Large N twisted partition functions in 3d-3d correspondence and holography

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We study the large N limit of twisted partition functions on $\mathcal{M}_{g,p}$, the S^1 bundle of degree p over a Riemann surface of genus g , for 3D $\mathcal{N} = 2$ superconformal field theories arising as low-energy limit of wrapped N M5-branes on hyperbolic 3-manifold M . We study contributions from two Bethe vacua which correspond to two canonical irreducible $SL(N, \mathbb{C})$ flat connections on M via 3D-3D correspondence. Using mathematical results on perturbative Chern-Simons invariants around the flat connections, we find universal expressions for the large N twisted partition functions contributed from the two Bethe vacua in term of the hyperbolic volume of M . The two large N partition functions perfectly match the on-shell actions for two Bolt-type solutions in the holographic dual AdS_4 gravity, respectively.

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I. INTRODUCTION AND SUMMARY

As a consistent theory of quantum gravity, string/M-theory is expected to provide microscopic understandings of quantum aspects of black holes (BHs). In a celebrated work [1], this hope was realized for extremal black holes in Minkowski spacetime, by reproducing the Bekenstein-Hawking entropy through counting D-brane bound states. Recently, this success was extended to black holes in asymptotically anti-de-Sitter spacetime. In particular, the entropy of magnetically charged supersymmetric black holes in AdS_4 supergravity can be explained using the holographic principle [2–11]. The AdS/CFT correspondence [12] says quantum gravity in asymptotically AdS_4 spacetime should be dual to a conformal field theory (CFT) on the 3-dimensional (3D) boundary. The field theories of interest have $\mathcal{N} \geq 2$ supersymmetry, and the black hole entropy on the gravity side turns out to be related to the so-called *topologically twisted indices* [13–16] in the dual field theory. They are the partition functions (ptns) on supersymmetric curved backgrounds $\mathcal{M}_{g,p=0} := \Sigma_g \times S^1$, with an appropriately chosen background magnetic flux coupled to R-symmetry current. The magnetic flux is turned on along the Riemann surface Σ_g of genus g . The twisted indices can be then computed using the supersymmetric

localization technique. Recently, the realm of localizable 3D manifolds has been further extended in [17], and we now have formulas for twisted partition functions $Z_{g,p}$, i.e., the ptn on degree- p S^1 -bundles $\mathcal{M}_{g,p}$ [see (4)] over Σ_g .

In this paper, we study holographic duality for a large class of 3D $\mathcal{N} = 2$ SCFTs $T_N[M]$, defined in (2) [see also (3)], arising from wrapped M5-branes on closed hyperbolic 3-manifolds M . The 3D theory is characterized by N , number of M5-branes, and the choice of a 3-manifold M . For each M , there is an associated AdS_4/CFT_3 correspondence. The holographic dual of the wrapped M5-brane 3D SCFT was studied in [18,19]. Since there are infinitely many such 3-manifolds [20], the wrapped M5-branes system provides a huge set of AdS_4/CFT_3 examples. We probe the holography using the twisted ptns. For $p = 0$, the ptn becomes a twisted index which counts ground states of M5-branes on $\Sigma_g \times M$. The counting is holographically dual to the microstates counting for a supersymmetric BH solution interpolating the asymptotic AdS_4 and its near-horizon limit $AdS_2 \times \Sigma_g$ [11].

The 3D-3D correspondence [21–23] provides a novel way of analyzing the 3D $\mathcal{N} = 2$ SCFTs. Schematically, the correspondence says

$$\begin{aligned} &(\text{supersymmetric ptns of } 3D T_N[M] \text{ theory}) \\ &= (SL(N, \mathbb{C}) \text{ Chern-Simons theory invariants on } M). \end{aligned} \quad (1)$$

Refer to [24–31] for more details on 3D-3D dictionary. Via the correspondence, some supersymmetric quantities of 3D $T_N[M]$ theory can be evaluated without relying on a field

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TABLE I. 3d-3d dictionaries for basic ingredients in twisted partition function computation. $S_{n=0,1}^\alpha$ are perturbative invariants of the complex Chern-Simons theory around a flat-connection $\mathcal{A}^\alpha$, see Eq. (15).

3D $T_N[M]$ theory	$SL(N, \mathbb{C})$ CS theory
Bethe vacuum α	Irreducible flat connection $\mathcal{A}^\alpha$
Handle gluing operator $\mathcal{H}^\alpha$	$\exp(-2S_1^\alpha)$
Fibering operator $\mathcal{F}^\alpha$	$\exp(iS_0^\alpha/(2\pi))$

theoretic description of the 3D SCFT. For example, as summarized in Table I, the twisted partition function $Z_{g,p}$ can be written in terms of basic perturbative invariants of the complex Chern-Simons (CS) theory. These invariants are mathematically well defined and have been extensively studied in math literature.

Combining the 3D-3D dictionary with mathematical results, we obtain the large N behavior of the twisted partition functions contributed from two distinguished Bethe vacua in the $T_N[M]$ theory. The Bethe vacua of our interest correspond to two irreducible $SL(N, \mathbb{C})$ flat connections on M in 3D-3D dual complex CS theory. These two flat connections also have a natural interpretation in terms of hyperbolic geometry, see Eq. (18). They also give global minimum and maximum of the absolute value of the fibering operator $\mathcal{F}$ appearing in the ptn computation (12), see (20). We confirm that the large N twisted ptns from the two Bethe-vacua nicely match the on-shell action for the two Bolt-type solutions [10] in the gravity dual respectively. The comparison is summarized in Table II, which is the main point of this paper.

II. 3D $T_N[M]$ THEORY

A large class of 3D $\mathcal{N} = 2$ SCFTs can be engineered through a twisted compactification of 6-dimensional SCFTs. They are labelled by the internal 3-manifolds M :

$T_N[M] :=$ (Effective 3D $\mathcal{N} = 2$ SCFT obtained from a twisted compactification of 6D $A_{N-1}(2, 0)$ theory on a 3-manifold M).

For simplicity, we assume M is a closed (compact) hyperbolic 3-manifold without boundary. To preserve supersymmetry, we perform a partial topological twisting along the

internal 3-manifold using $SO(3)$ vector subgroup of $SO(5)$ R-symmetry of the 6D theory. The twisting preserves a quarter of supersymmetries and the resulting 3D theory becomes a 3D $\mathcal{N} = 2$ SCFT with 4 supercharges. The 6D theory describes the low-energy effective world-volume theory of N coincident M5-branes in M-theory, and the 3D theory can be considered as an effective world-volume theory of N coincident M5-branes wrapped on the compact 3-manifold M , i.e.,

$$\begin{aligned} & N \text{ coincident M5-branes on } \mathbb{R}^3 \times M \\ & \text{in M-theory on } \mathbb{R}^3 \times (T^*M) \times \mathbb{R}^2 \\ & \xrightarrow{\text{IR world-volume theory of M5-branes}} T_N[M] \text{ on } \mathbb{R}^3. \end{aligned} \quad (3)$$

Here T^*M is the cotangent bundle of M , which is a local Calabi-Yau.

Let us comment on a subtle point in the setup. As emphasized in [32], in taking the twisted compactification we need to choose a connected subset of the vacuum moduli-space of the theory defined on $\mathbb{R}^3$, in order to have a genuine 3D SCFT. For a hyperbolic M , there is a natural choice (which is actually a single point) which is expected to become a discrete set of vacua when the theory is put on $\mathbb{R}^2 \times S^1$. This discrete set of vacua corresponds to a subset of irreducible $SL(N, \mathbb{C})$ flat connections on M . A field theoretic construction of the effective 3D gauge theory is proposed in [32], extending the beautiful construction in [23,33] for cusped 3-manifolds with at least one torus boundary component, by incorporating gauge theoretical operations corresponding to Dehn filling (removing torus boundaries) operations on 3-manifold.

III. $T_N[M]$ ON $\mathcal{M}_{g,p}$ FOR EVEN p

We now turn to the case where $T_N[M]$ is put on a large class of nontrivial backgrounds $\mathcal{M}_{g,p}$ [17]:

$$\begin{aligned} \mathcal{M}_{g,p} &:= (S^1\text{-bundle of degree } p \\ & \text{over a Riemann surface } \Sigma_g \text{ of genus } g), \\ \text{i.e. } S^1 \xrightarrow{p} \mathcal{M}_{g,p} &\rightarrow \Sigma_g. \end{aligned} \quad (4)$$

The metric can be written as

TABLE II. The M-theory is holographic dual to 3D $T_N[M]$ theory while the complex Chern-Simons theory is 3D-3D dual to the $T_N[M]$ theory. The 4d Newton constant G_4 is given in (11).

M-theory on $\text{AdS}_4 \times M \times \mathbb{S}^4$	$SL(N, \mathbb{C})$ CS theory on M
Bolt ₊ solution $I_{g,p}^{\text{Bolt}_+} = \frac{\pi(4(1-g)-p)}{8G_4}$	Flat connection $\mathcal{A}_N^{\text{geom}} F_{g,p}^{\text{geom}} = \frac{4(1-g)N^3 - pN^3}{12\pi} \text{vol}(M)$
Bolt ₋ solution $I_{g,p}^{\text{Bolt}_-} = \frac{\pi(4(1-g)+p)}{8G_4}$	Flat connection $\mathcal{A}_N^{\text{geom}} F_{g,p}^{\text{geom}} = \frac{4(1-g)N^3 + pN^3}{12\pi} \text{vol}(M)$

$$ds^2 = \beta^2(d\psi - pa(z, \bar{z}))^2 + 2g_{z\bar{z}}dzd\bar{z}, \quad (5)$$

where $z, \bar{z}$ are local coordinates on the Riemann surface and $\psi \sim \psi + 2\pi$ parametrizes the S^1 -fiber of length β . a is a 1-form on Σ_g whose curvature $F_a := da$ is normalized as

$$\frac{1}{2\pi} \int_{\Sigma_g} da = 1. \quad (6)$$

To preserve some supersymmetry, we turn on the following background gauge field coupled to $U(1)$ R-symmetry.

$$A^R = \beta\nu_R(d\psi - pa) + n_R(\pi^*a), \quad (7)$$

with proper quantization conditions for (ν_R, n_R) [10]. Here π^*a is a 1-form on $\mathcal{M}_{g,p}$ given as the pull-back of a using the projection map $\pi: \mathcal{M}_{g,p} \rightarrow \Sigma_g$.

For later comparison with the bolt-type solutions in the supergravity, we follow [10] and choose

$$\nu_R = \frac{1}{2}, \quad n_R = \frac{p}{2} + g - 1, \quad p \in 2\mathbb{Z}. \quad (8)$$

Throughout the paper, we restrict our attention to the choice in (8) and some formulas below may not work for other cases. For example, the large N computation in Table II give incorrect answer for the usual round S^3 case which is $\mathcal{M}_{g=0,p=1}$.

For small N the effective 3d theory $T_N[M]$ might witness emergent symmetries in addition to R-symmetry, as pointed out in [32,34]. When N is large enough, on the other hand, there is no accidental symmetry and the $U(1)$ R-symmetry in the IR should be simply inherited from the compact $SO(2)$ subgroup of $SO(5)$ R-symmetry in the 6D theory. It implies that the $U(1)$ R-charge, R , should be properly quantized

$$R(\mathcal{O}) \in \mathbb{Z}, \quad \text{for any state } \mathcal{O} \text{ of } T_N[M] \text{ on } \Sigma_g. \quad (9)$$

The Dirac quantization conditions for the $U(1)$ R-symmetry flux on Σ_g are

$$R(\mathcal{O}) \times n_R = R(\mathcal{O}) \times \left(\frac{p}{2} + g - 1 \right) \in \mathbb{Z},$$

for any state $\mathcal{O}$ of $T_N[M]$ on Σ_g . (10)

From (9), we see that the Dirac quantizations are always satisfied for even p . In summary, for large enough N we can put the 3D $T_N[M]$ theory on any $\mathcal{M}_{g \in \mathbb{Z}_{\geq 0}, p \in 2\mathbb{Z}}$ with supersymmetry preserving background gauge field, given in (7) and (8), coupled to the R-symmetry in the IR.

IV. HOLOGRAPHIC DUAL OF $T_N[M]$

The gravity dual description is given by the uplift of a certain magnetically charged AdS_4 solution in the maximally supersymmetric $D = 7$ gauged supergravity [18,19]. Schematically, the $D = 11$ solution is a product of AdS_4 , hyperbolic 3-manifold M , and a squashed 4-sphere $\tilde{S}^4$. Consistency of the truncation from $D = 11$ down to minimal $\mathcal{N} = 2$, $D = 4$ gauged supergravity is established in [35] and it is guaranteed that we may replace the AdS_4 part with any nontrivial $D = 4$ solution and we still have an exact $D = 11$ solution.

The computation of holographic free energy can be also first done in $D = 4$ setup, and substitute the Newton constant with [36]

$$G_4 = \frac{3\pi^2}{2N^3 \text{vol}(M)}. \quad (11)$$

Here, the hyperbolic volume is defined as

$$\begin{aligned} \text{vol}(M) &= (\text{hyperbolic volume of } M) \\ &:= (\text{volume measured in the unique hyperbolic metric}). \end{aligned}$$

The hyperbolic metric is normalized as $R_{\mu\nu} = -2g_{\mu\nu}$. Mostow's rigidity theorem [37] guarantees the uniqueness of the hyperbolic metric and thus the volume is actually a topological invariant.

As gravity duals of the boundary theory put on $\mathcal{M}_{g,p}$, we utilize the supersymmetric AdS-Taub-NUT and bolt solutions constructed in [38]. Since these solutions have nonvanishing Maxwell field, which in $D = 11$ uplift appears as a twisting of the R-symmetry angle in $\tilde{S}^4$, one might worry about a conflict with the quantization condition g, p . But it turns out, since the R-symmetry angle is part of $\tilde{S}^4$ and we have a standard periodicity of 2π , the regularity condition for $D = 4$ NUT/Bolt is enough. This is in line with the field theory side discussion, in particular (9). A comment is in order here, in comparison with the uplifts involving Sasaki-Einstein 7-manifolds. In that case, the periodicity of the R-symmetry angle from the regularity of Bolt solution should be compatible with the periodicity condition due to collapsing cycles in the Kähler-Einstein base manifold of the Sasaki-Einstein space. The readers are referred to [10] for more details, where the authors considered an explicit example of Sasaki-Einstein manifolds such as $V^{5,2} = SO(5)/SO(3)$.

V. TWISTED PARTITION FUNCTIONS OF $T_N[M]$ IN 3D-3D CORRESPONDENCE

The twisted partition function $Z_{g,p}$ on the $\mathcal{M}_{g,p}$ for general 3D $\mathcal{N} = 2$ SCFTs is given as the following finite sum [17,29]

$$Z_{g,p} = \sum_{\alpha} Z_{g,p}^{\alpha} := \sum_{\alpha} (\mathcal{H}^{\alpha})^{g-1} (\mathcal{F}^{\alpha})^p. \quad (12)$$

Here α labels the so-called Bethe vacua [39] of the 3D theory. It is obtained by extremizing the effective 2d twisted superpotential in the compactification on $\mathbb{R}^2 \times S^1$. The number of vacua is equal to the Witten index [40,41] of the 3D SCFT. $\mathcal{H}$ and $\mathcal{F}$ are called *handle-gluing* and *fibering* operators respectively. The explicit forms of $\mathcal{H}$ and $\mathcal{F}$ for any given ultraviolet (UV) Lagrangian are available in [17]. Let us emphasize that, the formula (12) applied to the case of S^3 partition function which corresponds to $(g, p) = (0, 1)$ is apparently *different* from the more familiar Coulomb branch integral expression [42]. But their equivalence is illustrated for a number of examples in [17].

Now let us specialize to the $T_N[M]$ theories in (2). The twisted ptns for these theories can be analyzed using the 3D-3D dictionaries summarized in Table I. Twisted ptns in 3D-3D correspondence were studied in [29,30]. In the table, the $\{S_n^{\alpha}\}_{n=0}^{\infty}$ represent terms in the loop expansion of the complex CS partition function around a flat-connection $\mathcal{A}^{\alpha}$ [43–46]:

$$Z_{\text{CS pert}}^{\alpha} := \int \frac{D(\delta\mathcal{A})}{(\text{gauge})} e^{-\frac{1}{2\hbar} \text{CS}[\mathcal{A}^{\alpha} + \delta\mathcal{A}; M]} \xrightarrow{\text{as } \hbar \text{ goes to } 0} \exp\left(\frac{1}{\hbar} S_0^{\alpha} + S_1^{\alpha} + \cdots + \hbar^{n-1} S_n^{\alpha} + \dots\right). \quad (13)$$

The Chern-Simons functional is

$$\text{CS}[\mathcal{A}, M] := \int_M \text{Tr}\left(\mathcal{A} \wedge d\mathcal{A} + \frac{2}{3} \mathcal{A}^3\right). \quad (14)$$

Note that the counterpart of $\mathcal{F}$ and $\mathcal{H}$ are simply *tree level* and *one-loop* contributions in perturbation theory. More explicitly, the perturbative coefficients are given as

$$S_0^{\alpha} = -\frac{1}{2} \text{CS}[\mathcal{A}^{\alpha}, M], \\ S_1^{\alpha} := \frac{1}{2} \log \text{Tor}_{R=\text{adjoint}}[\mathcal{A}^{\alpha}, M]. \quad (15)$$

$\text{Tor}_R[\mathcal{A}^{\alpha}, M]$ is the Ray-Singer torsion of an associated vector bundle in a representation $R \in \text{Hom}[SL(N, \mathbb{C}) \rightarrow GL(V_R)]$ twisted by a flat connection $\mathcal{A}^{\alpha}$. Here V_R is the vector space for representation R and $GL(V_R)$ is the general linear group on the V_R . The analytic torsion is defined as follows [45,47,48]

$$\text{Tor}_R[\mathcal{A}^{\alpha}, M] := \frac{[\det' \Delta_0(R, \mathcal{A}^{\alpha})]^{3/2}}{[\det' \Delta_1(R, \mathcal{A}^{\alpha})]^{1/2}}. \quad (16)$$

Here $\Delta_n(R, \mathcal{A}^{\alpha})$ is a Laplacian acting on V_R -valued n -form twisted by a flat connection $\mathcal{A}^{\alpha}$. $\det' \Delta$ denotes the zeta

function regularized determinant of the Laplacian Δ . For the one-loop part, the denominator comes from gauge field fluctuations $\delta\mathcal{A}$ while the numerator comes from the ghosts associated to a gauge fixing [49].

The 3D-3D dictionary in Table I can be derived combining several known results in literatures. The Bethe-vacua (vacua on $\mathbb{R}^2 \times S^1$) of 3D $T_N[M]$ theory are in one-to-one correspondence to a subset of irreducible flat-connections on M [21,32]. According to a dictionary of 3D-3D relation, the asymptotic expansion $Z_{\text{CS pert}}^{\alpha}$ in (14) is equal to the perturbative expansion of holomorphic block $B^{\alpha}(q)$ [21,27] associated to the Bethe-vacuum α in the limit $q \rightarrow 1$,

$$Z_{\text{CS pert}}^{\alpha}(\hbar) \simeq B^{\alpha}(q := e^{\hbar}), \quad \text{as an asymptotic expansion in } \hbar \rightarrow 0. \quad (17)$$

For general 3D $\mathcal{N} = 2$ theory, the asymptotic expansion coefficients S_0 and S_1 of holomorphic block are related to the operators $\mathcal{F}$ and $\mathcal{H}$ as given in Table I [50,51].

VI. LARGE N TWISTED PARTITION FUNCTIONS AROUND TWO BETHE-VACUA AND ITS HOLOGRAPHIC DUAL

For every hyperbolic 3-manifold M , there are two characteristic irreducible $SL(N, \mathbb{C})$ flat connections $\mathcal{A}_N^{\text{geom}}$ and $\overline{\mathcal{A}}_N^{\text{geom}}$ which can be constructed from the hyperbolic structure on M ,

$$\mathcal{A}_N^{\text{geom}} := \rho_N \cdot (\omega + ie), \quad \overline{\mathcal{A}}_N^{\text{geom}} := \rho_N \cdot (\omega - ie). \quad (18)$$

Here ω and e are respectively the spin-connection and vielbein of the unique hyperbolic metric on M . They are both locally $so(3)$ -valued 1-forms and the complex combinations $\omega \pm ie$ form an $SL(2, \mathbb{C})$ flat-connections on M . ρ_N is the N -dimensional irreducible representation of $sl(2, \mathbb{C}) = su(2)_{\mathbb{C}}$, and obviously $\rho_N \cdot (\omega \pm ie)$ become also irreducible $SL(N, \mathbb{C})$ flat connections. A crucial property of these two flat-connections is that they take the minimum (maximum) value of $\text{Im}[S_0]$ among all $SL(N, \mathbb{C})$ flat connections. Namely,

$$\text{Im}[S_0^{\overline{\text{geom}}}] < \text{Im}[S_0^{\alpha}] < \text{Im}[S_0^{\text{geom}}], \quad (19)$$

for any flat connection $\mathcal{A}^{\alpha}$ which is neither $\overline{\mathcal{A}}^{\text{geom}}$ nor $\mathcal{A}^{\text{geom}}$. Combined with the 3D-3D dictionary in Table I, it is implied

$$|\mathcal{F}^{\text{geom}}| < |\mathcal{F}^{\alpha}| < |\mathcal{F}^{\overline{\text{geom}}}|, \quad (20)$$

for any Bethe-vacuum α which is neither $(\overline{\text{geom}})$ nor (geom) .

Classical actions S_0^{α} for the two connections above can be computed as follows

$$\begin{aligned}
\text{Im}[S_0^{\text{geom}}] &= -\frac{1}{2} \text{Im}[CS(\mathcal{A}_N^{\text{geom}})] = -\frac{1}{2} \text{Im}[CS(\rho_N \cdot (\omega + ie))] \\
&= -\frac{1}{2} \frac{\text{Tr}[\rho_N \cdot (T^a) \rho_N \cdot (T^b)]}{\text{Tr}[T^a T^b]} \text{Im}[CS(\omega + ie)] \\
&= -\frac{1}{2} \frac{N^3 - N}{6} \text{Im}[CS(\omega + ie)] = \frac{N^3 - N}{6} \text{vol}(M), \quad \text{and} \\
\text{Im}[S_0^{\overline{\text{geom}}}] &= -\text{Im}[S_0^{\text{geom}}] = -\frac{N^3 - N}{6} \text{vol}(M). \tag{21}
\end{aligned}$$

In the second line, $T^a (a = 1, 2, 3)$ are Pauli matrices and $\rho_N \cdot (T^a)$ are generators in the N -dimensional irreducible representation. From a simple group theoretical fact

$$\frac{\text{Tr}[\rho_N \cdot (T^a) \rho_N \cdot (T^b)]}{\text{Tr}[T^a T^b]} = \frac{N^3 - N}{6}, \tag{22}$$

the expected N^3 -scaling of $T_N[M]$ theory follows. In the third line of (21), we use the fact that the imaginary part of Chern-Simons functional of $\mathcal{A} = \omega + ie$ is equal to the Einstein-Hilbert action with unit negative cosmology constant up to an overall numerical factor [52]. The action for the unique hyperbolic metric is twice of the hyperbolic volume of 3-manifold with a minus sign.

The large N asymptotic behavior of the 1-loop coefficients, S_1^{geom} and $S_1^{\overline{\text{geom}}}$, can be analyzed using a following mathematical theorem [53],

$$\begin{aligned}
&\log |\text{Tor}_{R=\rho_{2m+1}}[\mathcal{A}_{N=2}^{\text{geom}}, M]| \\
&\xrightarrow{\text{as } m \text{ goes to } \infty} -\frac{1}{\pi} m^2 \text{vol}(M) + o(m). \tag{23}
\end{aligned}$$

Here ρ_{2m+1} is the $(2m+1)$ -dimensional irreducible representation of $sl(2, \mathbb{C}) = su(2)_{\mathbb{C}}$. Combining the theorem with the following branching rule,

$$(\text{adjoint of } sl(N, \mathbb{C})) = \bigoplus_{m=1}^{N-1} \rho_{2m+1} \text{ of } sl(2, \mathbb{C}),$$

when the $sl(2, \mathbb{C})$ is embedded into $sl(N, \mathbb{C})$ via ρ_N , \tag{24}

we have following large N behavior of the 1-loop coefficients

$$\begin{aligned}
\text{Re}[S_1^{\text{geom}}] &= \frac{1}{2} \log |\text{Tor}_{R=\text{adjoint}}[\mathcal{A}_N^{\text{geom}}, M]| \\
&= \frac{1}{2} \sum_{m=1}^{N-1} \log |\text{Tor}_{R=\rho_{2m+1}}[\mathcal{A}_{N=2}^{\text{geom}}, M]| \\
&= -\frac{1}{2\pi} \text{vol}(M) \sum_{m=1}^{N-1} m^2 + o(m) = -\frac{N^3 + o(N^2)}{6\pi} \text{vol}(M), \\
&\text{and} \\
\text{Re}[S_1^{\overline{\text{geom}}}] &= \text{Re}[S_1^{\text{geom}}] = -\frac{N^3 + o(N^2)}{6\pi} \text{vol}(M). \tag{25}
\end{aligned}$$

Combining the 3D-3D dictionaries in Table I with the large N analysis in (21) and (25), we finally obtain the following universal large N behavior of the twisted ptns (12)

$$\begin{aligned}
F_{g,p}^{\text{geom}} &:= -\log |Z_{g,p}^{\text{geom}}(T_N[M])| \\
&= \frac{4(1-g)N^3 + pN^3}{12\pi} \text{vol}(M) + o(N^2), \\
F_{g,p}^{\overline{\text{geom}}} &:= -\log |Z_{g,p}^{\overline{\text{geom}}}(T_N[M])| \\
&= \frac{4(1-g)N^3 - pN^3}{12\pi} \text{vol}(M) + o(N^2). \tag{26}
\end{aligned}$$

They nicely match the on-shell actions $I_{g,p}^{\text{Bolt}_{\pm}}$ of $\text{Bolt}_{\pm}$ solution in [10]. The large N computations are summarized in Table II.

VII. CONCLUSION

In this paper, we probe a large class of $\text{AdS}_4/\text{CFT}_3$ associated to M5-branes wrapped on 3-manifolds by computing large N twisted ptns. For M2 and D2-branes and their Chern-Simons-matter theories, the large N computations have been performed already in [2–11].

A nicer feature of our analysis is that we map the large N analysis to a mathematical problem via 3D-3D correspondence which can be solved from known mathematical results, such as (19), (21) and (25). The results hold for any closed hyperbolic 3-manifold M and one does not need to perform the large N analysis for individual $\text{AdS}_4/\text{CFT}_3$ model associated to each M . We hope that the improvement made in our analysis may provide a better way of understanding the subleading corrections to the large N twisted ptns which might be related to quantum corrections to the Bekenstein-Hawking entropy.

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