WW fusion and Higgsstrahlung interplay in Higgs precision tests with the $e^-e^+ \rightarrow \nu\bar{\nu}h$ process

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The Higgs precision program at future lepton colliders aims at (sub) percent level precision measurement of the Higgs properties, shedding light on new physics through the Higgs lampost. Among the many exclusive Higgs channels that can be measured precisely, the WW fusion to Higgs with subsequent decays into the $b\bar{b}$ final state is of particular importance. This channel provides leading constraints on the Higgs total width in the κ framework and greatly improves the constraints in the EFT framework in conjunction with the Higgsstrahlung process. We argue in this paper that there are two effects involving the interplay between the WW -fusion and Higgsstrahlung processes which affect the physical information one can extract from the precision measurements. One is the interference between the two amplitudes that amounts to a -10% correction to the WW -fusion signal strength and results in a $4\text{--}5\sigma$ discrepancy between theory and measurement if not taken into account. The other takes place in the global fitting procedure where the ZH process is the dominant background with a cross section around 7 times larger than the WW -fusion signal. Although the ZH process can be measured to great precision at future lepton colliders, failing to take this interplay into consideration in the coupling extraction will result in constraints on κ_W , the Higgs coupling to W -boson pairs, that are 100% too aggressive.

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I. INTRODUCTION

In the summer of 2012, LHC established the existence of the Higgs boson, which completed the standard model (SM) [1,2]. However, instead of being an end, this discovery marks the start of a new journey. In spite of the great triumph of the SM, it is not a final theory of particle physics. For example, there is compelling evidence, both astronomically and cosmologically, for the existence of dark matter, while there is no particle candidate for dark matter in the SM. Moreover, the discovery of the Higgs boson has also left us with many open questions. As a fundamental scalar particle, the Higgs mass is quadratically sensitive to the new physics scale, which implies this new scale should not be too far above the electroweak (EW) scale. To extend the particle physics theory and settle these problems, different kinds of models have been developed, such as supersymmetry (SUSY) and the composite Higgs model. Since Higgs physics plays an essential role in

constraining and evaluating these models, a detailed understanding of the Higgs sector is obviously of great significance to the exploration of new physics. Therefore, a precision measurement program of Higgs physics will be the major focus of high-energy physics in the coming decades.

Due to the contributions of the LHC experiments, some of the Higgs properties, such as the Higgs couplings to W/Z bosons and gluons, have already been measured with a precision level at about $\mathcal{O}(10\%)$ [3–6]. This will be further improved with more data from LHC Run 2 and in the future luminosity upgrade program. However, because of the composite nature of the proton, the LHC can only provide a fuzzy picture of the Higgs. To achieve subpercent-level precision, an electron-positron collider will be required to operate as a Higgs factory, such as at the planned CEPC [7,8], FCC-ee [9], and ILC [10]. Running at a center-of-mass (c.m.) energy of about 250 GeV, such a Higgs factory will reach its peak value of the Higgs production cross section through the Higgsstrahlung process (or ZH process) $e^+e^- \rightarrow ZH$. Since the c.m. energy at an e^+e^- collider is precisely measurable, the Higgs boson mass and total Higgsstrahlung cross section can be reconstructed through the recoil mass method [8,11], and Higgs candidate events can be identified. In the selected $e^+e^- \rightarrow ZH$ events, branching ratios of different Higgs decay

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modes can then be measured by separating different decay final states.

The total Higgs boson decay width is not that straightforward to measure. The total decay width of a 125 GeV SM Higgs boson Γ_H is 4 MeV [12,13], which is too small to be measured directly. However, it can be indirectly derived from the total cross section and the branching ratios for some of its decay channels. One proper way is through the Higgs decaying to ZZ^* [8]:

$$\Gamma_H = \frac{\Gamma(H \rightarrow ZZ^*)}{\text{BR}(H \rightarrow ZZ^*)} \propto \frac{\sigma(ZH)}{\text{BR}(H \rightarrow ZZ^*)}. \quad (1)$$

Here, the partial decay width $\Gamma(H \rightarrow ZZ^*)$ depends on the Higgs-Z boson coupling $g(HZZ)$ which is then substituted equivalently by the cross section of the Higgsstrahlung process $\sigma(ZH)$. In this case, the performance of the $H \rightarrow ZZ^*$ measurement will determine the precision of Γ_H .

Alternatively, we can turn to the decay of the Higgs to a pair of b -quarks $H \rightarrow b\bar{b}$:

$$\Gamma_H = \frac{\Gamma(H \rightarrow b\bar{b})}{\text{BR}(H \rightarrow b\bar{b})}. \quad (2)$$

The partial width $\Gamma(H \rightarrow b\bar{b})$ can be obtained from the WW -fusion process with the Higgs decaying to a pair of b -jets $e^+e^- \rightarrow \nu\bar{\nu}H \rightarrow \nu\bar{\nu}b\bar{b}$:

$$\sigma(\nu\bar{\nu}H \rightarrow \nu\bar{\nu}b\bar{b}) \propto \Gamma(H \rightarrow WW^*) \times \text{BR}(H \rightarrow b\bar{b}) \quad (3)$$

$$\propto \text{BR}(H \rightarrow WW^*) \times \Gamma(H \rightarrow b\bar{b}). \quad (4)$$

Replacing $\Gamma(H \rightarrow b\bar{b})$ in Eq. (2) with the two measured quantities $\sigma(\nu\bar{\nu}H \rightarrow \nu\bar{\nu}b\bar{b})$ and $\text{BR}(H \rightarrow WW^*)$, the total width Γ_H is written as

$$\Gamma_H \propto \frac{\sigma(\nu\bar{\nu}H \rightarrow \nu\bar{\nu}b\bar{b})}{\text{BR}(H \rightarrow b\bar{b}) \times \text{BR}(H \rightarrow WW^*)}. \quad (5)$$

Here, the branching ratios $\text{BR}(H \rightarrow b\bar{b})$ and $\text{BR}(H \rightarrow WW^*)$ are extracted from the inclusive cross section $\sigma(ZH)$ and the cross sections of individual Higgs decay mode $\sigma(ZH) \times \text{BR}$, as mentioned before. The precision of the total width achieved with this method is limited by the small cross section $\sigma(\nu\bar{\nu}H \rightarrow \nu\bar{\nu}b\bar{b})$, which is the main concern of this paper. At an e^+e^- collider with c.m. energy about 250 GeV, the WW -fusion process is the second leading production process for a 125 GeV SM Higgs boson, after the Higgsstrahlung process [14–20]. And with $Z \rightarrow \nu\bar{\nu}$, the ZH channel interferes with the WW -fusion channel, contributing an irreducible background to the $\nu\bar{\nu}b\bar{b}$ final state. We first calculate the cross sections of

the two channels and their interference effect both analytically and numerically in Sec. II. Then, some important differential distributions of both the signal and the background channels are presented in Sec. III. Finally, we analyze the impact of this interference effect on the Higgs physics measurement at e^+e^- colliders.

II. THE INTERFERENCE EFFECTS

A. Analytic understanding

Figure 1 shows the Feynman diagram for the Higgsstrahlung process, which is one with s -channel Z boson exchange. Since left-handed and right-handed electrons have different gauge couplings, evaluating the diagram gives amplitudes, respectively, for $e_L^-e_R^+$ and $e_R^-e_L^+$ initial states:

$$i\mathcal{M}(e_L^-e_R^+) = \bar{v}_L(p_2) \frac{ie\gamma_\mu}{c_w s_w} \left(-\frac{1}{2} + s_w^2 \right) u_L(p_1) \times \frac{-ig^{\mu\nu}}{q^2 - m_z^2} \times 2i \frac{m_z^2}{v} g_{\nu\rho} \times \epsilon^{*\rho}(k) \quad (6)$$

$$= \frac{ie}{c_w s_w} \left(-\frac{1}{2} + s_w^2 \right) \frac{1}{q^2 - m_z^2} \frac{2m_z^2}{v} \times \bar{v}_L(p_2) \gamma^\mu u_L(p_1) \epsilon_\mu^*(k) \quad (7)$$

$$i\mathcal{M}(e_R^-e_L^+) = \bar{v}_R(p_2) \frac{ie\gamma_\mu}{c_w s_w} s_w^2 u_R(p_1) \times \frac{-ig^{\mu\nu}}{q^2 - m_z^2} \times 2i \frac{m_z^2}{v} g_{\nu\rho} \times \epsilon^{*\rho}(k) \quad (8)$$

$$= \frac{ie}{c_w s_w} s_w^2 \frac{1}{q^2 - m_z^2} \frac{2m_z^2}{v} \times \bar{v}_R(p_2) \gamma^\mu u_R(p_1) \epsilon_\mu^*(k), \quad (9)$$

where c_w and s_w are, respectively, $c_w \equiv \cos \theta_w$ and $s_w \equiv \sin \theta_w$. The squared amplitudes are

FIG. 1. Feynman diagram contributing to the amplitude of $e^+e^- \rightarrow ZH$ process.

$$|\mathcal{M}(e_L^- e_R^+)|^2 = \frac{e^2}{c_w^2 s_w^2} \left(-\frac{1}{2} + s_w^2 \right)^2 \left| \frac{1}{q^2 - m_z^2} \right|^2 \frac{4m_z^4}{v^2} \epsilon_\mu^* \epsilon_\nu \times \bar{v}_L(p_2) \gamma^\mu u_L(p_1) \bar{u}_L(p_1) \gamma^\nu v_L(p_2) \quad (10)$$

$$= \frac{e^2}{c_w^2 s_w^2} \left(-\frac{1}{2} + s_w^2 \right)^2 \left| \frac{1}{q^2 - m_z^2} \right|^2 \frac{4m_z^4}{v^2} \epsilon_\mu^* \epsilon_\nu \times 2(p_2^\mu p_1^\nu + p_2^\nu p_1^\mu - p_1 \cdot p_2 g^{\mu\nu} + i\epsilon^{\alpha\mu\beta\nu} p_{2\alpha} p_{1\beta}) \quad (11)$$

$$|\mathcal{M}(e_R^- e_L^+)|^2 = \frac{e^2}{c_w^2 s_w^2} s_w^4 \left| \frac{1}{q^2 - m_z^2} \right|^2 \frac{4m_z^4}{v^2} \epsilon_\mu^* \epsilon_\nu \times 2(p_2^\mu p_1^\nu + p_2^\nu p_1^\mu - p_1 \cdot p_2 g^{\mu\nu} - i\epsilon^{\alpha\mu\beta\nu} p_{2\alpha} p_{1\beta}). \quad (12)$$

Summing over final state polarization and averaging over initial state polarization, the unpolarized squared amplitude is

$$\frac{1}{4} \sum |\mathcal{M}|^2 = \frac{1}{4} \frac{e^2}{c_w^2 s_w^2} \left(-\frac{1}{2} + s_w^2 \right)^2 \left| \frac{1}{q^2 - m_z^2} \right|^2 \frac{4m_z^4}{v^2} \times \left(-g^{\mu\nu} + \frac{k^\mu k^\nu}{m_z^2} \right) 2(p_2^\mu p_1^\nu + p_2^\nu p_1^\mu - p_1 \cdot p_2 g^{\mu\nu} + i\epsilon^{\alpha\mu\beta\nu} p_{2\alpha} p_{1\beta}) \quad (13)$$

$$+ \frac{1}{4} \frac{e^2}{c_w^2 s_w^2} s_w^4 \left| \frac{1}{q^2 - m_z^2} \right|^2 \frac{4m_z^4}{v^2} \left(-g^{\mu\nu} + \frac{k^\mu k^\nu}{m_z^2} \right) \times 2(p_2^\mu p_1^\nu + p_2^\nu p_1^\mu - p_1 \cdot p_2 g^{\mu\nu} - i\epsilon^{\alpha\mu\beta\nu} p_{2\alpha} p_{1\beta}) \quad (14)$$

$$= \frac{1}{4} \left(2s_w^4 - s_w^2 + \frac{1}{4} \right) \frac{e^2}{c_w^2 s_w^2} \left| \frac{1}{q^2 - m_z^2} \right|^2 \frac{4m_z^4}{v^2} \times \left[2p_1 \cdot p_2 + \frac{4}{m_z^2} (k \cdot p_1)(k \cdot p_2) \right] \quad (15)$$

$$= \frac{1}{4} \left(2s_w^4 - s_w^2 + \frac{1}{4} \right) \frac{e^2}{c_w^2 s_w^2} \left| \frac{1}{s - m_z^2} \right|^2 \frac{4m_z^4 s}{v^2} \times \left[2 + \frac{|\vec{k}|^2}{m_z^2} (1 - \cos^2 \theta) \right], \quad (16)$$

where $|\vec{k}| = \frac{1}{2\sqrt{s}} \sqrt{s^2 - 2(m_1^2 + m_2^2)s + (m_1^2 - m_2^2)^2}$ is the final state momentum in the c.m. frame, θ is the polar angel of the final state, and $\sqrt{s}$ is the c.m. energy. Then the cross section is

$$\frac{d\sigma}{d\Omega} = \frac{1}{2s} \frac{|\vec{k}|}{16\pi^2 \sqrt{s}} \times \frac{1}{4} \sum |\mathcal{M}|^2 \quad (17)$$

$$\sigma = \frac{|\vec{k}|}{8\pi\sqrt{s}} \left(2s_w^4 - s_w^2 + \frac{1}{4} \right) \frac{\alpha}{c_w^2 s_w^2} \left| \frac{1}{s - m_z^2} \right|^2 \frac{m_z^4}{v^2} \times \int d\Omega \left(2 + \frac{|\vec{k}|^2}{m_z^2} - \frac{|\vec{k}|^2}{m_z^2} \cos^2 \theta \right) \quad (18)$$

$$= \frac{|\vec{k}|}{8\pi\sqrt{s}} \left(2s_w^4 - s_w^2 + \frac{1}{4} \right) \frac{\alpha}{c_w^2 s_w^2} \left| \frac{1}{s - m_z^2} \right|^2 \frac{m_z^4}{v^2} \times \left(8\pi + \frac{8\pi |\vec{k}|^2}{3 m_z^2} \right) \quad (19)$$

$$= \frac{\pi\alpha^2}{24} \left(\frac{2|\vec{k}|}{\sqrt{s}} \right) \frac{|\vec{k}|^2 + 3m_z^2}{(s - m_z^2)^2} \frac{1 - 4s_w^2 + 8s_w^4}{s_w^4(1 - s_w^2)^2}, \quad (20)$$

where α is the fine structure constant [21–24]. At $\sqrt{s} = 250$ GeV, the tree-level cross section of the Higgsstrahlung process is 241.556 fb. If we take the branching ratio of the Z boson invisible decay to be 20%, then $\sigma(ZH, Z \rightarrow \nu\bar{\nu}) = 241.556 \text{ fb} \times 20\% = 48.311 \text{ fb}$.

The Feynman diagram for the WW-fusion process is shown in Fig. 2. There is only one possible polarization configuration for the initial states of this process, $e_L^- e_R^+$. The polarization amplitude is

$$i\mathcal{M}(e_L^- e_R^+) = \bar{u}_L(k_1) \frac{ig}{\sqrt{2}} \gamma_\mu u_L(p_1) \times \frac{-ig^{\mu\nu}}{q_1^2 - m_w^2} \times 2i \frac{m_w^2}{v} g_{\nu\rho} \times \frac{-ig^{\rho\lambda}}{q_2^2 - m_w^2} \bar{v}_L(p_2) \frac{ig}{\sqrt{2}} \gamma_\lambda v_L(k_2) \quad (21)$$

$$= \frac{i4\pi\alpha m_w^2}{s_w^2 v} \frac{1}{q_1^2 - m_w^2} \frac{1}{q_2^2 - m_w^2} \times \bar{u}_L(k_1) \gamma^\mu u_L(p_1) \bar{v}_L(p_2) \gamma_\mu v_L(k_2). \quad (22)$$

The squared amplitude for the unpolarized beam is

FIG. 2. Feynman diagram contributing to the amplitude of WW-fusion process.

FIG. 3. Feynman diagram contributing to the amplitude of $e^+e^- \rightarrow ZH \rightarrow \nu\bar{\nu}H$ process.

$$\begin{aligned} \frac{1}{4} \sum |\mathcal{M}|^2 &= \frac{1}{4} \frac{16\pi^2 \alpha^2 m_w^4}{s_w^4 v^2} \left| \frac{1}{q_1^2 - m_w^2} \right|^2 \left| \frac{1}{q_2^2 - m_w^2} \right|^2 \\ &\times \bar{u}_L(k_1) \gamma^\mu u_L(p_1) \bar{u}_L(p_1) \gamma^\nu u_L(k_1) \\ &\times \bar{v}_L(p_2) \gamma_\mu v_L(k_2) \bar{v}_L(k_2) \gamma_\nu v_L(p_2) \end{aligned} \quad (23)$$

$$\begin{aligned} &= \frac{1}{4} \frac{16\pi^2 \alpha^2 m_w^4}{s_w^4 v^2} \left| \frac{1}{2p_1 k_1 + m_w^2} \right|^2 \left| \frac{1}{2p_2 k_2 + m_w^2} \right|^2 \\ &\times 16(p_1 k_2)(p_2 k_1) \end{aligned} \quad (24)$$

$$\begin{aligned} &= \frac{(4\pi\alpha)^3 m_w^2}{s_w^6} \left| \frac{1}{2p_1 k_1 + m_w^2} \right|^2 \left| \frac{1}{2p_2 k_2 + m_w^2} \right|^2 \\ &\times (p_1 k_2)(p_2 k_1). \end{aligned} \quad (25)$$

It is not straightforward to quote analytic results for the total cross section [17,18]. However, integrating over the three-body phase space numerically [25], we can get the result, which is $\sigma(WW \text{ fusion}) = 7.950 \text{ fb}$, at $\sqrt{s} = 250 \text{ GeV}$.

Figure 3 gives the Feynman diagram of another contribution to the $e^-e^+ \rightarrow \nu\bar{\nu}H$ process, which is $e^-e^+ \rightarrow ZH \rightarrow \nu\bar{\nu}H$. With the Z boson off-shell and $e_L^- e_R^+$ as initial states, this diagram will interfere with the WW -fusion term. The amplitude for the ZH channel is

$$\begin{aligned} i\mathcal{M}(e_L^- e_R^+) &= \bar{v}_L(p_2) \frac{ie\gamma_\mu}{c_w s_w} \left(-\frac{1}{2} + s_w^2 \right) u_L(p_1) \\ &\times \frac{-ig^{\mu\nu}}{q_1^2 - m_z^2} \times 2i \frac{m_z^2}{v} g_{\nu\rho} \times \frac{-ig^{\rho\lambda}}{q_2^2 - m_z^2 + im_z \Gamma_Z} \\ &\times \bar{u}_L(k_1) \frac{ie\gamma_\lambda}{2c_w s_w} v_L(k_2) \end{aligned} \quad (26)$$

$$\begin{aligned} &= \frac{i4\pi\alpha}{c_w^2 s_w^2} \left(-\frac{1}{2} + s_w^2 \right) \frac{m_z^2}{v} \frac{1}{q_1^2 - m_z^2} \\ &\times \frac{1}{q_2^2 - m_z^2 + im_z \Gamma_Z} \\ &\times \bar{v}_L(p_2) \gamma^\mu u_L(p_1) \bar{u}_L(k_1) \gamma_\mu v_L(k_2). \end{aligned} \quad (27)$$

Combining this amplitude and the WW -fusion amplitude in Eq. (22), we get the interference effect for unpolarized initial states:

$$\frac{1}{4} \sum |\mathcal{M}|_{\text{intf}}^2 = -\frac{1}{4} (\mathcal{M}_1^* \mathcal{M}_2 + \mathcal{M}_2^* \mathcal{M}_1) \quad (28)$$

$$\begin{aligned} &= -\frac{1}{4} \frac{16\pi^2 \alpha^2 m_w^2}{c_w^2 s_w^4 v} \left(-\frac{1}{2} + s_w^2 \right) \frac{m_z^2}{v} \frac{1}{s - m_z^2} \\ &\times \frac{1}{2p_1 k_1 + m_w^2} \frac{1}{2p_2 k_2 + m_w^2} \\ &\times \frac{4k_1 k_2 - 2m_z^2}{(2k_1 k_2 - m_z^2)^2 + m_z^2 \Gamma_Z^2} \\ &\times (-16)(p_1 k_2)(p_2 k_1) \end{aligned} \quad (29)$$

$$\begin{aligned} &= -\frac{(4\pi\alpha)^3 m_w^2}{2s_w^6 (1 - s_w^2)^2} \left(\frac{1 - 2s_w^2}{s - m_z^2} \right) \\ &\times \frac{1}{2p_1 k_1 + m_w^2} \frac{1}{2p_2 k_2 + m_w^2} \\ &\times \frac{4k_1 k_2 - 2m_z^2}{(2k_1 k_2 - m_z^2)^2 + m_z^2 \Gamma_Z^2} (p_1 k_2)(p_2 k_1). \end{aligned} \quad (30)$$

Integrating numerically, the contribution from the interference term at $\sqrt{s} = 250 \text{ GeV}$ is $\sigma_{\text{intf}} = -0.327 \text{ fb}$. Then the interference effect at $\sqrt{s} = 250 \text{ GeV}$ derived from these data is

$$\sigma_{\text{intf}} / \sigma(WW \text{ fusion}) = \frac{-0.327 \text{ fb}}{7.950 \text{ fb}} \approx -4.1\%. \quad (31)$$

The numerical values of the parameters used to compute those results are summarized below.

m_Z [GeV]	Γ_Z [GeV]	G_F [GeV ⁻²]	$\alpha_{em}(m_Z^2)^{-1}$	$\sin^2 \theta_W$	m_h [GeV]
91.1876 ± 0.0021	2.441404	$1.1663787(6) \times 10^{-5}$	127.940 ± 0.014	$0.23124(6)$	125.09 ± 0.24

B. Phenomenological analysis

Following the calculation above, we further perform a numerical study of this interference effect. For Higgs boson decay to $b\bar{b}$, the signal event is two b -tagged jets plus missing energy $\nu\bar{\nu}$. Apart from the irreducible background $ZH \rightarrow \nu\bar{\nu}b\bar{b}$, the main non-Higgs boson background is the SM ZZ production. Since our focus is on the impact of the interference term, and the ZZ background can be minimized by a selection cut on the invariant mass of the two b jets [7,8], it is neglected in the following analysis. We generate the signal and ZH background events for an $\sqrt{s} = 250$ GeV e^-e^+ collider with CalcHEP at the parton level [26] and impose the detector acceptance and resolution with our own code. Since the proposed future lepton colliders, in general, have similar detector performance, here we take the performance of the CEPC detector as the benchmark for our numerical analysis [8].

The b -tagging efficiency is conservatively chosen to be 80%. To achieve this identification efficiency, b jets are required to have $|\cos \theta_j| < 0.98$ and $E_j > 10$ GeV. Since these two b jets are Higgs decay products, the invariant mass is required to satisfy $|m_{jj} - m_h| < 5$ GeV. We mimic the detector resolution effect by adding Gaussian smearing effects on the four-momentum of final state particles. The energy resolution of jets is affected by the hadron calorimeter and performs approximately as

$$\frac{\delta E}{E} = \frac{0.3}{\sqrt{E/\text{GeV}}} \oplus 0.02. \quad (32)$$

Cross sections of the involved different channels before and after imposing the above detector simulation are

Process	No cuts (fb)	After cuts (fb)
$e^+e^- \rightarrow \nu\bar{\nu}H \rightarrow \nu\bar{\nu}b\bar{b}$	32.69	11.07
$e^+e^- \rightarrow ZH \rightarrow \nu\bar{\nu}b\bar{b}$	28.26	9.716
WW fusion, $H \rightarrow b\bar{b}$	4.618	1.448

Here, the branching ratio of Higgs decaying to $b\bar{b}$ is $\text{BR}(H \rightarrow b\bar{b}) = 58.09\%$ [6].

The interference effect corresponding to results before cuts is

$$\begin{aligned} \sigma_{\text{intf}}/\sigma_{WW} &= \frac{\sigma_{\text{tot}} - \sigma_{ZH} - \sigma_{WW}}{\sigma_{WW}} \\ &= \frac{(32.69 - 28.26 - 4.618) \text{ fb}}{4.618 \text{ fb}} \\ &= \frac{-0.188 \text{ fb}}{4.618 \text{ fb}} \approx -4.1\%, \end{aligned} \quad (33)$$

where σ_{tot} is the total cross section for the process $e^+e^- \rightarrow \nu\bar{\nu}H \rightarrow \nu\bar{\nu}b\bar{b}$, and σ_{ZH} , σ_{WW} , and σ_{intf} are, respectively, the three components: Higgsstrahlung, WW fusion, and the interference term. The interference effect corresponding to cross sections after cuts is

$$\begin{aligned} \sigma_{\text{intf}}/\sigma_{WW} &= \frac{\sigma_{\text{tot}} - \sigma_{ZH} - \sigma_{WW}}{\sigma_{WW}} \\ &= \frac{(11.07 - 9.716 - 1.448) \text{ fb}}{1.448 \text{ fb}} \\ &= \frac{-0.094 \text{ fb}}{1.448 \text{ fb}} \approx -6.5\%. \end{aligned} \quad (34)$$

Detector resolution effects have increased the significance of the interference term.

FIG. 4. (a) Angular distributions in $\cos \theta_H$ of WW -fusion term (red line) and the interference term (blue line); (b) the ratio between the angular distributions of the interference term and that of WW -fusion term $d\sigma_{\text{intf}}/d\sigma_{WW}$ for different $\cos \theta_H$.

FIG. 5. (a) Missing mass spectrums of WW -fusion term (red line) and the interference term (blue line); (b) the ratio between the missing mass distributions of the interference term and that of WW -fusion term $d\sigma_{\text{intf}}/d\sigma_{\text{WW}}$ for different M_{miss} .

III. DIFFERENTIAL DISTRIBUTIONS

In this section, we display some differential distributions of both the WW -fusion and the interference terms. Figures 4 and 5 are, respectively, the angular distributions and missing mass distributions for the WW -fusion process and the interference term at a c.m. energy $\sqrt{s} = 250$ GeV. Figure 6 shows missing mass distributions at a different c.m. energy $\sqrt{s} = 240$ GeV. The angular distribution of the interference term is uniformly negative and almost isotropic compared to that of the WW -fusion process, which peaks at $\theta_h \rightarrow 0$ and π . The missing mass distributions of the WW -fusion and the interference term are of comparable size as shown in Figs. 5 and 6. The differential cross section of the WW -fusion process is more pronounced with higher missing masses, while the interference term has two extreme values near the Z pole. However, in conformity to the analytical results of scattering amplitudes in the last section, the interference term flips sign at the Z pole. The two sides nearly cancel out and leave a net negative effect of a few percent (-4.1%) for the WW -fusion process at $\sqrt{s} = 250$ GeV, while at $\sqrt{s} = 240$ GeV, the net effect of the interference term is positive.

IV. IMPACT ON THE HIGGS PHYSICS

We show the impact of the interplay between the WW -fusion and the ZH channels in Fig. 7 in the κ framework. First, in many lepton collider studies [7,27–29], the $\nu\bar{\nu}b\bar{b}$ final state is treated as a pure signal from WW fusion with a SM background dominated by $ZH \rightarrow \nu\bar{\nu}b\bar{b}$ and no theoretical uncertainty. However, as κ_W and κ_Z are expected to be measured with comparable precision, $ZH \rightarrow \nu\bar{\nu}b\bar{b}$ should also be varied in the fitting procedure and not be treated as fixed background. In this figure, we show in green dashed curves the $\Delta\chi^2 = 1$ window on κ_W with the incorrect treatment of neglecting the variation from the HZZ coupling. In contrast, we show in yellow curves the correct two-dimensional $\Delta\chi^2 = 1$ window in the κ_Z and κ_W plane. As one can see, a flat direction is caused from this consideration for the $\nu\bar{\nu}H$ measurement. Furthermore, we can take advantage of the $ZH \rightarrow \ell\bar{\ell}b\bar{b}$ and $ZH \rightarrow q\bar{q}b\bar{b}$ precision cross section measurements to constrain the κ_Z direction, shown in the window bounded by blue curves. We note here one should not take at face value the combined $ZH, H \rightarrow b\bar{b}$ cross section precision, as it has a dominant contribution actually from the $ZH \rightarrow \nu\bar{\nu}b\bar{b}$ process to avoid double counting.

FIG. 6. (a) Missing mass spectrums and (b) the ratio of the interference term to WW -fusion term, same as Fig. 5 only at a different c.o.m energy $\sqrt{s} = 240$ GeV. The cross sections at $\sqrt{s} = 240$ GeV are, respectively, $\sigma_{\text{intf}} = 0.444$ fb and $\sigma_{\text{WW}} = 6.36$ fb. Here, the net effect of the interference term is positive.

FIG. 7. The interference effects reflected in the coupling extraction process in the κ framework.

Combining these measurements, one can determine a 68% C.L. precision on the κ_W of 2.8% instead of 1.4% with the incorrect treatment of neglecting the interplay between κ_W and κ_Z for this process.

Next, the quantum interference effect between the $ZH \rightarrow \nu\bar{\nu}b\bar{b}$ amplitude and the WW fusion with $H \rightarrow b\bar{b}$ amplitude yields roughly a 10% reduction for the WW -fusion cross section. Hence, neglecting this effect will result in a wrong expectation value of the measurement, shown as the dark red square in the figure, which is $4 - 5\sigma$ away from the true SM expectation.

V. CONCLUSIONS

In this work, we study the interplay between the WW -fusion channel and the ZH channel at e^-e^+ colliders both analytically and numerically. At $\sqrt{s} = 250$ GeV, which is usually chosen to be the running energy for a Higgs factory,

the quantum interference effect between the two channels amounts to roughly -10% of the WW -fusion cross section. If overlooked, this will result in a wrong expectation value of the measurement $4-5\sigma$ away from the true SM expectation. In addition, there is the interplay between κ_W and κ_Z to be considered in the global fitting procedure. Otherwise, we would derive constraints on κ_W at the 68% confidence level that are 100% too aggressive. Therefore, the two effects both need to be taken care of when dealing with the WW -fusion process.

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