)$, $B_s - \bar{B}_s$ mixing, $\text{Br}(B^+ \rightarrow K^+ \tau^+ \tau^-)$, and $\text{Br}(B \rightarrow X_s \gamma)$ experimental data, which are also consistent with the Planck [62] and PICO-60 limits [64]. Here, green (red) dots denote the PICO-60- and flavor-excluded (PICO-60-allowed and flavor-excluded) data points.

$$\begin{aligned} & \text{Br}(\tau \rightarrow \mu \nu_\tau \bar{\nu}_\mu) \\ &= \text{Br}(\tau \rightarrow \mu \nu_\tau \bar{\nu}_\mu)|^{\text{SM}} \left(1 + \frac{3g_{\mu\tau}^2 \log(M_W^2/M_{Z'}^2)}{4\pi^2 (1 - M_{Z'}^2/M_W^2)} \right)^2, \end{aligned} \quad (55)$$

where the branching ratio in the SM is given by [85]

$$\text{Br}(\tau \rightarrow \mu \nu_\tau \bar{\nu}_\mu)|^{\text{SM}} = (17.29 \pm 0.032)\%. \quad (56)$$

Now, by comparing the theoretical result with the experimentally measured value [78]

$$\text{Br}(\tau \rightarrow \mu \nu_\tau \bar{\nu}_\mu)|^{\text{Expt}} = (17.39 \pm 0.04)\%, \quad (57)$$

one can put bounds on the $M_{Z'}-g_{\mu\tau}$ parameter space.

Now, by correlating the theoretical predictions of $\text{Br}(\bar{B}^0 \rightarrow \bar{K}^0 \mu^+ \mu^-)$, $\text{Br}(B^+ \rightarrow K^+ \tau^+ \tau^-)$, and $\text{Br}(\tau \rightarrow \mu \nu_\tau \bar{\nu}_\mu)$ with the corresponding 3σ experimental data, we compute the allowed $M_{Z'}-g_{\mu\tau}$ parameter space. Since Z' does not couple to quarks, these gauge parameters cannot be constrained from $b \rightarrow s\gamma$ decay modes and $B_s - \bar{B}_s$ oscillation data. The constraint on the $M_-(y_{qR})^2$ parameter space is obtained from $\text{Br}(\bar{B}^0 \rightarrow \bar{K}^0 \mu^+ \mu^-)$, $\text{Br}(B^+ \rightarrow K^+ \tau^+ \tau^-)$, $\text{Br}(B \rightarrow X_s \gamma)$, and $B_s - \bar{B}_s$ mixing results. In addition, the branching ratio of the rare semileptonic $B \rightarrow K \nu_l \bar{\nu}_l$ process can play a vital role in restricting these parameters. Though the proposed model can allow $b \rightarrow s \nu_l \bar{\nu}_l$ decay modes, the contributions of μ and τ leptons cancel each other in the leading order of NP due to their equal and opposite $L_\mu - L_\tau$ charges. Since there is no $Z'\mu\tau$ coupling, the neutral and charged lepton-flavor-violating decay processes like $B \rightarrow K^{(*)} \mu^\mp \tau^\pm$, $\tau^- \rightarrow \mu^- \gamma$, and $\tau \rightarrow \mu \mu \mu$ do not play any role. It should be noted that the model considered here only provides an additional contribution to $b \rightarrow sll$ transitions through the C_9^{NP} coefficient, and hence the $B_s \rightarrow l^+ l^-$ processes cannot provide any strict bound on the new

parameters. In this analysis, we consider that the y_{qR} coupling is perturbative, i.e., $|y_{qR}| \lesssim \sqrt{4\pi}$. The left [right] panel of Fig. 12 denotes the parameter space in the $M_{Z'}-g_{\mu\tau}$ [$M_-(y_{qR})^2$] plane consistent with DM and flavor studies. From the left panel, one can see that the obtained parameter space survives the lower limit imposed on the ratio $M_{Z'}/g_{\mu\tau}$ by neutrino trident production [86,87], i.e., 540 GeV. It is also noted that the allowed region favored by the $(g-2)_\mu$ anomaly is completely excluded by the constraint from neutrino trident production [85]. In the right panel of Fig. 12, we show the $M_-(y_{qR})^2$ parameter space of Fig. 4 after a combined analysis made by imposing the DM and flavor experimental limits, with the surviving region shown in blue. In Table II we report the allowed regions of the parameters M_- and $(y_{qR})^2$ that are consistent with only DM studies (DM-I and -II) and both the DM and flavor sectors (DM + Flavor).

TABLE II. Predicted allowed ranges of the parameters M_- and $(y_{qR})^2$. Here, DM-I and DM-II represent the two regions in Fig. 4 consistent with only DM observables, while DM + Flavor denotes the region favored by both the dark matter and flavor studies.

Parameters	DM-I	DM-II	DM + Flavor
M_- [GeV]	103–560	561–988	103–560
$(y_{qR})^2$	0–3.51	1.94–2.56	0–1.26

TABLE III. Sample benchmark values chosen from the allowed parameter space of Fig. 12.

S.No	$(y_{qR})^2$	$g_{\mu\tau}$	$M_{Z'}$ [GeV]	M_- [GeV]
1.	1.016	0.336	669.84	291.22
2.	0.0003	0.94	763.29	496.7

FIG. 13. The left and right panels depict the relative contributions of annihilation channels for the benchmark-1 and benchmark-2 values from Table III, respectively. The vertical dashed lines represent the values of the DM mass corresponding to each benchmark.

To illustrate the relative contributions of the annihilation channels to the relic density for the parameter space depicted in Fig. 12, we choose two benchmark values (Table III), specifically the combination of the maximum $(y_{qR})^2$ and minimum $g_{\mu\tau}$ (benchmark-1), and vice versa (benchmark-2). For these values, we show the relative contribution of each S_1 -portal (Z' -portal) channel in the left (right) panel of Fig. 13. For benchmark-1 i.e., maximal $(y_{qR})^2$, the contribution (blue curve) from SLQ-portal channels [$d\bar{d}, s\bar{s}, b\bar{b}$ ($\sim 32\%$ each)] dominate over the $(Z', H_{1,2}, \eta)$ -portal contribution (green curve) for almost the entire DM mass region except near the resonances in the propagators Z' , H_1 , and H_2 . Similarly, for benchmark-2, i.e., maximal $g_{\mu\tau}$, the contribution from the Z' -portal channels [$\mu\bar{\mu}, \tau\bar{\tau}, \nu_\mu\bar{\nu}_\mu, \nu_\tau\bar{\nu}_\tau$ ($\sim 24\%$ each)] (blue curve) is stronger than the rest (green curve) with the exception of the region near the resonances of the propagators H_1 and

H_2 . The contribution from the channel with $Z'H_1$ in the final state is negligible due to the ζ^2 (Higgs mixing) factor, while the process with $Z'H_2$ as final-state particles is not kinematically allowed ($M_{H_2} = 2.4$ TeV) for the displayed DM mass range. In the rest of the parameter region of Fig. 12, however, the dominant contribution depends on all four parameters listed in Table III.

VII. IMPLICATIONS ON R_K AND $B_{(s)} \rightarrow K^*(\phi)\mu^+\mu^-$ PROCESSES

The constrained parameter space discussed in the previous section can have an impact on R_K and the observables of the $B \rightarrow V l^+ l^-$ process, where $V = K^*, \phi$ are the vector mesons, which subsequently decay into $K^* \rightarrow K\pi$ and $\phi \rightarrow K^+ K^-$ states. The $B \rightarrow V$ hadronic matrix elements of the local quark bilinear operators can be parametrized as [88,89]

$$\begin{aligned} \langle V(k) | \bar{s} \gamma_\mu (1 - \gamma_5) b | B(p) \rangle &= \epsilon_{\mu\nu\alpha\beta} \epsilon^{*\nu} p^\alpha q^\beta \frac{2V(q^2)}{M_B + M_V} - i \epsilon_\mu^* (M_B + M_V) A_1(q^2) \\ &+ i (\epsilon^* \cdot q) (2p - q)_\mu \frac{A_2(q^2)}{M_B + M_V} + i \frac{2M_V}{q^2} (\epsilon^* \cdot q) [A_3(q^2) - A_0(q^2)] q_\mu, \end{aligned} \quad (58)$$

where

$$A_3(s) = \frac{(M_B + M_V)}{2M_V} A_1(s) - \frac{(M_B - M_V)}{2M_V} A_2(s), \quad (59)$$

q^2 is the momentum transfer between the B and V mesons, i.e., $q_\mu = p_\mu - k_\mu$, and ϵ_μ is the polarization vector of the V meson. The full angular differential decay distribution for

the processes $B^0 \rightarrow (K^{*0} \rightarrow K^- \pi^+) l^+ l^-$ and $B_s \rightarrow (\phi \rightarrow K^+ K^-) l^+ l^-$ in terms of q^2 , θ_l , θ_V , and ϕ is given as [90–92]

$$\frac{d^4\Gamma}{dq^2 d\cos\theta_l d\cos\theta_V d\phi} = \frac{9}{32\pi} J(q^2, \theta_l, \theta_V, \phi), \quad (60)$$

where

$$\begin{aligned}
J(q^2, \theta_l, \theta_V, \phi) = & J_1^s \sin^2 \theta_V + J_1^c \cos^2 \theta_V + (J_2^s \sin^2 \theta_V + J_2^c \cos^2 \theta_V) \cos 2\theta_l \\
& + J_3 \sin^2 \theta_V \sin^2 \theta_l \cos 2\phi + J_4 \sin 2\theta_V \sin 2\theta_l \cos \phi + J_5 \sin 2\theta_V \sin \theta_l \cos \phi \\
& + (J_6^s \sin^2 \theta_V + J_6^c \cos^2 \theta_V) \cos \theta_l + J_7 \sin 2\theta_V \sin \theta_l \sin \phi \\
& + J_8 \sin 2\theta_V \sin 2\theta_l \sin \phi + J_9 \sin^2 \theta_V \sin^2 \theta_l \sin 2\phi,
\end{aligned} \tag{61}$$

where θ_l is the angle between l^- and $B_{(s)}$ in the dilepton frame, θ_V is defined as the angle between K^- and $B_{(s)}$ in the $K^- \pi^+ (K^- K^+)$ frame, and ϕ is the angle between the normal of the $K^- \pi^+ (K^- K^+)$ and dilepton planes. The complete expressions for $J_i^{s(c)}$, $i = 1, 2, \dots, 9$ as a function of transversity amplitudes are given in Appendix B [93]. The transversity amplitudes written in terms of the form factors and Wilson coefficients are [93]

$$\begin{aligned}
A_{\perp L,R} = & N\sqrt{2}\lambda \left[((C_9^{\text{eff}} + C_9^{\text{NP}}) \mp C_{10}) \frac{V(q^2)}{M_B + M_V} + \frac{2m_b}{q^2} C_7 T_1(q^2) \right], \\
A_{\parallel L,R} = & -N\sqrt{2}(M_B^2 - M_V^2) \left[((C_9^{\text{eff}} - C_9^{\text{NP}}) \mp C_{10}) \frac{A_1(q^2)}{M_B - M_V} + \frac{2m_b}{q^2} C_7 T_2(q^2) \right], \\
A_{0L,R} = & -\frac{N}{2M_V \sqrt{s}} \left[(C_9^{\text{eff}} - C_9^{\text{NP}}) \mp C_{10} \left((M_B^2 - M_V^2 - q^2)(M_B + M_V) A_1(q^2) - \lambda \frac{A_2(q^2)}{M_B + M_V} \right) \right. \\
& \left. + 2m_b C_7 \left((M_B^2 + 3M_V^2 - q^2) T_2(q^2) - \frac{\lambda}{M_B^2 - M_V^2} \right) \right], \\
A_t = & 2N \sqrt{\frac{\lambda}{q^2}} C_{10} A_0(q^2),
\end{aligned} \tag{62}$$

where

$$N = V_{tb} V_{ts}^* \left[\frac{G_F^2 \alpha_{\text{em}}^2}{3 \cdot 2^{10} \pi^5 M_B^3} q^2 \beta_l \sqrt{\lambda} \right]^{1/2}, \quad \lambda = \lambda(M_V^2, M_B^2, q^2). \tag{63}$$

The dilepton invariant mass spectrum for $B \rightarrow V l^+ l^-$ decay after integrating over all angles [90] is given by

$$\frac{d\Gamma}{dq^2} = \frac{3}{4} \left(J_1 - \frac{J_2}{3} \right), \tag{64}$$

where $J_i = 2J_i^s + J_i^c$. The most interesting observables in these decay modes are the lepton nonuniversality parameter defined as

$$R_V = \frac{\text{Br}(B \rightarrow V \mu^+ \mu^-)}{\text{Br}(B \rightarrow V e^+ e^-)}, \tag{65}$$

and the form-factor-independent observables [94]

$$P'_4 = \frac{J_4}{\sqrt{-J_2^s J_2^c}}, \quad P'_5 = \frac{J_5}{2\sqrt{-J_2^s J_2^c}}. \tag{66}$$

Now that we are familiar with the different observables and the allowed values of the new parameters, we perform a numerical analysis in the full dilepton mass region, i.e., $4m_l^2 \leq q^2 \leq (M_B - M_V)^2$, excluding the regions around

$q^2 \sim m_{J/\psi}^2$ and $m_{\psi'}^2$. The cuts are employed to remove the dominant charmonium resonance $(c\bar{c}) = J/\psi, \psi'$ backgrounds from $B \rightarrow V(c\bar{c}) \rightarrow V l^+ l^-$. In Fig. 14, we show the behavior of R_K (top-left panel), R_{K^*} (top-right panel), and R_ϕ (bottom panel) with respect to q^2 in the full kinematically accessible physical region. In these figures, the blue dashed lines stand for the SM contribution, the orange bands represent the allowed region of parameters shown in Table II favored by both DM and flavor studies (DM + Flavor), and cyan (magenta) bands represent only the DM case, i.e., DM-I (DM-II). The bin-wise experimental values of $R_{K^{(*)}}$ are shown in black. From the top-left panel of Fig. 14, it can be seen that the result obtained in the $q^2 \in [1, 6]$ GeV² bin, by using the constraints from only DM observables is consistent with the experimental data and can be explained within 1σ for the DM + Flavor case. The measured value of R_{K^*} in the $q^2 \in [0.045, 1.1]$ GeV² bin can be accommodated within 1σ (DM-I) and 2σ (DM-II and DM + Flavor), while the $q^2 \in [1.1, 6]$ GeV² bin result can be described within 1σ (DM-II) and 2σ (DM + Flavor) and is consistent with experimental data for the DM-I case. Although there is no experimental evidence for R_ϕ , the additional NP contributions arising from the allowed parameter space of all cases (DM-I, SM-II, and DM + Flavor) provide significant deviations from the SM

FIG. 14. The LNU parameters R_K (top-left panel), R_{K^*} (top-right panel), and R_ϕ (bottom panel) for varying q^2 in the $L_\mu - L_\tau$ model. Here, the blue dashed lines represent the SM prediction, while the cyan (magenta) bands stand for the NP contributions from the dark matter studies, i.e., DM-I (DM-II). The orange bands represent the contributions from both the flavor and DM sectors (DM + Flavor). The experimental data points (with 2σ error bars) [1] are shown as black lines.

predictions, implying the presence of lepton universality violation in the $B_s \rightarrow \phi \mu^+ \mu^-$ process. In Table IV, we present our predicted values of $R_{K^{(*)}}$ and R_ϕ for different bins. The variation of the optimized observables $-P'_4$

(top-left panel) and P'_5 (top-right panel) of $B^0 \rightarrow K^{*0} \mu^+ \mu^-$ process with q^2 are depicted in Fig. 15. The bottom panels of this figure show analogous plots for the $B_s \rightarrow \phi \mu^+ \mu^-$ process in both the high- and low-recoil limits. It should be

TABLE IV. Predicted numerical values of LNU parameters (R_V) and $P'_{4,5}$ observables of the $B_{(s)} \rightarrow V \mu^+ \mu^-$ ($V = K^*, \phi$) processes in the high- and low-recoil limits. The values of R_K are also presented in both the low- and high- q^2 regime.

	Observables	Values for SM	Values for DM-I	Values for DM-II	Values for DM + Flavor
$B^0 \rightarrow K^{*0} \mu^+ \mu^-$	$R_{K^*} q^2 \in [0.045, 1.1] \text{ GeV}^2$	0.949	0.793–0.949	0.852–0.863	0.881–0.949
	$R_{K^*} q^2 \in [1.1, 6] \text{ GeV}^2$	0.993	0.671–0.993	0.786–0.811	0.848–0.993
	$R_{K^*} q^2 \geq 14.18 \text{ GeV}^2$	0.998	0.754–0.998	0.832–0.851	0.879–0.998
	$P'_5 q^2 \in [1, 6] \text{ GeV}^2$	-0.542 ± 0.044	$-0.82 \rightarrow -0.542$	$-0.655 \rightarrow -0.635$	$-0.612 \rightarrow -0.542$
	$P'_5 q^2 \geq 14.18 \text{ GeV}^2$	-0.695 ± 0.056	$-1.165 \rightarrow -0.695$	$-0.894 \rightarrow -0.86$	$-0.818 \rightarrow -0.695$
	$P'_4 q^2 \in [1, 6] \text{ GeV}^2$	0.563 ± 0.045	$-0.239 \rightarrow 0.563$	$0.195 - 0.257$	$0.334 \rightarrow 0.563$
	$P'_4 q^2 \geq 14.18 \text{ GeV}^2$	1.2 ± 0.096	$0.787 - 1.2$	$1.03 - 1.06$	$1.098 - 1.2$
	$R_\phi q^2 \in [0.045, 1.1] \text{ GeV}^2$	0.9499	0.753–0.9499	0.828–0.844	0.866–0.9499
$B_s \rightarrow \phi \mu^+ \mu^-$	$R_\phi q^2 \in [1.1, 6] \text{ GeV}^2$	0.994	0.642–0.994	0.77–0.8	0.839–0.994
	$R_\phi q^2 \geq 14.18 \text{ GeV}^2$	0.998	0.731–0.998	0.82–0.84	0.871–0.998
	$P'_5 q^2 \in [1, 6] \text{ GeV}^2$	-0.511 ± 0.041	$-0.766 \rightarrow -0.551$	$-0.618 \rightarrow -0.599$	$-0.578 \rightarrow -0.511$
	$P'_5 q^2 \geq 14.18 \text{ GeV}^2$	-0.645 ± 0.052	$-1.1 \rightarrow -0.645$	$-0.839 \rightarrow -0.805$	$-0.765 \rightarrow -0.645$
	$P'_4 q^2 \in [1, 6] \text{ GeV}^2$	0.596 ± 0.048	$-0.202 \rightarrow 0.596$	$0.228 - 0.29$	$0.367 \rightarrow 0.596$
	$P'_4 q^2 \geq 14.18 \text{ GeV}^2$	1.233 ± 0.099	$0.822 - 1.233$	$1.059 - 1.092$	$1.13 - 1.233$
	$R_K q^2 \in [1, 6] \text{ GeV}^2$	1.0004	0.643–1.0004	0.71–0.766	0.834–1.0004
	$R_K q^2 \geq 14.18 \text{ GeV}^2$	1.002	0.645–1.002	0.71–0.768	0.836–1.002

FIG. 15. Top panels: The variation of P'_4 (left) and P'_5 (right) of the $B^0 \rightarrow K^{*0} \mu^+ \mu^-$ process with respect to q^2 . Bottom panels: The behavior of P'_4 (left) and P'_5 (right) for the $B_s \rightarrow \phi \mu^+ \mu^-$ process. The bin-wise experimental data points with error bars are shown in black [4]. Note that $P'_4|_{\text{LHCb}} = -P'_4$.

noted that $P'_4|_{\text{LHCb}} = -P'_4$. In the low- q^2 region, our predictions for the $-P'_4$ observable of the $B^0 \rightarrow K^{*0} \mu^+ \mu^-$ process are in very good agreement with the LHCb data. For the $B^0 \rightarrow K^{*0} \mu^+ \mu^-$ decay mode, the model can accommodate the P'_5 observable within 1σ of the experimental data, in the high- and very low- q^2 regions. We notice considerable deviation between the SM results and the presented $L_\mu - L_\tau$ model on the $P'_{4,5}$ observables for the $B_s \rightarrow \phi \mu^+ \mu^-$ decay modes. The numerical values of all of these observables are given in Table IV. We find that our results for the angular observables of the $B \rightarrow VII$ process obtained from the DM-I parameter space are almost consistent with the corresponding experimentally measured data.

VIII. SUMMARY AND CONCLUSION

In summary, we have studied Majorana dark matter in a new version of the $U(1)_{L_\mu - L_\tau}$ gauge extension of the Standard Model. The model is free from triangle gauge anomalies with the inclusion of three neutral fermions with $L_\mu - L_\tau$ charges 0, 1, and -1 . A scalar singlet with charge $+2$ under the new $U(1)$ is added to spontaneously break the $L_\mu - L_\tau$ gauge symmetry, thereby giving masses to the new fermions and the neutral boson Z' associated with the gauge extension. In addition, the scalar sector is enriched with an

inert doublet and a $(\bar{3}, 1, 1/3)$ scalar leptoquark to obtain the neutrino masses at the one-loop level and address the flavor anomalies, respectively. All of the new fermions, the leptoquark, and the inert doublet have charge -1 under Z_2 symmetry. Choosing the lightest mass eigenstate of the new fermion spectrum as dark matter, we thoroughly studied Majorana dark matter in the context of the relic density and direct detection. The channels contributing to the relic density are mediated by the scalar leptoquark, Z' , and inert doublet components. As Z' does not couple to quarks, the Z' -mediated tree-level interaction for direct detection is not permitted. Only leptoquark-portal channels contribute to the spin-dependent WIMP-nucleon cross section. By imposing the Planck limit on the relic density and well known PICO-60 and LUX bounds on the spin-dependent cross section, we have constrained the new parameters of the model. We have also computed the spin-dependent and spin-independent contributions of one-loop diagrams involving the scalar leptoquark, but found that they have zero impact on the model parameters. We have also shown the mechanism that radiatively generates light neutrino masses using the inert doublet. The viable parameter region consistent with neutrino oscillation data was also addressed.

We have further restricted the new parameters from the quark and lepton sectors, i.e., by comparing the theoretical predictions of $\text{Br}(\tau \rightarrow \mu \nu_\tau \bar{\nu}_\mu)$, $\text{Br}(B \rightarrow X_s \gamma)$,

$\text{Br}(B^+ \rightarrow K^+ \tau^+ \tau^-)$, $\text{Br}(\bar{B}^0 \rightarrow \bar{K}^0 \mu^+ \mu^-)$, and $B_s - \bar{B}_s$ mixing with their corresponding 3σ experimental data. The neutral and charged lepton-flavor-violating decay processes are absent due to zero $Z' \tau \mu$ coupling, and the vanishing $Z' q \bar{q}$ coupling restricts the involvement of Z' in $B_s - \bar{B}_s$ mixing and $b \rightarrow s \gamma$ processes at the one-loop level. We then investigated the implications on the observables $P'_{4,5}$, $R_{K^{(*)}}$, and R_ϕ of the $B_{(s)} \rightarrow K^{(*)}(\phi) l^+ l^-$ decay modes in the full kinematically allowed q^2 region for two cases, i.e., dark-matter- and flavor-allowed, or only dark-matter-allowed parameter space. We observed that our model can explain the LNU parameter R_K very well. The observable R_{K^*} obtained from the allowed parameter space consistent with only dark matter was found to be within its 1σ and from the allowed region compatible with both the dark matter and flavor sectors is within 2σ experimental limit. In the presence of new physics, lepton universality violation is observed in the $B_s \rightarrow \phi \mu^+ \mu^-$ process, and thus it can be probed by LHCb. We noticed that the proposed $L_\mu - L_\tau$ model is also able to explain the LHCb experimental data of the famous optimized $P'_{4,5}$ observables of the $B^0 \rightarrow K^{*0} \mu^+ \mu^-$ process. We also found that the form-factor-independent observables for the $B_s \rightarrow \phi \mu^+ \mu^-$ decay modes have sizable deviations from their Standard Model values. We observed that the parameter region satisfying only dark matter observables for $M_- \leq 560$ GeV has a strong impact on the flavor anomalies. To conclude, we have comprehensively studied Majorana dark matter, neutrino phenomenology, and flavor anomalies in a $U(1)_{L_\mu - L_\tau}$ gauge-extended model. This simple framework survives all of the current experimental limits on dark matter and flavor observables, and can be probed in upcoming high-luminosity experiments.

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APPENDIX A: LOOP FUNCTIONS

The loop functions required to compute the $b \rightarrow s ll$ decays are given by [20,81]

$$\begin{aligned} \mathcal{V}_{sb}(\chi_-, \chi_+) &= \cos^2 \alpha \sin^2 \alpha (1 + 4\sqrt{\chi_- \chi_+} j(\chi_-, \chi_+) \\ &\quad - 2k(\chi_-, \chi_+)) \\ &\quad + 2\sin^2 \alpha I(\chi_+) + 2\cos^2 \alpha I(\chi_-), \end{aligned} \quad (\text{A1})$$

where

$$\begin{aligned} f(\chi_1, \chi_2, \chi_3, \dots) &\equiv \frac{f(\chi_1, \chi_3, \dots) - f(\chi_2, \chi_3, \dots)}{\chi_1 - \chi_2}, \\ f &= j, \kappa, \end{aligned} \quad (\text{A2})$$

with

$$j(\chi) = \frac{\chi \log \chi}{\chi - 1}, \quad (\text{A3})$$

$$\kappa(\chi) = \frac{\chi^2 \log \chi}{\chi - 1}, \quad (\text{A4})$$

$$I(\chi) = \frac{-3\chi^2 + 4\chi - 1 + 2\chi^2 \log \chi}{8(\chi - 1)^2}. \quad (\text{A5})$$

Equations (A2)–(A4) are used to investigate $B_s - \bar{B}_s$ mixing.

APPENDIX B: J_i COEFFICIENTS OF $B_{(s)} \rightarrow VII$ PROCESSES

The expressions for the J_i coefficients of $B_{(s)} \rightarrow VII$ processes in terms of transversity amplitudes are given by [91,93]

$$\begin{aligned} J_1^s &= \frac{(2 + \beta_l^2)}{4} [|A_\perp^L|^2 + |A_\parallel^L|^2 + (L \rightarrow R)] \\ &\quad + \frac{4m_l^2}{q^2} \text{Re}(A_\perp^L A_\perp^{R*} + A_\parallel^L A_\parallel^{R*}), \end{aligned} \quad (\text{B1})$$

$$J_1^c = |A_0^L|^2 + |A_0^R|^2 + \frac{4m_l^2}{q^2} [|A_t|^2 + 2\text{Re}(A_0^L A_0^{R*})] + \beta_l^2 |A_S|^2, \quad (\text{B2})$$

$$J_2^s = \frac{\beta_l^2}{4} [|A_\perp^L|^2 + |A_\parallel^L|^2 + (L \rightarrow R)], \quad (\text{B3})$$

$$J_2^c = -\beta_l^2 [|A_0^L|^2 + (L \rightarrow R)], \quad (\text{B4})$$

$$J_3 = \frac{1}{2} \beta_l^2 [|A_\perp^L|^2 - |A_\parallel^L|^2 + (L \rightarrow R)], \quad (\text{B5})$$

$$J_4 = \frac{1}{\sqrt{2}} \beta_l^2 [\text{Re}(A_0^L A_\parallel^{L*}) + (L \rightarrow R)], \quad (\text{B6})$$

$$\begin{aligned} J_5 &= \sqrt{2} \beta_l \left[\text{Re}(A_0^L A_\perp^{L*}) - (L \rightarrow R) \right. \\ &\quad \left. - \frac{m_l}{\sqrt{q^2}} \text{Re}(A_\parallel^L A_S^{*} + A_\parallel^R A_S^{*}) \right], \end{aligned} \quad (\text{B7})$$

$$J_6^s = 2\beta_l [\text{Re}(A_\perp^L A_\perp^{L*}) - (L \rightarrow R)], \quad (\text{B8})$$

$$J_6^c = 4\beta_l \frac{m_l}{\sqrt{q^2}} \text{Re}[A_0^L A_S^* + (L \rightarrow R)], \quad (\text{B9})$$

$$J_7 = \sqrt{2}\beta_l \left[\text{Im}(A_0^L A_{\parallel}^{L*}) - (L \rightarrow R) + \frac{m_l}{\sqrt{q^2}} \text{Im}(A_{\perp}^L A_S^* + A_{\perp}^R A_S^*) \right], \quad (\text{B10})$$

$$J_8 = \frac{1}{\sqrt{2}}\beta_l^2 [\text{Im}(A_0^L A_{\perp}^{L*}) + (L \rightarrow R)], \quad (\text{B11})$$

$$J_9 = \beta_l^2 [\text{Im}(A_{\parallel}^{L*} A_{\perp}^L) + (L \rightarrow R)], \quad (\text{B12})$$

where

$$A_i A_j^* = A_i^L(q^2) A_j^{*L}(q^2) + A_i^R(q^2) A_j^{*R}(q^2) \quad (i, j = 0, \parallel, \perp), \quad (\text{B13})$$

in shorthand notation.

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