Unimodular gravity theory with external sources in a Lorentz-symmetry breaking scenario

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This paper is dedicated to the study of interactions between stationary field sources for the linearized unimodular gravity or the Weyl plus transverse diffeomorphism (WTDIFF) theory in a model that exhibits Lorentz-symmetry breaking due to the presence of the linearized topological Chern-Simons term in $3 + 1$ dimensions, where the Lorentz symmetry breaking is caused by a single background vector v^μ . Since the background vector is very tiny, we treat it perturbatively up to second order and we focus on physical phenomena which have no counterpart in standard WTDIFF theory. We consider effects related to field sources describing pointlike particles and cosmic strings. We show that in a Lorentz-violating scenario the interaction between external sources lead to numerically different results for the linearized Einstein-Hilbert and WTDIFF theories; however, both results are qualitatively similar and can be equalized after a rescaling of the Lorentz-breaking source term which makes an experimental distinction impossible at leading order in perturbation theory as far as point particles and cosmic strings are concerned.

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I. INTRODUCTION

Theories with Lorentz-symmetry breaking have been under intensive investigation as an attempt to find a consistent description of quantum gravity. Most of these investigations have been done in the context of the standard model extension (SME) [1,2] that incorporates in the standard model the full set of gauge-invariant, renormalizable Lorentz violation interactions. Some aspects of Lorentz violation have been investigated, e.g., in Maxwell electrodynamics [3–9], QED [10,11], linearized gravity [12–23], short-range experiments in pure gravity [24–29].

Here we are interested in the issue of Lorentz violation in linearized gravity. There are two covariant descriptions of massless spin-2 particles in $3 + 1$ dimensions via a symmetric rank-2 tensor: the linearized Einstein-Hilbert (LEH) theory [30] and the Weyl plus transverse diffeomorphism (WTDIFF) invariant model [31–34]. The WTDIFF model is the linearized truncation of unimodular gravity [35–38] which, in its turn, corresponds to the Einstein-Hilbert theory with the replacement $g_{\mu\nu} \rightarrow \hat{g}_{\mu\nu}/(-\hat{g})^{1/4}$. The WTDIFF model can be obtained from the usual LEH theory by the

singular replacement $h_{\mu\nu} \rightarrow h_{\mu\nu} - \eta_{\mu\nu}h/4$, and the external source is just the traceless piece of the energy-momentum tensor $T_{\mu\nu}$, i.e., $T_{\mu\nu} \rightarrow T_{\mu\nu} - \eta_{\mu\nu}T/4$ [39,40].

One remarkable feature of the WTDIFF theory is the fact that this theory leads to the same results obtained in the LEH theory for the interactions between external sources [39–41], at least, as far as the Lorentz symmetry of the linearized theory is preserved. From this point of view, it is natural to ask whether such equivalence remains in a Lorentz-violating scenario. One might try to distinguish WTDIFF from LEH based on Lorentz-violating phenomena as we pursue here. Moreover, there are physical phenomena produced by the presence of external field sources, with no counterpart in standard (Lorentz-preserving) WTDIFF and LEH theories. Studies of this kind have not yet been considered in the literature, to the best of the authors' knowledge, and deserve investigations not only for their theoretical aspects, but also because of their possible experimental relevance in the search for Lorentz-symmetry breaking.

This paper is devoted to this subject, where we search for effects produced by external sources. At leading order in perturbation theory, we show that it is not possible to distinguish the WTDIFF-LV and LEH-LV theories through interactions between external sources. Specifically, we consider the WTDIFF Lagrangian modified by the *CPT*-breaking linearized topological Chern-Simons term in $3 + 1$ dimensions [12], where the Lorentz-symmetry breaking is due to the presence of a single background vector v^μ . Since the background vector is very tiny, we treat it

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perturbatively up to leading order. We show that a spontaneous torque on a classical rigid halter emerges, and we investigate some phenomena due to the presence of cosmic strings and show that the string can interact with a pointlike particle as well as with another cosmic string in the Lorentz-symmetry breaking scenario considered.

The paper is structured as follows; in Sec. II, we describe some general aspects of the models we deal with along the paper and present our basic formula for the computation of the energy of the system. In Sec. III, we consider effects due to the presence of pointlike stationary particles. Section IV is dedicated to the study of physical phenomena due to the presence of cosmic strings, and Sec. V is dedicated to our final remarks and conclusions.

II. PROPAGATORS AND VACUUM ENERGY IN THE PRESENCE OF SOURCES

First, let us consider the following Lagrangian density in $3 + 1$ dimensions,

$$\mathcal{L} = -\frac{\sqrt{-g}}{2k^2}R - \frac{1}{2}\epsilon^{\mu\nu\kappa\lambda}v_\mu\Gamma_{\lambda\sigma}^\rho\left(\partial_\nu\Gamma_{\rho\kappa}^\sigma + \frac{2}{3}\Gamma_{\nu\alpha}^\sigma\Gamma_{\kappa\rho}^\alpha\right), \quad (1)$$

where the first term is the well-known Einstein-Hilbert Lagrangian and the second one is the so-called topological Chern-Simons term in four dimensions, $g = \det g_{\mu\nu}$, $g_{\mu\nu}$ is the spacetime metric, as standard in the literature, we define the gravitational coupling as $k^2 = 8\pi G$, R stands for the scalar curvature,

$$R = g^{\mu\nu}(\partial_\nu\Gamma_{\mu\lambda}^\lambda - \partial_\lambda\Gamma_{\mu\nu}^\lambda + \Gamma_{\mu\lambda}^\tau\Gamma_{\tau\nu}^\lambda - \Gamma_{\mu\nu}^\tau\Gamma_{\tau\lambda}^\lambda), \quad (2)$$

where

$$\Gamma_{\mu\nu}^\lambda = \frac{1}{2}g^{k\lambda}(\partial_\mu g_{k\nu} + \partial_\nu g_{\mu k} - \partial_k g_{\mu\nu}), \quad (3)$$

is the affine connection, $\epsilon^{\mu\nu\kappa\lambda}$ stands for the Levi-Civita tensor ($\epsilon^{0123} = 1$), and $v^\mu = (v^0, \mathbf{v})$ is the background vector.

We will restrict our attention to a graviton propagating on a flat Minkowski geometry whose metric tensor is $\eta^{\mu\nu} = (+, -, -, -)$. We use

$$g_{\mu\nu}(x) = \eta_{\mu\nu} + kh_{\mu\nu}(x), \quad (4)$$

where $kh_{\mu\nu}$ is the symmetric spin-2 field and represents, as usually, a small perturbation around flat Minkowski spacetime.

The model (1) at linearized level (quadratic in $h_{\mu\nu}$) is invariant under linearized general coordinate transformations,

$$\delta h_{\mu\nu}(x) = \partial_\mu\xi_\nu(x) + \partial_\nu\xi_\mu(x), \quad (5)$$

where $\xi_\nu(x)$ are the gauge parameters. So, in order to avoid singularities it is necessary to fix this gauge invariance, a common choice in the perturbative gravity literature is the De Donder gauge-fixing term,

$$\mathcal{L}_{gf} = \frac{1}{2\alpha}F_\mu F^\mu, \quad (6)$$

with

$$F^\mu = \partial_\nu\left(h^{\mu\nu} - \frac{1}{2}\eta^{\mu\nu}h\right), \quad (7)$$

and $h = \eta^{\mu\nu}h_{\mu\nu}$. Therefore, by using the weak-field approximation (4) and taking into account (6), the quadratic Lagrangian in the spin-2 field reads

$$\begin{aligned} \mathcal{L}_{\text{EH-LV}} &= -\frac{1}{2}\left(\frac{1}{2}h_{\mu\nu}\Box h^{\mu\nu} - \frac{1}{2}h\Box h + h\partial_\mu\partial_\nu h^{\mu\nu} - h^{\mu\nu}\partial_\mu\partial_\nu h_{\lambda\nu}\right) \\ &\quad + \frac{1}{2\alpha}\left(-h^{\mu\nu}\partial_\mu\partial^\lambda h_{\lambda\nu} + h^{\mu\nu}\partial_\mu\partial_\nu h - \frac{1}{4}h\Box h\right) \\ &\quad - \frac{k^2}{4}[\epsilon^{\mu\nu\kappa\lambda}v_\mu(h_{\lambda}^\rho\Box\partial_\nu h_{\rho\kappa} - h_{\lambda\rho}\partial_\nu\partial^\rho h_{\sigma\kappa})] \\ &\quad + \frac{1}{2}kh_{\mu\nu}T^{\mu\nu}, \end{aligned} \quad (8)$$

where in the last line we have inserted an external source, $T^{\mu\nu}$, that must be conserved $\partial_\mu T^{\mu\nu} = 0$ in order that the source term $\int d^4x h_{\mu\nu}T^{\mu\nu}$ be invariant under linearized reparametrizations (5).

Now let us jump to the WTDIFF model. There are only two ways of describing massless spin-2 particles covariantly in terms of a symmetric rank-2 tensor. Besides the usual linearized Einstein-Hilbert theory $\mathcal{L}_{\text{EH}}$, which is the far more popular theory, we have a model invariant under Weyl (W) transformations and transverse reparametrizations (TDIFF). There are older [42,43] and more recent [39,41] references on this subject. In practice, the so-called WTDIFF model can be obtained from the Einstein-Hilbert theory (8) in $D = 3 + 1$ via the singular replacement by a traceless field, i.e., $h_{\mu\nu} \rightarrow h_{\mu\nu} - \eta_{\mu\nu}h/4$. From (8), we obtain

$$\begin{aligned} \mathcal{L}_{\text{WT-LV}} &= -\frac{1}{2}\left(\frac{1}{2}h_{\mu\nu}\Box h^{\mu\nu} - \frac{3}{16}h\Box h + \frac{1}{2}h\partial_\mu\partial_\nu h^{\mu\nu} - h^{\mu\nu}\partial_\mu\partial_\nu h_{\lambda\nu}\right) \\ &\quad + \frac{1}{2\alpha}\left(-h^{\mu\nu}\partial_\mu\partial^\lambda h_{\lambda\nu} + h^{\mu\nu}\partial_\mu\partial_\nu h - \frac{1}{4}h\Box h\right) \\ &\quad - \frac{k^2}{4}[\epsilon^{\mu\nu\kappa\lambda}v_\mu(h_{\lambda}^\rho\Box\partial_\nu h_{\rho\kappa} - h_{\lambda\rho}\partial_\nu\partial^\rho h_{\sigma\kappa})] \\ &\quad + \frac{1}{2}kh_{\mu\nu}\hat{T}^{\mu\nu}. \end{aligned} \quad (9)$$

Notice that the singular redefinition has automatically generated a redefinition of the source:

$$T_{\mu\nu} \rightarrow \hat{T}_{\mu\nu} = T_{\mu\nu} - \eta_{\mu\nu} \frac{T}{4}. \quad (10)$$

The new source is defined out of the old conserved source $T_{\mu\nu}$, but it is not conserved by itself. It satisfies instead the constraints

$$\eta^{\mu\nu} \hat{T}_{\mu\nu} = 0; \quad \partial^\mu \hat{T}_{\mu\nu} = -\partial_\nu T/4. \quad (11)$$

The new constraints make the source term $\int d^4x h_{\mu\nu} \hat{T}^{\mu\nu}$ invariant under transverse reparametrizations and Weyl transformations:

$$\delta h_{\mu\nu} = \partial_\mu \xi_\nu^T + \partial_\nu \xi_\mu^T + \eta_{\mu\nu} \phi; \quad \partial^\mu \xi_\mu^T = 0, \quad (12)$$

which are the symmetries of $\mathcal{L}_{\text{WT-LV}}$, except for the gauge fixing term of course, where we have not made any field redefinition; otherwise, the Weyl symmetry would not be broken by such a term. We have made use of the fact that the Lorentz-violating term is invariant under Weyl transformations and (arbitrary) reparametrizations.

Now we are going to write down the propagators corresponding to $\mathcal{L}_{\text{EH-LV}}$ and $\mathcal{L}_{\text{WT-LV}}$ and the respective source-source term. The presence of the LV term makes the calculation complicate. Fortunately, it has been worked out before in [14]. Both Lagrangians (8) and (9) fit in the form

$$\mathcal{L} = C_1(\partial^\mu h_{\mu\nu})^2 + C_2 \partial^\mu h \partial^\alpha h_{\alpha\mu} + h C_3(\Box)h + h^{\mu\nu} C_4(\Box)h_{\mu\nu} + h_{\lambda\alpha} \Box \theta^{\alpha\lambda} S^{\nu\lambda} h_{\rho\nu}, \quad (13)$$

where C_1, C_2 are constants, $C_3(\Box), C_4(\Box)$ are functions of $\Box$, and $S^{\nu\lambda} = k^2 \epsilon^{\nu\lambda\beta\sigma} v_\beta \partial_\sigma$. If we write down (13) in the form $\mathcal{L} = h^{\mu\nu} G_{\mu\nu,\alpha\beta} h^{\alpha\beta}$, the differential operator $G_{\mu\nu,\alpha\beta}$ can be decomposed on the basis of the operators [14] displayed in our Appendix. Suppressing indices, we have

$$G = C_4 P_{SS}^{(2)} + \left(C_4 - \frac{C_1 \Box}{2} \right) P_{SS}^{(1)} + [C_3 + C_4 - (C_1 + C_2) \Box] P_{WW}^{(0)} + (3C_3 + C_4) P_{SS}^{(0)} + \sqrt{3} \left(C_3 - \frac{C_2 \Box}{2} \right) (P_{SW}^{(0)} + P_{WS}^{(0)}) - \frac{\Box}{2} (S_\sigma + S_\tau). \quad (14)$$

Using (A16), we have the inverse operator

$$G^{-1} = \frac{P_{SS}^{(2)}}{C_4} + \frac{P_{SS}^{(1)}}{C_4 - \frac{C_1 \Box}{2}} + \frac{3C_3 + C_4}{K^{(0)}} P_{WW}^{(0)} + \frac{[C_3 + C_4 - (C_1 + C_2) \Box]}{K^{(0)}} P_{SS}^{(0)} - \sqrt{3} \left(C_3 - \frac{C_2 \Box}{2} \right) (P_{SW}^{(0)} + P_{WS}^{(0)}) + G_{LV}^{-1}, \quad (15)$$

where the spin-0 determinant $K^{(0)}$ and the Lorentz-violation piece are given, respectively, by

$$K^{(0)} = [C_3 + C_4 - (C_1 + C_2) \Box] (3C_3 + C_4) - 3 \left(C_3 - \frac{C_2 \Box}{2} \right)^2, \quad (16)$$

$$G_{LV}^{-1} = \frac{4(iS_\tau + 2S_\tau^2)}{\Box[1 + 4f(p)]} + \frac{4(iS_\sigma + 2S_\sigma^2)}{\Box[1 + 16f(p)]}, \quad (17)$$

where $f(p)$ is given in (A13) and the operators $S_\sigma, S_\sigma^2, S_\tau$, and S_τ^2 are also displayed in the Appendix. In the cases of EH and WTDIFF, we have, respectively,

$$C_1 = 1 + \frac{1}{2\alpha}; \quad C_2 = -\left(1 + \frac{1}{2\alpha}\right); \quad C_3 = -\frac{\Box}{2} - \frac{\Box}{8\alpha}; \quad C_4 = \frac{\Box}{2}; \quad K^{(0)} = \frac{\Box^2}{8\alpha}, \quad (18)$$

$$C_1 = 1 + \frac{1}{2\alpha}; \quad C_2 = -\frac{1}{2} - \frac{1}{2\alpha}; \quad C_3 = -\frac{3\Box}{16} - \frac{\Box}{8\alpha}; \quad C_4 = \frac{\Box}{2}; \quad K^{(0)} = \frac{\Box^2}{32\alpha}. \quad (19)$$

Explicitly, we have

$$G_{\text{EH-LV}}^{-1} = \frac{P_{SS}^{(2)}}{\Box/2} - \frac{P_{SS}^{(1)}}{\Box/(4\alpha)} - \left(\frac{3}{\Box} + \frac{8\alpha}{\Box} \right) P_{WW}^{(0)} - \frac{P_{SS}^{(0)}}{\Box} - \frac{\theta\omega + \omega\theta}{\Box} + G_{LV}^{-1}, \quad (20)$$

$$G_{\text{WT-LV}}^{-1} = \frac{P_{SS}^{(2)}}{\Box/2} - \frac{P_{SS}^{(1)}}{\Box/(4\alpha)} - \left(\frac{12}{\Box} + \frac{2\alpha}{\Box} \right) P_{WW}^{(0)} - \left(\frac{4}{\Box} + \frac{6\alpha}{\Box} \right) P_{SS}^{(0)} - \left(\frac{4}{\Box} + \frac{2\alpha}{\Box} \right) (\theta\omega + \omega\theta) + G_{LV}^{-1}. \quad (21)$$

Notice that since G_{LV}^{-1} only depends upon the Lorentz-violating term and the coefficient C_4 in the operator G , see (A1) and (A16), we end up with the same Lorentz-violating term in G^{-1} for both EH and WTDIFF models. After we saturate $G_{\text{EH-LV}}^{-1}$ with conserved sources, it turns out that all terms containing the derivatives $\omega_{\mu\nu}$ drop out. We have in momentum space

$$\begin{aligned} T^{\mu\nu}(-p) (G_{\text{EH-LV}}^{-1})_{\mu\nu,\alpha\beta} T^{\alpha\beta}(p) &= T^{\mu\nu}(-p) \left[\left(\frac{2P_{SS}^{(2)}}{p^2} - \frac{P_{SS}^{(0)}}{p^2} \right) + G_{LV}^{-1} \right]_{\mu\nu,\alpha\beta} T^{\alpha\beta}(p) \\ &= T^{\mu\nu}(-p) \left[\frac{\eta_{\mu\alpha} \eta_{\nu\beta} + \eta_{\mu\beta} \eta_{\nu\alpha}}{p^2} - \left(\frac{2}{3} + \frac{1}{3} \right) \frac{\eta_{\mu\nu} \eta_{\alpha\beta}}{p^2} + G_{LV}^{-1} \right]_{\mu\nu,\alpha\beta} \\ &\quad \times T^{\alpha\beta}(p), \end{aligned} \quad (22)$$

$$\begin{aligned}
&= T^{\mu\nu}(-p) \left[\frac{\eta_{\mu\alpha}\eta_{\nu\beta} + \eta_{\mu\beta}\eta_{\nu\alpha}}{p^2} - \frac{\eta_{\mu\nu}\eta_{\alpha\beta}}{p^2} \right]_{\mu\nu,\alpha\beta} T^{\alpha\beta}(p) \\
&+ T^{\mu\nu}(-p)(G_{\text{LV}}^{-1})_{\mu\nu,\alpha\beta} T^{\alpha\beta}(p). \quad (23)
\end{aligned}$$

Notice the factor $1/3$ in (22) which comes from $P_{SS}^{(0)}$; it is responsible for the vDVZ [44,45] mass discontinuity since it does not appear in the propagator of the massive theory, no matter how small is the mass.

Now, if we saturate $G_{\text{WT-LV}}^{-1}$ with the traceless sources $\hat{T}_{\mu\nu}$, which satisfy (11), we have once again no contribution from the spin-1 sector, though this is more subtle now,

$$\begin{aligned}
\hat{T}^{\mu\nu}(-p)(P_{SS}^{(1)})_{\mu\nu,\alpha\beta} \hat{T}^{\alpha\beta}(p) &= 2\hat{T}^{\mu\nu}(-p)\theta_{\mu\alpha}\omega_{\nu\beta} \hat{T}^{\alpha\beta}(p) \\
&= \frac{1}{2} \hat{T}^{\mu\nu}(-p)\theta_{\mu\alpha}P_{\nu}p^{\alpha}T = 0, \quad (24)
\end{aligned}$$

where the last equality holds due to the transverse property of $\theta_{\mu\alpha}$. It turns out that adding up all spin-0 contributions, we derive the known result [36] that no one can tell a difference between the source-source terms in LEH and WTDIFF theories as far as Lorentz-preserving terms are considered, namely from (21),

$$\begin{aligned}
&\hat{T}^{\mu\nu}(-p)(G_{\text{WT-LV}}^{-1})_{\mu\nu,\alpha\beta} \hat{T}^{\alpha\beta}(p) \\
&= \hat{T}^{\mu\nu}(-p) \left[\frac{2P_{SS}^{(2)}}{p^2} - \left(\frac{12}{p^2} + \frac{4}{3p^2} - \frac{8}{p^2} \right) \omega\omega + G_{\text{LV}}^{-1} \right]_{\mu\nu,\alpha\beta} \hat{T}^{\alpha\beta}(p) \\
&= T^{\mu\nu}(-p) \left[\frac{\eta_{\mu\alpha}\eta_{\nu\beta} + \eta_{\mu\beta}\eta_{\nu\alpha}}{p^2} - \frac{\eta_{\mu\nu}\eta_{\alpha\beta}}{p^2} \right]_{\mu\nu,\alpha\beta} T^{\alpha\beta}(p) \\
&+ \hat{T}^{\mu\nu}(-p)(G_{\text{LV}}^{-1})_{\mu\nu,\alpha\beta} \hat{T}^{\alpha\beta}(p). \quad (25)
\end{aligned}$$

Notice that the only difference between (23) and (25) is the replacement of $T^{\mu\nu}$ by $\hat{T}^{\mu\nu}$ in the Lorentz-violating term.

Paving the way for the next section, we present here the formulas for the vacuum energy. Since we have a quadratic Lagrangian in the field variables, the contribution due to the stationary sources to the vacuum energy of the system is given in the WTDIFF case by [46]

$$E_{\text{WT-LV}} = \frac{k^2}{4t} \int \int d^4x d^4y \hat{T}^{\alpha\beta}(x) \hat{D}_{\alpha\beta,\gamma\delta}(x, y) \hat{T}^{\gamma\delta}(y), \quad (26)$$

where t is a time variable and the limit $t \rightarrow \infty$ is implicit. The integration in y^0 is from $-t/2$ to $t/2$. We have introduced the propagator in the coordinate space:

$$\hat{D}_{\alpha\beta,\gamma\delta}(x, y) = \int \frac{d^4p}{(2\pi)^4} \hat{D}_{\alpha\beta,\gamma\delta}(p) e^{-ip \cdot (x-y)}. \quad (27)$$

Splitting the momentum space propagator into Lorentz-preserving and Lorentz-violating parts, we have from (25),

$$\begin{aligned}
E_{\text{WT-LV}} &= \frac{k^2}{4t} \int \frac{d^4p}{(2\pi)^4} \int \int d^4x d^4y e^{-ip \cdot (x-y)} \\
&\times [T^{\alpha\beta}(x) D_{\alpha\beta,\gamma\delta}^{(0)}(p) T^{\gamma\delta}(y) \\
&+ \hat{T}^{\alpha\beta}(x) D_{\alpha\beta,\gamma\delta}^{(\text{LV})}(p) \hat{T}^{\gamma\delta}(y)], \quad (28)
\end{aligned}$$

where

$$\begin{aligned}
D_{\alpha\beta,\gamma\delta}^{(0)}(p) &= \frac{\eta_{\alpha\gamma}\eta_{\beta\delta} + \eta_{\alpha\delta}\eta_{\beta\gamma}}{p^2} - \frac{\eta_{\alpha\beta}\eta_{\gamma\delta}}{p^2}; \\
D_{\alpha\beta,\gamma\delta}^{(\text{LV})}(p) &= \frac{4(iS_{\tau} + 2S_{\tau}^2)_{\alpha\beta,\gamma\delta}}{p^2[1 + 4f(p)]} + \frac{4(iS_{\sigma} + 2S_{\sigma}^2)_{\alpha\beta,\gamma\delta}}{p^2[1 + 16f(p)]}. \quad (29)
\end{aligned}$$

From (28), we can compute the interaction energy between different spin-2 field sources for the WTDIFF-LV model. The same expression (28) can be used for the EH-LV case if we replace $\hat{T}^{\mu\nu} \rightarrow T^{\mu\nu}$ in the Lorentz-violating term.

As the Lorentz-symmetry breaking must be very tiny, the background vector must be small. Therefore, throughout the paper, we will treat v^{μ} perturbatively up to second order, which is the leading order in the background vector.

III. POINTLIKE PARTICLES

In this section, we study the interaction energy between two stationary pointlike particles in $3 + 1$ dimensions. The external source which describes this system is given by

$$[T^{\alpha\beta}(x)]^I = M_1 \eta^{\alpha 0} \eta^{\beta 0} \delta^3(\mathbf{x} - \mathbf{a}_1) + M_2 \eta^{\alpha 0} \eta^{\beta 0} \delta^3(\mathbf{x} - \mathbf{a}_2), \quad (30)$$

$$\begin{aligned}
[\hat{T}^{\alpha\beta}(x)]^I &= M_1 \left(\eta^{\alpha 0} \eta^{\beta 0} - \frac{\eta^{\alpha\beta}}{4} \right) \delta^3(\mathbf{x} - \mathbf{a}_1) \\
&+ M_2 \left(\eta^{\alpha 0} \eta^{\beta 0} - \frac{\eta^{\alpha\beta}}{4} \right) \delta^3(\mathbf{x} - \mathbf{a}_2), \quad (31)
\end{aligned}$$

where the location of the particles are specified by the vectors $\mathbf{a}_1$ and $\mathbf{a}_2$, and the parameters M_1 and M_2 are the particles masses.

Substituting Eqs. (30) and (31) into Eq. (28) and discarding the self-interacting contributions (that is, the interactions of a given point particle with itself), we obtain

$$\begin{aligned}
E_{\text{WT-LV}}^I = & \frac{k^2 M_1 M_2}{2t} \int \frac{d^4 p}{(2\pi)^4} \int \int d^4 x d^4 y \delta^3(\mathbf{x} - \mathbf{a}_1) \delta^3(\mathbf{y} - \mathbf{a}_2) e^{-ip \cdot (x-y)} \left[D_{00,00}^{(0)}(p) + \frac{9}{16} D_{00,00}^{(\text{LV})}(p) \right. \\
& + \frac{3}{16} \left(D_{00,11}^{(\text{LV})}(p) + D_{00,22}^{(\text{LV})}(p) + D_{00,33}^{(\text{LV})}(p) \right) + \frac{3}{16} \left(D_{11,00}^{(\text{LV})}(p) + D_{22,00}^{(\text{LV})}(p) + D_{33,00}^{(\text{LV})}(p) \right) \\
& + \frac{1}{16} \left(D_{11,11}^{(\text{LV})}(p) + D_{22,22}^{(\text{LV})}(p) + D_{33,33}^{(\text{LV})}(p) + D_{11,22}^{(\text{LV})}(p) + D_{22,11}^{(\text{LV})}(p) + D_{11,33}^{(\text{LV})}(p) \right. \\
& \left. \left. + D_{33,11}^{(\text{LV})}(p) + D_{22,33}^{(\text{LV})}(p) + D_{33,22}^{(\text{LV})}(p) \right) \right]. \quad (32)
\end{aligned}$$

By using the Eqs. (29), (A7)–(A13) up to second order in v^μ , computing the integrals in the following order, $d^3 \mathbf{x}$, $d^3 \mathbf{y}$, dx^0 , dp^0 and dy^0 , using the Fourier representation for the Dirac delta function $\delta(p^0) = \int dx^0 / (2\pi) \exp(-ipx^0)$, and identifying the time interval as $t = \int_{-\frac{t}{2}}^{\frac{t}{2}} dy^0$, we arrive at

$$\begin{aligned}
E_{\text{WT-LV}}^I = & -\frac{k^2 M_1 M_2}{2} \left[\int \frac{d^3 \mathbf{p}}{(2\pi)^3} \frac{e^{i\mathbf{p} \cdot \mathbf{a}}}{\mathbf{p}^2} \right. \\
& - 10k^4 (\mathbf{v} \cdot \nabla_{\mathbf{a}})^2 \int \frac{d^3 \mathbf{p}}{(2\pi)^3} \frac{e^{i\mathbf{p} \cdot \mathbf{a}}}{\mathbf{p}^2} \\
& \left. + 2k^4 [(v^0)^2 - 5\mathbf{v}^2] \int \frac{d^3 \mathbf{p}}{(2\pi)^3} e^{i\mathbf{p} \cdot \mathbf{a}} \right], \quad (33)
\end{aligned}$$

where $\mathbf{a} = \mathbf{a}_1 - \mathbf{a}_2$ is the distance between the two massive particles and we defined the differential operator

$$\nabla_{\mathbf{a}} = \left(\frac{\partial}{\partial a^1}, \frac{\partial}{\partial a^2}, \frac{\partial}{\partial a^3} \right). \quad (34)$$

The last term inside the brackets of Eq. (33) is the Dirac delta function $\delta^3(\mathbf{a})$ and provided that $\mathbf{a}$ is nonzero, this term vanishes.

Using the fact that

$$\int \frac{d^3 \mathbf{p}}{(2\pi)^3} \frac{e^{i\mathbf{p} \cdot \mathbf{a}}}{\mathbf{p}^2} = \frac{1}{4\pi a}, \quad (35)$$

where $a = |\mathbf{a}|$, and performing some manipulations, we obtain

$$E_{\text{WT-LV}}^I = -\frac{GM_1 M_2}{a} \left[1 - 10 \left(\frac{8\pi G}{a} \right)^2 \left(\frac{3(\mathbf{v} \cdot \mathbf{a})^2}{a^2} - \mathbf{v}^2 \right) \right]. \quad (36)$$

Equation (36) is a perturbative result and gives the interaction energy between two pointlike particles mediated by the spin-2 field with the specific Lorentz-violating coupling contained in Eq. (8). The v^μ -dependent contribution in (36) is a correction to the usual gravitational interaction due the Lorentz-symmetry breaking, leading to

an anisotropic interaction between the particles. If we take $v^\mu \rightarrow 0$, the expression reduces to the standard gravitational interaction. The same happens for the case where $v^\mu = (v^0, 0)$, i.e., if $\mathbf{v} = 0$. However, for the particular case where the distance vector $\mathbf{a}$ is perpendicular to the background vector $\mathbf{v}$, Eq. (36) still exhibits effects due to Lorentz-symmetry breaking,

$$E_{\text{WT-LV}}^I(\mathbf{v} \cdot \mathbf{a} = 0) = -\frac{GM_1 M_2}{a} \left[1 + 10 \left(\frac{8\pi G}{a} \right)^2 \mathbf{v}^2 \right]. \quad (37)$$

The force between the two particles can be calculated from Eq. (36), resulting in

$$\begin{aligned}
\mathbf{F}_{\text{WT-LV}}^I = & -\nabla_{\mathbf{a}} E_{\text{WT-LV}}^I \\
= & -\frac{GM_1 M_2}{a^2} \left\{ \left[1 + 30 \left(\frac{8\pi G}{a} \right)^2 \left(\mathbf{v}^2 - \frac{5(\mathbf{v} \cdot \mathbf{a})^2}{a^2} \right) \right] \hat{a} \right. \\
& \left. + 60 \left(\frac{8\pi G}{a} \right)^2 \frac{(\mathbf{v} \cdot \mathbf{a})}{a} \mathbf{v} \right\}, \quad (38)
\end{aligned}$$

where $\hat{a}$ is a unit vector pointing in the direction of the vector $\mathbf{a}$. The interaction force in (38) shows us more explicitly the anisotropies generated by the Lorentz-symmetry breaking.

We remark that the authors of [47] have considered the effect of Lorentz- and *CPT*-breaking terms of mass dimension five (three derivatives) on a couple of moving point masses. Their Lorentz-violating terms include ours; see (8). In the static limit, their inverse cubic force contribution $F \sim a^{-3}$ disappears in agreement with (38) where our leading Lorentz-breaking contribution only appears at order¹ a^{-4} .

An important consequence of the anisotropies in expression (36) is the emergence of a spontaneous torque on a rigid halter. In order to investigate this effect, we consider a typical rigid hater composed by two particles of masses M_1 and M_2 , respectively, placed at the positions $\mathbf{a}_1 = \mathbf{R} + \frac{\mathbf{A}}{2}$

¹The case of moving point particles as well as the use of more general Lorentz-violating terms is now in progress.

and $\mathbf{a}_2 = \mathbf{R} - \frac{\mathbf{A}}{2}$, where we take the distance vector $\mathbf{A}$ fixed (and small). From Eq. (36), we obtain

$$E_{\text{WT-LV}}^{\text{halter}} = -\frac{GM_1M_2}{A} \left[1 - 10 \left(\frac{8\pi G}{A} \right)^2 \mathbf{v}^2 (3\cos^2(\theta) - 1) \right], \quad (39)$$

where $A = |\mathbf{A}|$ and θ is the angle between the vectors $\mathbf{A}$ and $\mathbf{v}$. Notice that $0 \leq \theta \leq 2\pi$. The energy (39) leads to a spontaneous torque on the rigid halter, as follows:

$$\begin{aligned} \tau_{\text{WT-LV}}^{\text{halter}} &= -\frac{\partial E_{\text{WT-LV}}^{\text{halter}}}{\partial \theta} \\ &= 30 \frac{GM_1M_2}{A} \left(\frac{8\pi G}{A} \right)^2 \mathbf{v}^2 \sin(2\theta). \end{aligned} \quad (40)$$

This spontaneous torque on the rigid halter is an exclusive effect due to the Lorentz-violating background. If, $v^\mu = 0$ the torque vanishes, as it should, as well as for the specific cases $\theta = 0, \pi, \pi/2, 2\pi$. When $\theta = \pi/4$, the torque exhibits a maximum value.

For the LEH-LV theory, we proceed as previously, replacing $\hat{T}^{\alpha\beta}(x) \rightarrow T^{\alpha\beta}(x)$ and $\hat{T}^{\gamma\delta}(y) \rightarrow T^{\gamma\delta}(y)$ in the second term between brackets on the right-hand side of the Eq. (28), obtaining for the interaction energy between two pointlike particles up to second order in v^μ , the result

$$\begin{aligned} E_{\text{EH-LV}}^I &= \frac{k^2 M_1 M_2}{2t} \int \frac{d^4 p}{(2\pi)^4} \int \int d^4 x d^4 y \delta^3(\mathbf{x} - \mathbf{a}_1) \\ &\quad \times \delta^3(\mathbf{y} - \mathbf{a}_2) e^{-ip \cdot (x-y)} [D_{00,00}^{(0)}(p) + D_{00,00}^{(\text{LV})}(p)] \\ &= -\frac{GM_1M_2}{a} \left[1 - 16 \left(\frac{8\pi G}{a} \right)^2 \left(\frac{3(\mathbf{v} \cdot \mathbf{a})^2}{a^2} - \mathbf{v}^2 \right) \right]. \end{aligned} \quad (41)$$

Comparing (36) with (41), we verify that for WTDIFF-LV and LEH-LV theories, the interaction energy between two point particles is numerically different already at second order in the background vector v^μ . An opposite situation occurs in standard WTDIFF and LEH theories where the energies are the same ones. The Lorentz-violating terms in (36) and (41) are qualitatively similar,

and we could turn the overall factor 10 into 16 by scaling v^μ into $(4/\sqrt{10})v^\mu$, at least at leading order in perturbation theory. It is not clear whether such simple rescaling will fit the remaining terms beyond the quadratic approximation in v^μ . This is under investigation.

IV. COSMIC STRINGS

Opposite from the pointlike particles of the last section, we now focus on long objects, namely, cosmic strings. Their contribution to the interaction energy, at leading order in perturbation theory, can only appear due to the Lorentz-violating terms.

We start this section considering the interaction energy between a pointlike particle and an cosmic string, both of them stationary. The cosmic string shall be taken to flow parallel to the z axis, along the straight line located at $\mathbf{A} = (A^1, A^2, 0)$. The pointlike particle is concentrated at position $\mathbf{s}$. This system is described by the external source,

$$\begin{aligned} [T^{\alpha\beta}(x)]^{II} &= M\eta^{\alpha 0}\eta^{\beta 0}\delta^3(\mathbf{x} - \mathbf{s}) \\ &\quad + \mu(\eta^{\alpha 0}\eta^{\beta 0} - \eta^{\alpha 3}\eta^{\beta 3})\delta^2(\mathbf{x}_\perp - \mathbf{A}), \end{aligned} \quad (42)$$

where the first term on the right-hand side of the above equation stands for the external field source produced by the pointlike charge, and the second one is the source produced by the cosmic string [48]. The mass parameter M and the linear mass density μ are the coupling constants between the field and the delta functions, and $\mathbf{x}_\perp = (x^1, x^2, 0)$, is the position vector perpendicular to the cosmic string. For the traceless source, we have

$$\begin{aligned} [\hat{T}^{\alpha\beta}(x)]^{II} &= M \left(\eta^{\alpha 0}\eta^{\beta 0} - \frac{\eta^{\alpha\beta}}{4} \right) \delta^3(\mathbf{x} - \mathbf{s}) \\ &\quad + \mu \left(\eta^{\alpha 0}\eta^{\beta 0} - \eta^{\alpha 3}\eta^{\beta 3} - \frac{\eta^{\alpha\beta}}{2} \right) \delta^2(\mathbf{x}_\perp - \mathbf{A}). \end{aligned} \quad (43)$$

Substituting the sources (42) and (43) in (28) and discarding the self-interacting terms, which do not contribute to the interaction force between the string and the particle (the self-interacting terms are proportional to M^2 or μ^2), we have

$$\begin{aligned} E_{\text{WT-LV}}^{II} &= \frac{k^2 M \mu}{2t} \int \frac{d^4 p}{(2\pi)^4} \int \int d^4 x d^4 y \delta^3(\mathbf{x} - \mathbf{s}) \delta^2(\mathbf{y}_\perp - \mathbf{A}) e^{-ip \cdot (x-y)} [D_{00,00}^{(0)}(p) - D_{00,33}^{(0)}(p) + \frac{3}{8} D_{00,00}^{(\text{LV})}(p) \\ &\quad + \frac{3}{8} (D_{00,11}^{(\text{LV})}(p) + D_{00,22}^{(\text{LV})}(p) + D_{00,33}^{(\text{LV})}(p)) + \frac{1}{8} (D_{11,00}^{(\text{LV})}(p) + D_{22,00}^{(\text{LV})}(p) + D_{33,00}^{(\text{LV})}(p)) \\ &\quad + \frac{1}{8} (D_{11,11}^{(\text{LV})}(p) + D_{22,22}^{(\text{LV})}(p) + D_{33,33}^{(\text{LV})}(p) + D_{11,22}^{(\text{LV})}(p) + D_{22,11}^{(\text{LV})}(p) + D_{11,33}^{(\text{LV})}(p) \\ &\quad + D_{33,11}^{(\text{LV})}(p) + D_{22,33}^{(\text{LV})}(p) + D_{33,22}^{(\text{LV})}(p)) - \frac{3}{4} D_{00,33}^{(\text{LV})}(p) - \frac{1}{4} (D_{11,33}^{(\text{LV})}(p) + D_{22,33}^{(\text{LV})}(p) + D_{33,33}^{(\text{LV})}(p))], \end{aligned} \quad (44)$$

where the integration limits for y^0 are as in the previous section. Substituting the propagators (29), using the Eqs. (A7)–(A13), take into account only the contributions up to second order in v^μ , and evaluating the integrals: $d^2\mathbf{y}_\perp$, $d^3\mathbf{x}$, dy^3 , dp^3 , dx^0 , dp^0 and dy^0 , we obtain

$$E_{\text{WT-LV}}^{II} = 2k^6 M\mu \left[(\mathbf{v}_\perp \cdot \nabla_{\mathbf{a}_\perp})^2 \int \frac{d^2\mathbf{p}_\perp}{(2\pi)^2} \frac{e^{i\mathbf{p}_\perp \cdot \mathbf{a}_\perp}}{\mathbf{p}_\perp^2} + [\mathbf{v}_\perp^2 + 2(v^3)^2] \int \frac{d^2\mathbf{p}_\perp}{(2\pi)^2} e^{i\mathbf{p}_\perp \cdot \mathbf{a}_\perp} \right], \quad (45)$$

where v^3 is the projection of the background vector $\mathbf{v}$ along the string, and defined $\mathbf{p}_\perp = (p^1, p^2, 0)$, $\mathbf{v}_\perp = (v^1, v^2, 0)$, the distance between the particle and the cosmic string $\mathbf{a}_\perp = (s^1 - A^1, s^2 - A^2, 0)$, and the differential operator

$$\nabla_{\mathbf{a}_\perp} = \left(\frac{\partial}{\partial a^1}, \frac{\partial}{\partial a^2}, 0 \right). \quad (46)$$

Provided that $\mathbf{a}_\perp$ is nonzero, the last term inside the brackets in (45) vanishes. The remaining integral is divergent, in order to circumvent this problem we proceed as in Refs. [8,49,50], introducing a mass regulator parameter, as follows:

$$E_{\text{WT-LV}}^{II} = 2k^6 M\mu (\mathbf{v}_\perp \cdot \nabla_{\mathbf{a}_\perp})^2 \lim_{m \rightarrow 0} \int \frac{d^2\mathbf{p}_\perp}{(2\pi)^2} \frac{e^{i\mathbf{p}_\perp \cdot \mathbf{a}_\perp}}{\mathbf{p}_\perp^2 + m^2}. \quad (47)$$

Using the fact that [49]

$$\int \frac{d^2\mathbf{q}_\perp}{(2\pi)^2} \frac{e^{i\mathbf{q}_\perp \cdot \mathbf{a}_\perp}}{\mathbf{q}_\perp^2 + m^2} = \frac{1}{2\pi} K_0(ma_\perp), \quad (48)$$

and acting with the differential operator (46), we arrive at

$$E_{\text{WT-LV}}^{II} = \frac{k^6 M\mu}{\pi} \lim_{m \rightarrow 0} \left[\frac{(\mathbf{v}_\perp \cdot \mathbf{a}_\perp)^2}{a_\perp^2} m^2 K_2(ma_\perp) - \frac{\mathbf{v}_\perp^2}{a_\perp} m K_1(ma_\perp) \right], \quad (49)$$

where $a_\perp = |\mathbf{a}_\perp|$ and $K_0(ma_\perp)$, $K_1(ma_\perp)$, $K_2(ma_\perp)$ stand for the K-Bessel functions.

Using the fact that [51]

$$m K_1(ma_\perp) \xrightarrow{m \rightarrow 0} \frac{1}{a_\perp}, \quad m^2 K_2(ma_\perp) \xrightarrow{m \rightarrow 0} \frac{2}{a_\perp^2}, \quad (50)$$

we obtain

$$E_{\text{WT-LV}}^{II} = \frac{512\pi^2 G^3 M\mu}{a_\perp^2} \left[2 \frac{(\mathbf{v}_\perp \cdot \mathbf{a}_\perp)^2}{a_\perp^2} - \mathbf{v}_\perp^2 \right]. \quad (51)$$

This interaction energy is an effect due solely to the Lorentz-violating background up to lowest order of the background vector, having no counterpart in standard WTDIFF theory. Clearly, if the background four-vector v^μ is zero, there is no interaction energy. The same happens for the case where $\mathbf{v}_\perp = 0$.

The interaction force can be obtained from Eq. (51) as follows,

$$\begin{aligned} \mathbf{F}_{\text{WT-LV}}^{II} &= -\nabla_{\mathbf{a}_\perp} E_{\text{WT-LV}}^{II} \\ &= -\frac{1024\pi^2 G^3 M\mu}{a_\perp^3} \left\{ \left[\mathbf{v}_\perp^2 - 4 \frac{(\mathbf{v}_\perp \cdot \mathbf{a}_\perp)^2}{a_\perp^2} \right] \hat{a}_\perp + 2 \frac{(\mathbf{v}_\perp \cdot \mathbf{a}_\perp)}{a_\perp} \mathbf{v}_\perp \right\}, \end{aligned} \quad (52)$$

where $\hat{a}_\perp$ is an unit vector pointing on the direction of vector $\mathbf{a}_\perp$.

As a final comment, we point out that from Eq. (51), one can also obtain a torque on the cosmic string by fixing the point particle. Denoting by ϕ the angle between $\mathbf{v}_\perp$ and $\mathbf{a}_\perp$, we obtain

$$\begin{aligned} \tau_{\text{WT-LV}}^{II} &= -\frac{\partial E_{\text{WT-LV}}^{II}}{\partial \phi} \\ &= \frac{1024\pi^2 G^3 M\mu}{a_\perp^2} \mathbf{v}_\perp^2 \sin 2(\phi). \end{aligned} \quad (53)$$

We notice that the torque (53) is an effect due solely to the Lorentz-violating background. It does not appear in standard WTDIFF theory. If $\phi = 0, \pi/2, \pi, 2\pi$ or $\mathbf{v}_\perp = 0$, the torque vanishes. For $\phi = \pi/4$ the torque has a maximum intensity.

For LEH-LV theory, we employ the same steps as above, we obtain for the interaction energy between the cosmic string and the pointlike particle the expression

$$\begin{aligned} E_{\text{EH-LV}}^{II} &= \frac{k^2 M\mu}{2t} \int \frac{d^4 p}{(2\pi)^4} \int \int d^4 x d^4 y \delta^3(\mathbf{x} - \mathbf{s}) \\ &\quad \times \delta^2(\mathbf{y}_\perp - \mathbf{A}) e^{-ip \cdot (x-y)} [D_{00,00}^{(0)}(p) - D_{00,33}^{(0)}(p) \\ &\quad + D_{00,00}^{(\text{LV})}(p) - D_{00,33}^{(\text{LV})}(p)] = 4E_{\text{WT-LV}}^{II}, \end{aligned} \quad (54)$$

showing that we have a numerically different result for both WTDIFF-LV and LEH-LV theories although they can be related via a scaling of the Lorentz-breaking term once again.

The next and last example is given by two parallel cosmic strings. We take a coordinate system where the first string lies along the straight line located at $\mathbf{A}_1 = (A_1^1, A_1^2, 0)$, with linear mass density μ_1 , and the second string lies along the line that crosses the xy plane at $\mathbf{A}_2 = (A_2^1, A_2^2, 0)$, with linear mass density μ_2 . The corresponding external source is given by

$$[T^{\alpha\beta}(x)]^{III} = \mu_1(\eta^{\alpha 0}\eta^{\beta 0} - \eta^{\alpha 3}\eta^{\beta 3})\delta^2(\mathbf{x}_\perp - \mathbf{A}_1) + \mu_2(\eta^{\alpha 0}\eta^{\beta 0} - \eta^{\alpha 3}\eta^{\beta 3})\delta^2(\mathbf{x}_\perp - \mathbf{A}_2), \quad (55)$$

$$[\hat{T}^{\alpha\beta}(x)]^{III} = \mu_1\left(\eta^{\alpha 0}\eta^{\beta 0} - \eta^{\alpha 3}\eta^{\beta 3} - \frac{\eta^{\alpha\beta}}{2}\right)\delta^2(\mathbf{x}_\perp - \mathbf{A}_1) + \mu_2\left(\eta^{\alpha 0}\eta^{\beta 0} - \eta^{\alpha 3}\eta^{\beta 3} - \frac{\eta^{\alpha\beta}}{2}\right)\delta^2(\mathbf{x}_\perp - \mathbf{A}_2). \quad (56)$$

Substituting the sources (55) and (56) in (28), discarding the self-interacting terms, using the Eqs. (29), (A7)–(A13), proceeding as in the previous cases, and identifying the length of the cosmic string as $L = \int dx^3$, we can show that the interaction energy between the two cosmic strings up to second order in v^μ is given by

$$E_{\text{WT-LV}}^{III} = 4k^6\mu_1\mu_2L(\mathbf{v}_\perp \cdot \nabla_{\mathbf{a}_\perp})^2 \lim_{m \rightarrow 0} \int \frac{d^2\mathbf{p}_\perp}{(2\pi)^2} \frac{e^{i\mathbf{p}_\perp \cdot \mathbf{a}_\perp}}{\mathbf{p}_\perp^2 + m^2} = \frac{1024\pi^2 G^3 \mu_1 \mu_2 L}{a_\perp^2} \left[2 \frac{(\mathbf{v}_\perp \cdot \mathbf{a}_\perp)^2}{a_\perp^2} - \mathbf{v}_\perp^2 \right], \quad (57)$$

where we have identified the distance between the strings as $\mathbf{a}_\perp = \mathbf{A}_1 - \mathbf{A}_2$, and $a_\perp = |\mathbf{a}_\perp|$.

It can be seen that the energy given above vanishes in the limit $v^\mu = 0$, where we do not have Lorentz-symmetry breaking.

Similarly, the energy (57) leads to an interaction force between two cosmic strings as well as to a torque on one string when we fix the other one.

For the LEH-LV theory, we obtain the expression

$$E_{\text{EH-LV}}^{III} = 4E_{\text{WT-LV}}^{III}, \quad (58)$$

where the numerical difference between the results for both theories was already expected.

V. CONCLUSIONS AND PERSPECTIVES

In this paper, we have investigated the interactions between stationary sources for the WTDIFF and LEH theories in the presence of the Lorentz-violating and *CPT*-breaking topological Chern-Simons term in 3 + 1 dimensions. All the results have been obtained perturbatively up to second order in v^μ . We have shown the emergence of an spontaneous torque on a classical rigid halter. We have also investigated interactions with one or two cosmic strings. We have shown that a cosmic string has a nontrivial interaction with a pointlike particle as well as with another cosmic string. All those phenomena would not appear in the absence of the Lorentz-violating term and have not been explored before in the literature.

We have also shown that there is a numerical difference in the interactions between stationary external sources

regarding WTDIFF-LV and LEH-LV theories. At leading order in perturbation theory, the differences can be extinguished after a convenient scaling of the Lorentz-violating source term which makes their experimental detection impossible in the cases analyzed here. It is not clear, however, if such scaling will keep working beyond the leading order in perturbation theory. This is under investigation as well as the investigation of more general nonminimal Lorentz-violating term; see [28].

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APPENDIX: A GENERALIZED PROPAGATOR

Here, we give a summary of technical details about the propagator found in [14] which includes a class of Lorentz-violating terms. We use a slightly different notation. The Lagrangian densities for symmetric rank-2 fields can be written as $\mathcal{L} = h_{\alpha\beta} G^{\alpha\beta,\mu\nu} h_{\mu\nu}$. Let us assume that the differential operator $G^{\alpha\beta,\mu\nu}$, suppressing indices, can be decomposed as

$$G = AP_{SS}^{(2)} + BP_{SS}^{(1)} + A_{SS}P_{SS}^{(0)} + A_{WW}P_{WW}^{(0)} + A_{SW}(P_{WS}^{(0)} + P_{SW}^{(0)}) + aS_\tau + bS_\sigma + cS_\tau^2 + dS_\sigma^2, \quad (A1)$$

where the spin-*s* operators $P_{IJ}^{(s)}$ acting on symmetric rank-2 tensors in $D = 4$ are given by

$$(P_{SS}^{(2)})^{\lambda\mu}_{\alpha\beta} = \frac{1}{2}(\theta^\lambda_\alpha \theta^\mu_\beta + \theta^\mu_\alpha \theta^\lambda_\beta) - \frac{\theta^{\lambda\mu} \theta_{\alpha\beta}}{3}, \quad (A2)$$

$$(P_{SS}^{(1)})^{\lambda\mu}_{\alpha\beta} = \frac{1}{2}(\theta^\lambda_\alpha \omega^\mu_\beta + \theta^\mu_\alpha \omega^\lambda_\beta + \theta^\lambda_\beta \omega^\mu_\alpha + \theta^\mu_\beta \omega^\lambda_\alpha), \quad (A3)$$

$$(P_{SS}^{(0)})^{\lambda\mu}_{\alpha\beta} = \frac{1}{3}\theta^{\lambda\mu} \theta_{\alpha\beta}, \quad (P_{WW}^{(0)})^{\lambda\mu}_{\alpha\beta} = \omega^{\lambda\mu} \omega_{\alpha\beta}, \quad (A4)$$

$$(P_{SW}^{(0)})^{\lambda\mu}_{\alpha\beta} = \frac{1}{\sqrt{3}}\theta^{\lambda\mu} \omega_{\alpha\beta}, \quad (P_{WS}^{(0)})^{\lambda\mu}_{\alpha\beta} = \frac{1}{\sqrt{3}}\omega^{\lambda\mu} \theta_{\alpha\beta}, \quad (A5)$$

$$\omega_{\mu\nu} = \frac{\partial_\mu \partial_\nu}{\square}, \quad \theta_{\mu\nu} = \eta_{\mu\nu} - \frac{\partial_\mu \partial_\nu}{\square}, \quad (A6)$$

while in the Lorentz-violating sector, we have the spin-2 operators,

$$(S_\tau)_{\alpha\beta,\gamma\delta} = \frac{1}{2}(\tau_{\alpha\gamma}S_{\beta\delta} + \tau_{\alpha\delta}S_{\beta\gamma} + \tau_{\beta\gamma}S_{\alpha\delta} + \tau_{\beta\delta}S_{\alpha\gamma}), \quad (\text{A7})$$

$$(S_\tau^2)_{\alpha\beta,\gamma\delta} = f(p) \left[\frac{1}{2}(\theta_{\alpha\gamma}\tau_{\beta\delta} + \theta_{\alpha\delta}\tau_{\beta\gamma} + \theta_{\beta\gamma}\tau_{\alpha\delta} + \theta_{\beta\delta}\tau_{\alpha\gamma}) - (\tau_{\alpha\gamma}\tau_{\beta\delta} + \tau_{\alpha\delta}\tau_{\beta\gamma}) \right], \quad (\text{A8})$$

$$(S_\sigma)_{\alpha\beta,\gamma\delta} = \frac{1}{2}(\sigma_{\alpha\gamma}S_{\beta\delta} + \sigma_{\alpha\delta}S_{\beta\gamma} + \sigma_{\beta\gamma}S_{\alpha\delta} + \sigma_{\beta\delta}S_{\alpha\gamma}), \quad (\text{A9})$$

$$(S_\sigma^2)_{\alpha\beta,\gamma\delta} = f(p)[\sigma_{\alpha\gamma}\sigma_{\beta\delta} + \sigma_{\alpha\delta}\sigma_{\beta\gamma} - (S_{\alpha\gamma}S_{\beta\delta} + S_{\alpha\delta}S_{\beta\gamma})], \quad (\text{A10})$$

with

$$\tau_{\alpha\beta} = \frac{1}{f(p)} \left[(v \cdot p)(v_\alpha p_\beta + v_\beta p_\alpha) - p^2 v_\alpha v_\beta - \frac{(v \cdot p)^2 p_\alpha p_\beta}{p^2} \right], \quad (\text{A11})$$

$$\sigma_{\alpha\beta} = \theta_{\alpha\beta} - \tau_{\alpha\beta}, \quad S_{\alpha\beta} = k^2 \varepsilon_{\alpha\beta\rho\phi} v^\rho p^\phi, \quad (\text{A12})$$

$$f(p) = k^4[(v \cdot p)^2 - v^2 p^2]; \quad p_\mu = -i\partial_\mu. \quad (\text{A13})$$

From the algebra,

$$(P^{(s)})_{IJ}(P^{(r)})^{JK} = \delta^{rs}(P^{(s)})_I^K; \quad S_\sigma P_{IJ}^{(s)} = \delta^{s2} S_\sigma; \quad S_\tau P_{IJ}^{(s)} = \delta^{s2} S_\tau \quad (\text{A14})$$

$$S_\sigma \cdot S_\tau = 0; \quad S_\sigma^3 = 4f S_\sigma; \quad S_\tau^3 = f S_\tau. \quad (\text{A15})$$

The reader can check that the inverse of the operator (A1) is [14]

$$G^{-1} = \frac{P_{SS}^{(2)}}{A} + \frac{P_{SS}^{(1)}}{B} + \frac{A_{WW}P_{SS}^{(0)} + A_{SS}P_{WW}^{(0)} - A_{SW}(P_{WS}^{(0)} + P_{SW}^{(0)})}{K^{(0)}} + \frac{a}{D_1} S_\tau + \frac{b}{D_2} S_\sigma + \frac{c(A + cf) - a^2}{AD_1} S_\tau^2 + \frac{d(A + 4df) - b^2}{AD_2} S_\sigma^2, \quad (\text{A16})$$

where f is given in (A13) and

$$D_1 = a^2 f - (A + cf)^2; \quad D_2 = 4b^2 f - (A + 4df)^2; \quad K^{(0)} = A_{WW}A_{SS} - A_{WS}^2. \quad (\text{A17})$$

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