$\bar{D}^*\bar{D}^*\Sigma_c$ three-body system

M. Pavon Valderrama*

School of Physics and Nuclear Energy Engineering, International Research Center for Nuclei and Particles in the Cosmos and Beijing Key Laboratory of Advanced Nuclear Materials and Physics, Beihang University, Beijing 100191, China

(Received 27 November 2018; published 15 February 2019)

In the molecular picture the hidden-charm, pentaquarklike $P_c(4450)$ resonance is a $\bar{D}^*\Sigma_c$ bound state with quantum numbers $I = \frac{1}{2}$ and $J^P = \frac{3}{2}^-$. If this happens to be the case, it will be natural to expect the existence of $\bar{D}^*\bar{D}^*\Sigma_c$ three-body bound states. The most probable quantum numbers for a bound $\bar{D}^*\bar{D}^*\Sigma_c$ trimer are the isoscalar $J^P = \frac{1}{2}^+, \frac{5}{2}^+$ and the isovector $J^P = \frac{3}{2}^+, \frac{5}{2}^+$ configurations. Calculations within a contact-range theory indicate a trimer binding energy $B_3 \sim 3\text{--}5$ MeV and $14\text{--}16$ MeV for the isoscalar $\frac{1}{2}^+$ and $\frac{5}{2}^+$ states and $B_3 \sim 1\text{--}3$ MeV and $3\text{--}5$ MeV for the isovector $\frac{3}{2}^+$ and $\frac{5}{2}^+$ states, respectively, with B_3 relative to the $\bar{D}^*P_c(4450)$ threshold. These predictions are affected by a series of error sources that we discuss in detail.

DOI: 10.1103/PhysRevD.99.034010

I. INTRODUCTION

The discovery of two hidden-charm pentaquarklike resonances by the LHCb [1], the $P_c(4380)$ and $P_c(4450)$, begs the question of what is their nature. These resonances are indeed different: while the $P_c(4380)$ is relatively broad, the $P_c(4450)$ is narrow and is located close to a few meson-baryon thresholds, namely the $\bar{D}\Lambda_c(2595)$, $\bar{D}^*\Sigma_c$, $\bar{D}^*\Sigma_c^*$, and $J/\psi p$ thresholds. This coincidence makes the $P_c(4450)$ a good candidate for a hadronic molecule, a type of exotic hadron theorized a few decades ago [2,3]. In particular, the $P_c(4450)$ has been suggested to be a $\bar{D}^*\Sigma_c$ [4–6], a $\bar{D}^*\Sigma_c^*$ molecule [7,8] [in these two cases probably with a small admixture of $\bar{D}\Lambda_c(2595)$ [9,10]], and a $\chi_{c1}p$ molecule [11]. Of these possibilities, the most natural one probably is $\bar{D}^*\Sigma_c$. First, the interaction between a heavy baryon and antimeson is expected to be mediated by the exchange of light mesons, providing a mechanism to justify the existence of attraction. Second, the binding energy of the $\Sigma_c\bar{D}^*$, $B_2 = 12 \pm 3$ MeV, translates into a bound state that is not excessively compact, which is compatible with the molecular hypothesis. The expected size is of the order of $1/\sqrt{2\mu B_2} \sim 1.2$ fm, with μ being the reduced mass of the molecule. Besides the molecular hypothesis there are other

competing explanations for the nature of the P_c^* , which include a genuine pentaquark [12–18], threshold effects [19,20] (for a detailed discussion see Ref. [21]), baryo-charmonia [22], a molecule bound by *color chemistry* [23], or a soliton [24]. The molecular hypothesis relies on the quantum numbers of the pentaquark to be $J^P = \frac{3}{2}^-$ or alternatively $\frac{5}{2}^-$. Experimentally J^P is not uniquely determined yet, with $\frac{3}{2}^+$ and $\frac{5}{2}^+$ also probable [1]. Until the quantum numbers of the P_c^* are known, the discussion about its nature will remain theoretical.

If the $P_c(4450)$ (the P_c^* from now on) is indeed a $J^P = \frac{3}{2}^-$ $\bar{D}^*\Sigma_c$ bound state, its location will provide useful information about the dynamics of this two-body system. This information can be used to deduce the existence of new molecules. For instance, from heavy quark spin symmetry it is sensible to expect the existence of a $J^P = \frac{5}{2}^-$ $\bar{D}^*\Sigma_c^*$ partner of the P_c^* with a mass of 5515 MeV [25]. In this manuscript we argue that if the P_c^* turns out to be molecular the dynamics of the $\bar{D}^*\Sigma_c$ two-body system imply the existence of a series of $\bar{D}^*\bar{D}^*\Sigma_c$ three-body bound states. Concrete calculations in a contact-range theory lead to the following predictions:

- (i) a $J = \frac{1}{2}$, $I = 0$ trimer with $B_3 \sim 3\text{--}5$ MeV,
- (ii) a $J = \frac{3}{2}$, $I = 1$ trimer with $B_3 \sim 1\text{--}3$ MeV,
- (iii) a $J = \frac{5}{2}$, $I = 0$ trimer with $B_3 \sim 14\text{--}16$ MeV,
- (iv) a $J = \frac{5}{2}$, $I = 1$ trimer with $B_3 \sim 3\text{--}5$ MeV,

where the trimer binding energy B_3 is relative to the hadron-dimer threshold, i.e., the $\bar{D}^*P_c^*$ threshold. There are a series of uncertainties related to the previous predictions of which the most crucial one is whether the P_c^* is really a $\bar{D}^*\Sigma_c$ bound state. Until the nature of the P_c^* is

*mpavon@buaa.edu.cn

Published by the American Physical Society under the terms of the [Creative Commons Attribution 4.0 International license](#). Further distribution of this work must maintain attribution to the author(s) and the published article's title, journal citation, and DOI. Funded by SCOAP³.

settled, these trimers will remain a theoretical possibility. Conversely the experimental production of these trimers or their detection in the lattice will strongly suggest a molecular nature for the P_c^* . It is worth noticing that analogous trimer predictions have been made for other molecular candidates. For instance, from the hypothesis that the $X(3873)$ is a $D^*\bar{D}$ molecule it is possible to theorize about $D^*\bar{D}K$, $D^*\bar{D}^*K$ [26], $D^*D^*\bar{D}$, and $D^*D^*\bar{D}^*$ trimers [27]. Within more phenomenological approaches there has also been a growing interest in the possibility of three-body systems in the heavy sector, for instance, $D^*\bar{D}^*\rho$ [28], $B^*\bar{B}^*\rho$ [29], BDD , $BDD\bar{D}$, $B^*D\bar{D}$, etc. [30,31], just to mention a few recent works.

The manuscript is structured as follows: in Sec. II we write the Faddeev equations for the $\bar{D}^*\bar{D}^*\Sigma_c$ system. In Sec. III we consider the $\bar{D}^*\bar{D}^*\Sigma_c$ system in the unitary limit, i.e., when the binding energy of the $\bar{D}^*\Sigma_c$ system approaches 0. In Sec. IV we present our predictions of three-body states. Finally in Sec. V we summarize the results of this manuscript.

II. FADDEEV EQUATIONS FOR THE $\Sigma_c\bar{D}^*\bar{D}^*$ SYSTEM

In this section we write the Faddeev decomposition and equations for the $\bar{D}^*\bar{D}^*\Sigma_c$ three-body system. We consider the S-wave three-body states only, as they have the highest chances of binding. We make the following assumptions:

- (i) the $\bar{D}\bar{D}$, $\bar{D}\bar{D}^*$, and $\bar{D}^*\bar{D}^*$ pairs do not interact,
- (ii) the $\bar{D}^*\Sigma_c$ only interacts in the P_c^* channel,
- (iii) the $\bar{D}^*\Sigma_c$ interaction in the P_c^* channel can be described in terms of a contact-range interaction.

This leads to a considerable simplification of the Faddeev equations. Equivalently we can consider the $\bar{D}^*\bar{D}^*\Sigma_c$ system from the effective field theory point of view. In this case we say that the $\bar{D}^*\Sigma_c$ contact-range interaction is leading order, while all the other interactions are subleading and can be ignored at leading order. We review the previous set of assumptions at the end of the manuscript.

A. The equations for $\bar{D}^*\bar{D}^*\Sigma_c$

We begin by writing the three-body wave function in terms of Faddeev components for the $\bar{D}^*\bar{D}^*\Sigma_c$ system,

$$\Psi_{3B} = \sum_{\beta} [\phi_{\beta}(\vec{k}_{23}, \vec{p}_1) + \zeta_{\beta} \phi_{\beta}(\vec{k}_{31}, \vec{p}_2)] \times |S_{12} \otimes \frac{1}{2}\rangle_S |I_{12} \otimes 1\rangle_I, \quad (1)$$

where we have labeled the two D^* mesons as particles 1 and 2 and the Σ_c as particle 3. The summation index $\beta = (S_{12}, I_{12})$ refers to the spin and isospin of the $\bar{D}^*\bar{D}^*$ subsystem. The spin and isospin piece of the wave function is indicated by

$$|S_{12} \otimes \frac{1}{2}\rangle_S |I_{12} \otimes 1\rangle_I, \quad (2)$$

where S and I are the total spin and isospin of the $\bar{D}^*\bar{D}^*\Sigma_c$ three-body system. The sign $\zeta_{\beta} = \pm 1$ indicates whether the spatial part of the $\bar{D}^*\bar{D}^*$ wave function is symmetric or antisymmetric. We ignore the $\zeta_{\beta} = -1$ configurations: they require the orbital angular momentum of the 12 subsystem to be $L_{12} \geq 1$, which implies that they are suppressed for the positive parity states (they depend on the P-wave $\bar{D}^*\Sigma_c$ interaction). We define the Jacobi momenta as follows,

$$\vec{k}_{ij} = \frac{m_j \vec{k}_i - m_i \vec{k}_j}{m_i + m_j}, \quad (3)$$

$$\vec{p}_k = \frac{1}{M_T} [(m_i + m_j) \vec{k}_k - m_k (\vec{k}_i + \vec{k}_j)], \quad (4)$$

with m_1, m_2, m_3 being the masses of particles 1, 2, 3, $M_T = m_1 + m_2 + m_3$ the total mass, and ijk an even permutation of 123. In particular, we have $m_1 = m_2 = m(\bar{D}^*)$ and $m_3 = m(\Sigma_c)$. At this point it is helpful to make the following observation about the Faddeev components: in principle, there are up to three Faddeev components for each channel β . Of these, the first two Faddeev components are symmetric or antisymmetric (as indicated by the sign ζ_{β}) because particles 1 and 2 are identical: $\phi_{\beta}(\vec{k}, \vec{p})$ denotes these components. Finally, the third Faddeev component vanishes because we have considered that the $\bar{D}^*\bar{D}^*$ subsystem does not interact.

We assume a contact-range $\bar{D}^*\Sigma_c$ potential of the type

$$V_{\Sigma_c\bar{D}^*} = C_{\sigma\tau} g(k)g(k'), \quad (5)$$

where $C_{\sigma\tau}$ is a coupling constant and $g(p)$ is a regulator function. The subscripts σ and τ indicate the spin and isospin channel of the $\bar{D}^*\Sigma_c$ subsystem, i.e., $\sigma = J_{23}$ and $\tau = I_{23}$. The $\bar{D}^*\Sigma_c$ T-matrix can be written as

$$T_{23} = t_{\sigma\tau}(Z)g(k)g(k'), \quad (6)$$

depending on the spin and isospin channel, where Z represents the energy of the few-body system with respect to the few-body mass threshold. With this two-body T-matrix, the Faddeev component ϕ_{β} admits the ansatz

$$\phi_{\beta}(k, p) = \frac{g(k)}{Z - \frac{k^2}{2\mu_{23}} - \frac{p^2}{2\mu_1}} a_{\beta}(p), \quad (7)$$

with the reduced masses defined as

$$\frac{1}{\mu_{ij}} = \frac{1}{m_i} + \frac{1}{m_j}, \quad (8)$$

TABLE I. The coupling factors $\lambda_{\sigma\tau}^{\beta\gamma}$ to be used in the coupled-channel Faddeev equations. The P_c^* channel corresponds to $\sigma = \frac{3}{2}$ and $\tau = \frac{1}{2}$, i.e., $\lambda_{\frac{31}{22}}$. In the last column we write the λ to be included in the single-channel Faddeev equation we use in this work, Eq. (15). For uncoupled channels $\lambda = \lambda_{\frac{31}{22}}$, while for coupled channels λ is given by the eigenvalues of $\lambda_{\frac{31}{22}}^{\beta\gamma}$.

J^P	I	$\beta = (S_{12}, I_{12})$	$\lambda_{\frac{11}{22}}$	$\lambda_{\frac{13}{22}}$	$\lambda_{\frac{31}{22}}$	$\lambda_{\frac{33}{22}}$	λ
$\frac{1}{2}^+$	0	(0,1)	$\frac{1}{3}$	0	$\frac{2}{3}$	0	$\frac{2}{3}$
$\frac{1}{2}^+$	1	$\{(0,1), (1,0)\}$	$\begin{pmatrix} \frac{2}{9} & \frac{2}{9} \\ \frac{2}{9} & \frac{2}{9} \end{pmatrix}$	$\begin{pmatrix} \frac{1}{9} & -\frac{2}{9} \\ -\frac{2}{9} & \frac{1}{9} \end{pmatrix}$	$\begin{pmatrix} \frac{4}{9} & -\frac{2}{9} \\ -\frac{2}{9} & \frac{1}{9} \end{pmatrix}$	$\begin{pmatrix} \frac{2}{9} & \frac{2}{9} \\ \frac{2}{9} & \frac{2}{9} \end{pmatrix}$	$\frac{5}{9}, 0$
$\frac{3}{2}^+$	0	(2,1)	$\frac{5}{6}$	0	$\frac{1}{6}$	0	$\frac{1}{6}$
$\frac{3}{2}^+$	1	$\{(1,0), (2,1)\}$	$\begin{pmatrix} \frac{1}{18} & \frac{\sqrt{10}}{18} \\ \frac{\sqrt{10}}{18} & \frac{5}{9} \end{pmatrix}$	$\begin{pmatrix} \frac{1}{9} & -\frac{\sqrt{10}}{18} \\ -\frac{\sqrt{10}}{18} & \frac{5}{18} \end{pmatrix}$	$\begin{pmatrix} \frac{5}{18} & -\frac{\sqrt{10}}{18} \\ -\frac{\sqrt{10}}{18} & \frac{1}{9} \end{pmatrix}$	$\begin{pmatrix} \frac{5}{9} & -\frac{\sqrt{10}}{18} \\ -\frac{\sqrt{10}}{18} & \frac{1}{18} \end{pmatrix}$	$\frac{7}{18}, 0$
$\frac{5}{2}^+$	0	(2,1)	0	0	1	0	1
$\frac{5}{2}^+$	1	(2,1)	0	0	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{2}{3}$

$$\frac{1}{\mu_k} = \frac{1}{m_k} + \frac{1}{m_i + m_j}. \quad (9)$$

From the previous, we find that $a_\beta(p)$ follows the integral equation

$$a_\beta(p_1) = \left[\sum \lambda_{\sigma\tau}^{\beta\gamma} t_{\sigma\tau}(Z_{23}) \right] \times \int \frac{d^3 p_2}{(2\pi)^3} B_{12}^0(\vec{p}_1, \vec{p}_2) a_\gamma(p_2), \quad (10)$$

where β and γ refer to the S_{12} and I_{12} spin and isospin channels and $Z_{23} = Z - \frac{p_1^2}{2m_1} - \frac{M_T}{m_2 + m_3}$. The driving term B_{12}^0 is given by

$$B_{ij}^0(\vec{p}_i, \vec{p}_j) = \frac{g(q_i)g(q_j)}{Z - \frac{p_1^2}{2m_1} - \frac{p_2^2}{2m_2} - \frac{p_3^2}{2m_3}}, \quad (11)$$

with $\vec{p}_1 + \vec{p}_2 + \vec{p}_3 = 0$ and

$$\vec{q}_k = \frac{m_j \vec{p}_i - m_i \vec{p}_j}{m_j + m_i}. \quad (12)$$

The integral equation can be discretized, in which case finding the bound state solution reduces to an eigenvalue problem.

Regarding the $\bar{D}^*\Sigma_c$ potential, the only $\sigma\tau$ channel for which we know the interaction is the P_c^* channel, i.e., $\sigma = \frac{3}{2}$ and $\tau = \frac{1}{2}$. For simplicity we only consider the interaction in the P_c^* channel and neglect the interaction in all other channels,

$$t_{\sigma\tau}(Z) = t_{P_c^*}(Z) \delta_{\sigma\frac{3}{2}} \delta_{\tau\frac{1}{2}}. \quad (13)$$

Equivalently, in the effective field theory language this means that we are considering the P_c^* interaction as leading order and the interaction in all the other channels as subleading corrections. This point is important as it entails a simplification in the Faddeev equations: coupled-channel equations become single-channel ones.

To understand this simplification we first notice that there are nine possible quantum numbers for the S-wave $\bar{D}^*\bar{D}^*\Sigma_c$ system: three spin states $J = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}$ compounded by three isospin states $I = 0, 1, 2$. Seven of these configurations involve a single channel, i.e., a single possible $\beta = (S_{12}, I_{12})$ configuration for the spin and isospin of the $\bar{D}^*\bar{D}^*$ subsystem. Then there are two quantum numbers that involve coupled channels: $J = \frac{1}{2}, \frac{3}{2}$ with $I = 1$. Now if we ignore all the $\bar{D}^*\Sigma_c$ interactions with the exception of the P_c^* channel, we can simply make the substitution

$$\lambda_{\sigma\tau}^{\beta\gamma} t_{\sigma\tau} \rightarrow \lambda_{P_c^*}^{\beta\gamma} t_{P_c^*}. \quad (14)$$

The matrix $\lambda_{P_c^*}^{\beta\gamma}$ can be diagonalized, in which case the coupled-channel equation reduces to a single-channel one,

$$a(p_1) = \lambda t_{P_c^*}(Z_{23}) \int \frac{d^3 p_2}{(2\pi)^3} B_{12}^0(\vec{p}_1, \vec{p}_2) a(p_2), \quad (15)$$

where λ is one of the eigenvalues of $\lambda_{P_c^*}^{\beta\gamma}$. In fact this is the equation that we use in all cases to find the binding energies of the $\bar{D}^*\bar{D}^*\Sigma_c$ trimers. The factors λ for the different trimer quantum numbers can be found in Table I.

III. THE $\bar{D}^*\bar{D}^*\Sigma_c$ SYSTEM IN THE UNITARY LIMIT

Here we consider the unitary limit of the previous set of Faddeev equations. The unitary limit refers to the limit in

which a two-body system is bound at threshold, which is interesting from a theoretical perspective because of its relation with the Efimov effect [32]. Actually the $\bar{D}^*\Sigma_c$ system in the P_c^* channel is far from the unitary limit: its expected size $1/\sqrt{2\mu_{23}B_2} \sim 1.2$ fm is comparable with the typical hadronic size (0.5–1.0 fm). Yet from a theoretical point of view the discussion about the unitary limit is relevant because of the following reasons: (i) the relation that exists among the Efimov effect [32], Thomas collapse [33] and the requirement of three-body forces to compensate for the latter [34,35] and (ii) the $\bar{D}^*\Sigma_c$ system might be closer to the unitary limit for quantum numbers different than the ones of the P_c^* channel.

As already seen, the eigenvalue equation of the $\bar{D}^*\bar{D}^*\Sigma_c$ system always reduces to

$$a(p_1) = \lambda \tau(Z_{23}) \int \frac{d^3 p_2}{(2\pi)^3} B_{12}^0(\vec{p}_1, \vec{p}_2) a(p_2), \quad (16)$$

with λ being the factor listed in Table I, which ranges from $\frac{1}{6}$ to 1 for the isoscalar and isovector trimers. Now we take the unitary limit, where we have

$$\tau(Z_{23}) \rightarrow -\frac{2\pi}{\mu_{23}} \sqrt{\frac{\mu_{23}}{\mu_1}} \frac{1}{p_1}, \quad (17)$$

$$\int \frac{d^2 p_2}{4\pi} B_{12}^0 \rightarrow -\frac{m_3}{2p_1 p_2} \log \left[\frac{p_1^2 + p_2^2 + \frac{2m}{m+m_3} p_1 p_2}{p_1^2 + p_2^2 - \frac{2m}{m+m_3} p_1 p_2} \right], \quad (18)$$

with $m = m(D^*)$, $m_3 = m(\Sigma_c)$, and where μ_{23} and μ_1 are defined in Eqs. (8) and (9). From this we arrive at

$$p_1 a(p_1) = \frac{\lambda}{2\pi} \frac{m_3}{\mu_{23}} \sqrt{\frac{\mu_{23}}{\mu_1}} \frac{1}{p_1} \int_0^\infty dp_2 p_2 a(p_2) \times \log \left[\frac{p_1^2 + p_2^2 + \frac{2m}{m+m_3} p_1 p_2}{p_1^2 + p_2^2 - \frac{2m}{m+m_3} p_1 p_2} \right], \quad (19)$$

which after the change of variable $p^2 a(p) = b(p)$ transforms into

$$b(p) = \frac{\lambda}{2\pi} \frac{m_3}{\mu_{23}} \sqrt{\frac{\mu_{23}}{\mu_1}} \int_0^\infty dx \frac{b(xp)}{x} \times \log \left[\frac{1 + x^2 + \frac{2m}{m+m_3} x}{1 + x^2 - \frac{2m}{m+m_3} x} \right]. \quad (20)$$

This equation admits power-law solutions of the type $b(p) = p^s$, in which case we end up with an eigenvalue equation for s ,

$$1 = \frac{\lambda}{2\pi} \frac{m_3}{\mu_{23}} \sqrt{\frac{\mu_{23}}{\mu_1}} \int_0^\infty dx x^{s-1} \log \left[\frac{1 + x^2 + \frac{2m}{m+m_3} x}{1 + x^2 - \frac{2m}{m+m_3} x} \right]. \quad (21)$$

This integral can be evaluated analytically [36,37], in which case we arrive at the eigenvalue equation

$$1 = \lambda J_{\text{Efimov}}(s, \alpha), \quad (22)$$

where J_{Efimov} is given by

$$J_{\text{Efimov}}(s, \alpha) = \frac{1}{\sin 2\alpha} \frac{2 \sin \alpha s}{s \cos \frac{\pi}{2} s}, \quad (23)$$

and with the angle α determined as

$$\alpha = \arcsin \left(\frac{1}{1 + \frac{m_3}{m}} \right), \quad (24)$$

with m and m_3 being the masses of the charmed meson and baryon, respectively.

The Efimov effect [32] happens when the eigenvalue equation, i.e., Eq. (22), admits complex solutions of the type $s = \pm i s_0$, which leads to a $b(p)$ that oscillates,

$$b(p) \propto \sin \left[s_0 \log \left(\frac{p}{\Lambda_3} \right) + \varphi \right], \quad (25)$$

with Λ_3 being a momentum scale and φ a phase. From the form of $b(p)$ we deduce that the system is invariant under the discrete scaling transformation $p \rightarrow e^{\pi/s_0} p$, which for the case of the binding energy reads as $B_3 \rightarrow e^{2\pi/s_0} B_3$. The condition for having the Efimov geometric spectrum is

$$\lambda \geq \lambda_c = \frac{\sin 2\alpha}{2\alpha}, \quad (26)$$

which for the $\bar{D}^*\bar{D}^*\Sigma_c$ system gives us $\lambda_c \simeq 0.861$. From this we deduce that the $J = \frac{5}{2}$, $I = 0$ trimer is potentially Efimov-like, while all the others are not (but only for the set of assumptions laid out at the beginning of Sec. II). For the Efimov-like trimer we have $s_0 \simeq 0.363$, from which the discrete scaling factor is $e^{\pi/s_0} \simeq 5711$ for momenta and $e^{2\pi/s_0} \simeq 3.262 \times 10^7$ for the trimer binding energy.¹ Even with a molecular P_c^* close to the unitary limit, the factors are too big to be realistically observed. Thus the analysis presented here is mostly academical, except for one thing: if the forces binding the trimer are short ranged, the possibility of the Efimov effect is related to the necessity

¹Notice that the discrete scaling factor for the $\bar{D}^*\bar{D}^*\Sigma_c$ system is different than in the three boson system, where the Efimov effect was originally proposed. In the three boson system $s_0 \simeq 1.00624$ and $e^{\pi/s_0} \simeq 22.7$, which can be easily reproduced from Eq. (22) by simply setting $\lambda = 2$ and $m = m_3$. For $\lambda = 1$ and $m = m_3$ the factor for the standard heteronuclear Efimov effect with equal masses (i.e., a system of three particles with identical masses but only two resonantly particle pairs) $s_0 \simeq 0.4137$ and $e^{\pi/s_0} \simeq 1986.1$, is reproduced. Details on how the Efimov effect manifests in different systems can be consulted in Refs. [38,39].

of a three-body force for the system to be properly renormalized [34,35]. The reason is the relation between the Efimov effect and the Thomas collapse [33]: that is, as the range of the two-body potential shrinks, the trimer binding energy grows, eventually diverging. However, for the $\bar{D}^*\bar{D}^*\Sigma_c$ trimer, the observation of this collapse requires interaction ranges that are orders of magnitude smaller than the typical hadron size.

Finally, though the Efimov effect is absent in the $\bar{D}^*\bar{D}^*\Sigma_c$ trimers under the assumptions we are making, it might still happen in other heavy hadron systems. For instance, the $B^*B^*\Sigma_c$ system might be a better candidate for the Efimov effect because the associated discrete scaling factors are larger: for $m = m(B^*)$, $m_3 = m(\Sigma_c)$, and $\lambda = 1$ we obtain $s_0 \simeq 0.652$ and $e^{\pi/s_0} \simeq 123.4$.

IV. PREDICTIONS

The basic building block of the three-body calculation is the $\bar{D}^*\Sigma_c$ interaction in the channel with the quantum numbers of the P_c^* : $J^P = \frac{3}{2}^-$ and $I = \frac{1}{2}$. We describe the $\bar{D}^*\Sigma_c$ system in terms of a contact-range potential of the type

$$V(\bar{D}^*\Sigma_c) = C(\Lambda) f\left(\frac{k}{\Lambda}\right) f\left(\frac{k'}{\Lambda}\right), \quad (27)$$

where $C(\Lambda)$ is a coupling constant and $f(\Lambda)$ a regulator function. Here we use a Gaussian regulator $f(x) = e^{-x^2}$ and a cutoff window $\Lambda = 0.5\text{--}1.0$ GeV. The regulator choice is arbitrary, where we have chosen a Gaussian regulator mostly for convenience and compatibility with previous works [25,27] (we comment on the regulator dependence later). The cutoff choice corresponds to the expected momenta at which the description of the P_c^* as a $\bar{D}^*\Sigma_c$ bound state will break down. In the molecular interpretation the P_c^* is a $\bar{D}^*\Sigma_c$ bound state with a binding energy of $B_2 = 12 \pm 3$ MeV. This binding energy is simply the difference $m(D^*) + m(\Sigma_c) - m(P_c^*)$, where we take the isospin symmetric limit for the D^* and Σ_c masses and with the uncertainty basically corresponding to the experimental error in $m(P_c^*)$. We can determine the strength of the coupling $C(\Lambda)$ from the condition of reproducing the binding energy B_2 . For this we use the two-body eigenvalue equation

$$1 + C(\Lambda) \int \frac{d^3q}{(2\pi)^3} \frac{f^2(\frac{q}{\Lambda})}{B_2 + \frac{q^2}{2\mu_{23}}} = 0, \quad (28)$$

where B_2 is the two-body binding energy and μ_{23} the reduced mass of the $\bar{D}^*\Sigma_c$ system.

Once the contact-range coupling is obtained from the two-body eigenvalue equation, we can solve the three-body eigenvalue equation with $C_{\frac{31}{2}} = C(\Lambda)$, $g(k) = f(k/\Lambda)$ and the appropriate factor λ . Concrete calculations lead to the

TABLE II. Predictions for the binding energy B_3 (in units of MeV) of the $\bar{D}^*\bar{D}^*\Sigma_c$ trimers for different quantum numbers and for a cutoff $\Lambda = 0.5\text{--}1.0$ GeV, where B_3 is relative to the $\bar{D}^*P_c^*$ threshold (i.e., $B_3 > 0$ indicates that the trimer binds). The errors in B_3 are a consequence of the uncertainty in the $\bar{D}^*\Sigma_c$ binding energy, $B_2 = 12 \pm 3$ MeV.

J^P	I	$B_3(\Lambda = 0.5 \text{ GeV})$	$B_3(\Lambda = 1.0 \text{ GeV})$	λ
$\frac{1}{2}^+$	0	$4.8^{+1.5}_{-1.4}$	$3.1^{+1.1}_{-1.0}$	$\frac{2}{3}$
$\frac{1}{2}^+$	1	$2.6^{+1.0}_{-0.9}$	$1.3^{+0.6}_{-0.5}$	$\frac{5}{9}$
$\frac{3}{2}^+$	0	...	...	$\frac{1}{6}$
$\frac{3}{2}^+$	1	$0.5^{+0.3}_{-0.2}$	...	$\frac{7}{18}$
$\frac{5}{2}^+$	0	14 ± 3	16^{+3}_{-4}	1
$\frac{5}{2}^+$	1	$4.8^{+1.5}_{-1.4}$	$3.1^{+1.1}_{-1.0}$	$\frac{2}{3}$

predictions of Table II, where the binding energy B_3 is shown for different trimer configurations. The binding energy B_3 is defined with respect to the dimer-particle threshold; i.e., the mass of the trimers is

$$M = 2m + m_3 - B_2 - B_3, \quad (29)$$

with m and m_3 being the mass of the $\bar{D}^*$ meson and Σ_c baryon, respectively. The binding energy does not directly depend on the quantum numbers of the trimer, but indirectly by means of the factor λ as can be appreciated in Table II. This dependence on the coefficient λ is shown explicitly in Fig. 1.

The trimer binding energies are affected by a series of uncertainties, which we discuss below. The results of Table II already contain two error sources, the binding energy of the P_c^* ($B_2 = 12 \pm 3$ MeV) and the cutoff window ($\Lambda = 0.5\text{--}1.0$ GeV). Assuming that the P_c^* is indeed molecular, the next most important source of uncertainty is the choice of which interactions are leading and subleading. Previously we have assumed that the only

FIG. 1. Binding energy B_3 of the $\bar{D}^*\bar{D}^*\Sigma_c$ trimer as a function of the coefficient λ , which parametrizes the overlap of the trimer quantum numbers with the $\bar{D}^*\Sigma_c$ system in the P_c^* channel. The bars represent the error coming from the uncertainty in the binding energy of the P_c^* , $B_2 = 12 \pm 3$ MeV.

leading order interaction is the $\bar{D}^*\Sigma_c$ short-range potential in the P_c^* channel. We review this assumption in detail in the following lines, where the different possibilities are named scenarios A, B, and C.

We begin with the $\bar{D}^*\bar{D}^*$ interaction. Törnqvist pointed out [40] that the flavor exotic configurations of this system are less likely to bind in general. But this conclusion is probably incomplete because it relies on one-pion exchange while ignoring the short-range contributions to the $\bar{D}^*\bar{D}^*$ interaction. In this regard several works [41–44] ([45–47]) have indicated the possibility of a isoscalar flavor exotic 1^+ tetraquark below (above) the $\bar{D}\bar{D}^*$ threshold. If close enough to the $\bar{D}^*\bar{D}^*$ threshold, it might contribute to the dynamics of this two-body system, suggesting a strong attraction in the $J = 1, I = 0$ channel. It turns out that this contribution will reduce/increase the binding of the isovector $J^P = \frac{1}{2}^+/ \frac{3}{2}^+$ trimer. The reason for this reduction/increase of the binding energy is the $\lambda_{\sigma\tau}^{\beta\gamma}$ coefficients in Table I: for the isovector $J^P = \frac{1}{2}^+/ \frac{3}{2}^+$ three-body state, attraction in the isoscalar $\bar{D}^*\bar{D}^*$ subsystem forces the trimer into a configuration that has less/more overlap with the P_c^* channel of the $\bar{D}^*\Sigma_c^*$ subsystem. Scenario A represents the possibility of a shallow isoscalar $J = 1 \bar{D}^*\bar{D}^*$ bound state located at threshold, where predictions for the binding energy of the two affected trimers can be found in Table III.

Next we consider the $\bar{D}^*\Sigma_c$ interaction in channels different than the P_c^* ($I = \frac{1}{2}, J^P = \frac{3}{2}^-$). Phenomenology, in particular, the hidden-gauge model [48], indicates that the interaction in the $I = \frac{1}{2}, J^P = \frac{1}{2}^-$ channel could very well be as attractive as in the P_c^* channel. Scenario B will refer to this possibility. If this is the case the three isoscalar trimers will be degenerate, having the same binding energy as the isoscalar $J^P = \frac{5}{2}^+$ trimer; see Table III. The $\bar{D}^*\Sigma_c^*$ interaction in the $I = \frac{3}{2}$ channel has received less attention, though it could also be strong and attractive [7]. Scenario C

TABLE III. Different scenarios for the predictions for the binding energy B_3 of the $\bar{D}^*\bar{D}^*\Sigma_c$ trimers. Scenario A refers to the existence of a $\bar{D}^*\bar{D}^*$ bound state at threshold. Scenario B is when the $I = \frac{1}{2}$ and $J^P = \frac{1}{2}^-$ and $\frac{3}{2}^- \bar{D}^*\Sigma_c^*$ are degenerate. Scenario C assumes that the $I = \frac{3}{2} \bar{D}^*\Sigma_c^*$ interaction is strong, generating a bound state at threshold for both $J^P = \frac{1}{2}^-$ and $\frac{3}{2}^-$. For each scenario only the quantum numbers affected are shown.

Scenario	J^P	I	B_3
A	$\frac{1}{2}^+$	1	0.5–1.9
A	$\frac{3}{2}^+$	1	0.1–0.9
B	$\frac{1}{2}^+$	0	14–16
B	$\frac{3}{2}^+$	0	14–16
C/A + C	$\frac{1}{2}^+$	1	6.7–7.3
C/A + C	$\frac{3}{2}^+$	1	6.7–7.3
C/A + C	$\frac{5}{2}^+$	1	6.7–7.3

considers that there is a $I = \frac{3}{2} \bar{D}^*\Sigma_c^*$ bound state at threshold for both $J^P = \frac{1}{2}^-$ and $\frac{3}{2}^-$ (while also assuming scenario B). In this scenario the isovector trimers will be degenerate and more bound than expected; see Table III. The degeneration is again a consequence of the $\lambda_{\sigma\tau}^{\beta\gamma}$ coefficients of Table I: the isovector trimers can always be in a configuration with the same relative contribution from the $I = \frac{1}{2}$ and $\frac{3}{2} \bar{D}^*\Sigma_c^*$ channels. In addition the hypothesis of $\bar{D}^*\Sigma_c^*$ spin degeneracy in scenario B means that the trimers are effectively decoupled from the isoscalar $J = 1 \bar{D}^*\bar{D}^*$ interaction. That is, the combination of scenarios A and C leads to the same predictions as scenario C alone; see again Table III.

Another aspect of the $\bar{D}^*\Sigma_c$ interaction is its long-range piece, which is given by one-pion exchange (OPE). Here we have considered OPE to be subleading. This assumption can be analyzed with the formalism of Refs. [49,50], which investigated the range of momenta for which OPE is perturbative in the heavy meson-meson and heavy baryon-baryon systems. By adapting the methods of Refs. [49,50] to the heavy meson-baryon case, i.e., to $\bar{D}^*\Sigma_c$, we arrive at the conclusion that in the P_c^* channel OPE is perturbative for $p < \Lambda_{\text{OPE}} \simeq 270\text{--}450$ MeV in the chiral limit ($m_\pi = 0$, with m_π being the pion mass). The uncertainty is a consequence of the axial coupling constant of the pion with the charmed meson Σ_c , which is not known experimentally; see Ref. [51] for a recent summary. For the physical pion mass, $m_\pi \simeq 140$ MeV, we expect $\Lambda_{\text{OPE}} \simeq 410\text{--}710$ MeV; see Refs. [49,50] for a detailed explanation. If we take into account that the binding momentum of a molecular P_c^* is about 160 MeV, it is natural to expect OPE to be subleading, although it will provide important corrections to the leading order description. The binding momentum of the $\frac{5}{2}^+$ trimer, the most bound three-body state we are predicting, is of the same order: $\sqrt{2\mu_1 B_3} \sim 200\text{--}210$ MeV (or, in terms of size $1/\sqrt{2\mu_1 B_3} \sim 0.9\text{--}1.0$ fm). Owing to the similarity in the scales involved, we expect the conclusions derived in the two-body sector to apply in the three-body sector, in agreement with our original assumption.

Finally, though the isoscalar $\frac{5}{2}^+$ trimer depends only on the interaction in the P_c^* channel, it actually contains an additional source of uncertainty. The Efimov effect [32] can happen in this trimer, as shown by the analysis of the $\bar{D}^*\bar{D}^*\Sigma_c$ system in the unitary limit; see Sec. III. The $\bar{D}^*\Sigma_c$ two-body system in the P_c^* channel is actually far from the unitary limit, but the analysis is still relevant because of the relation between the Efimov effect and Thomas collapse [33]. The idea is that, even though a molecular P_c^* is not in the unitary limit, for $\Lambda \rightarrow \infty$ the isoscalar $\frac{5}{2}^+$ trimer will collapse; i.e., its binding energy will diverge ($B_3 \rightarrow \infty$). This collapse can be prevented with the inclusion of a three-body force, without which the trimer binding energy predictions will not be formally cutoff independent [34,35]. In practice, owing to the large discrete scaling

factor of $e^{\pi/s_0} \simeq 5711$, the divergence of the trimer binding requires fantastically large cutoffs to be noticed. For instance, the cutoff required for the first nonphysical isovector $J^P = \frac{5}{2}^+$ trimer to appear is $\Lambda \sim 62$ GeV, which is pretty large. Thus it is not surprising that the cutoff uncertainty for this trimer is not particularly big, about 2 MeV in the $\Lambda = 0.5\text{--}1.0$ GeV cutoff window. Alternatively we can check the model dependence of the $J^P = \frac{5}{2}^+$ trimer prediction by using a different regulator, for instance, a delta shell in coordinate space,

$$V(r; \bar{D}^*\Sigma_c) = C(R_c) \frac{\delta(r - R_c)}{4\pi R_c^2}, \quad (30)$$

where R_c is a cutoff radius that we take of the order of the typical hadronic size: $R_c = 0.5\text{--}1.0$ fm. The delta-shell potential is actually a separable interaction, where its momentum space form is

$$V(\bar{D}^*\Sigma_c) = C(R_c) \frac{\sin(kR_c)}{kR_c} \frac{\sin(k'R_c)}{k'R_c}. \quad (31)$$

With this regulator the prediction for the location of the $\frac{5}{2}^+$ trimer binding is $B_3 = 14 \pm 3$ MeV (17^{+3}_{-4} MeV) for $R_c = 1.0$ fm (0.5 fm), i.e., almost identical to the Gaussian regulator predictions. Despite the previous checks caution is advised because the formal requirement of a three-body force, even if the related divergence happens at really large cutoffs, indicates the existence of systematic uncertainties that are not being taken into account.

V. CONCLUSIONS

The hypothesis that the P_c^* is a $I = \frac{1}{2}, J^P = \frac{3}{2}^- \bar{D}^*\Sigma_c$ molecule implies the existence of a few $\bar{D}^*\bar{D}^*\Sigma_c$ trimers. Calculations in a contact-range theory indicate that the most bound of these trimers has the quantum numbers $I = 0, J^P = \frac{5}{2}^+$ and a binding energy $B_3 \sim 14\text{--}16$ MeV. There are other three or four trimer configurations that are likely to bind, but they are expected to be considerably less bound: two states with $B_3 \sim 3\text{--}5$ MeV with quantum numbers $I = 0, J^P = \frac{1}{2}^+$ and $I = 1, J^P = \frac{5}{2}^+$, a state with $B_3 \sim 1\text{--}3$ MeV and quantum numbers $I = 1, J^P = \frac{1}{2}^+$ and maybe a state on the verge of binding with $I = 1, J^P = \frac{3}{2}^+$; see Table II for details. These predictions are affected by a

series of uncertainties, which mostly stem from the fact that we do not know too much about the $\bar{D}^*\Sigma_c$ interaction except in the P_c^* channel (and even this is dependent on the nature of the P_c^*). These uncertainties are taken into account on the basis of considering different hypotheses about the $\bar{D}^*\bar{D}^*$ and $\bar{D}^*\Sigma_c$ interactions, which are summarized in Table III. From the previous considerations we arrive at the conclusion that the most solid prediction is that of the isoscalar $J^P = \frac{5}{2}^+$ trimer.

We have also considered the $\bar{D}^*\bar{D}^*\Sigma_c$ system in the unitary limit, i.e., when the P_c^* is located at the $\bar{D}^*\Sigma_c$ threshold. In this situation the $J^P = \frac{5}{2}^+$ trimer will be Efimov-like and will have a geometric spectrum with a scaling factor of 3.3×10^7 for the binding energies. This scaling factor is fantastically large, which implies that there would be no practical way to observe it even if we could tune the $\bar{D}^*\Sigma_c$ scattering length. Yet the possibility of the Efimov effect in the isoscalar $J^P = \frac{5}{2}^+$ trimer implies that a three-body force should be included in the calculations at leading order. The practical importance of this three-body force might be tangential, as reflected by the mild cutoff dependence of the isoscalar $J^P = \frac{5}{2}^+$ trimer binding in the cutoff window chosen in this work, $\Lambda = 0.5\text{--}1.0$ GeV.

Without knowing the nature of the P_c^* , the predictions of this work will remain theoretical: if the P_c^* is not molecular but a compact pentaquark instead the $\bar{D}^*\bar{D}^*\Sigma_c$ trimers are not expected to bind. The experimental production of these trimers is expected to be difficult, with the lattice probably providing a more convenient way to investigate them. Finally we notice that the existence of $\Sigma_c\Sigma_c\bar{D}^*$ bound trimers is also likely, but their location will be subjected to even larger uncertainties as a consequence of the $\Sigma_c\Sigma_c$ interaction, which is not particularly well known but probably strong.

ACKNOWLEDGMENTS

I thank Li-Sheng Geng, Kanchan P. Khemchandani, and Alberto Martinez Torres for discussions and a critical and careful reading of this manuscript. This work is partly supported by the National Natural Science Foundation of China under Grants No. 11522539 and No. 11735003, the Fundamental Research Funds for the Central Universities, and the Thousand Talents Plan for Young Professionals.

-
- [1] R. Aaij *et al.* (LHCb Collaboration), *Phys. Rev. Lett.* **115**, 072001 (2015).
[2] M. Voloshin and L. Okun, *JETP Lett.* **23**, 333 (1976).
[3] A. De Rujula, H. Georgi, and S. Glashow, *Phys. Rev. Lett.* **38**, 317 (1977).

- [4] L. Roca, J. Nieves, and E. Oset, *Phys. Rev. D* **92**, 094003 (2015).
[5] J. He, *Phys. Lett. B* **753**, 547 (2016).
[6] C. W. Xiao and U. G. Meiner, *Phys. Rev. D* **92**, 114002 (2015).

- [7] R. Chen, X. Liu, X.-Q. Li, and S.-L. Zhu, *Phys. Rev. Lett.* **115**, 132002 (2015).
- [8] H.-X. Chen, W. Chen, X. Liu, T. G. Steele, and S.-L. Zhu, *Phys. Rev. Lett.* **115**, 172001 (2015).
- [9] T. J. Burns, *Eur. Phys. J. A* **51**, 152 (2015).
- [10] L. Geng, J. Lu, and M. P. Valderrama, *Phys. Rev. D* **97**, 094036 (2018).
- [11] U.-G. Meiner and J. A. Oller, *Phys. Lett. B* **751**, 59 (2015).
- [12] D. Diakonov, V. Petrov, and M. V. Polyakov, *Z. Phys. A* **359**, 305 (1997).
- [13] R. L. Jaffe and F. Wilczek, *Phys. Rev. Lett.* **91**, 232003 (2003).
- [14] S. G. Yuan, K. W. Wei, J. He, H. S. Xu, and B. S. Zou, *Eur. Phys. J. A* **48**, 61 (2012).
- [15] L. Maiani, A. D. Polosa, and V. Riquer, *Phys. Lett. B* **749**, 289 (2015).
- [16] R. F. Lebed, *Phys. Lett. B* **749**, 454 (2015).
- [17] G.-N. Li, X.-G. He, and M. He, *J. High Energy Phys.* **12** (2015) 128.
- [18] Z.-G. Wang, *Eur. Phys. J. C* **76**, 70 (2016).
- [19] F.-K. Guo, U.-G. Meiner, W. Wang, and Z. Yang, *Phys. Rev. D* **92**, 071502 (2015).
- [20] X.-H. Liu, Q. Wang, and Q. Zhao, *Phys. Lett. B* **757**, 231 (2016).
- [21] M. Bayar, F. Aceti, F.-K. Guo, and E. Oset, *Phys. Rev. D* **94**, 074039 (2016).
- [22] V. Kubarovsky and M. B. Voloshin, *Phys. Rev. D* **92**, 031502 (2015).
- [23] A. Mironov and A. Morozov, *Pis'ma Zh. Eksp. Teor. Fiz.* **102**, 302 (2015) [*JETP Lett.* **102**, 271 (2015)].
- [24] N. N. Scoccola, D. O. Riska, and M. Rho, *Phys. Rev. D* **92**, 051501 (2015).
- [25] M.-Z. Liu, F.-Z. Peng, M. Sanchez Sanchez, and M. P. Valderrama, *Phys. Rev. D* **98**, 114030 (2018).
- [26] X.-L. Ren, B. B. Malabarba, L.-S. Geng, K. P. Khemchandani, and A. Martinez Torres, *Phys. Lett. B* **785**, 112 (2018).
- [27] M. P. Valderrama, *Phys. Rev. D* **98**, 034017 (2018).
- [28] M. Bayar, X.-L. Ren, and E. Oset, *Eur. Phys. J. A* **51**, 61 (2015).
- [29] M. Bayar, P. Fernandez-Soler, Z.-F. Sun, and E. Oset, *Eur. Phys. J. A* **52**, 106 (2016).
- [30] J. M. Dias, V. R. Debastiani, L. Roca, S. Sakai, and E. Oset, *Phys. Rev. D* **96**, 094007 (2017).
- [31] J. M. Dias, L. Roca, and S. Sakai, *Phys. Rev. D* **97**, 056019 (2018).
- [32] V. Efimov, *Phys. Lett.* **33B**, 563 (1970).
- [33] L. H. Thomas, *Phys. Rev.* **47**, 903 (1935).
- [34] P. F. Bedaque, H. W. Hammer, and U. van Kolck, *Phys. Rev. Lett.* **82**, 463 (1999).
- [35] P. F. Bedaque, H. W. Hammer, and U. van Kolck, *Nucl. Phys. A* **646**, 444 (1999).
- [36] K. Helfrich, H. W. Hammer, and D. S. Petrov, *Phys. Rev. A* **81**, 042715 (2010).
- [37] K. Helfrich and H. W. Hammer, *J. Phys. B* **44**, 215301 (2011).
- [38] H.-W. Hammer and L. Platter, *Annu. Rev. Nucl. Part. Sci.* **60**, 207 (2010).
- [39] P. Naidon and S. Endo, *Rep. Prog. Phys.* **80**, 056001 (2017).
- [40] N. A. Tornqvist, *Z. Phys. C* **61**, 525 (1994).
- [41] J. Carlson, L. Heller, and J. A. Tjon, *Phys. Rev. D* **37**, 744 (1988).
- [42] B. A. Gelman and S. Nussinov, *Phys. Lett. B* **551**, 296 (2003).
- [43] J. Vijande, A. Valcarce, and N. Barnea, *Phys. Rev. D* **79**, 074010 (2009).
- [44] P. Junnarkar, N. Mathur, and M. Padmanath, *arXiv*: 1810.12285.
- [45] M. Karliner and J. L. Rosner, *Phys. Rev. Lett.* **119**, 202001 (2017).
- [46] E. J. Eichten and C. Quigg, *Phys. Rev. Lett.* **119**, 202002 (2017).
- [47] T. Mehen, *Phys. Rev. D* **96**, 094028 (2017).
- [48] J.-J. Wu, R. Molina, E. Oset, and B. S. Zou, *Phys. Rev. Lett.* **105**, 232001 (2010).
- [49] M. P. Valderrama, *Phys. Rev. D* **85**, 114037 (2012).
- [50] J.-X. Lu, L.-S. Geng, and M. P. Valderrama, *arXiv*: 1706.02588.
- [51] H.-Y. Cheng and C.-K. Chua, *Phys. Rev. D* **92**, 074014 (2015).