Testing scalar versus vector dark matter

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We investigate and compare two simple models of dark matter (DM): a vector and a scalar DM model. Both models require the presence of two physical Higgs bosons h_1 and h_2 which come from mixed components of the standard Higgs doublet H and a complex singlet S . In the vector model, the extra $U(1)$ symmetry is spontaneously broken by the vacuum of the complex field S . This leads to a massive gauge boson X^μ that is a DM candidate stabilized by the dark charge conjugation symmetry $S \rightarrow S^*$, $X^\mu \rightarrow -X^\mu$. On the other hand, in the scalar model the gauge group remains the standard one. The DM field A is the imaginary component of S and the stabilizing symmetry is also the dark charge conjugation $S \rightarrow S^*$ ($A \rightarrow -A$). In this case, in order to avoid spontaneous breaking, the $U(1)$ symmetry is broken explicitly, but softly, in the scalar potential. The possibility to disentangle the two models has been investigated. We have analyzed collider, cosmological, DM direct and indirect detection constraints and shown that there are regions in the space spanned by the mass of the nonstandard Higgs boson and the mass of the DM particle where the experimental bounds exclude one of the models. We have also considered possibility to disentangle the models at e^+e^- collider and concluded that the process $e^+e^- \rightarrow Z + \text{DM}$ provides a useful tool to distinguish the models.

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I. INTRODUCTION

The Higgs boson was discovered at CERN's Large Hadron Collider (LHC) by the ATLAS [1] and CMS [2] collaborations thus turning one important page in our knowledge of the Universe by not only discovering a new particle but also to hint very strongly that there is a mechanism of spontaneous symmetry breaking giving mass to both gauge bosons and fermions. Over the years, it has become increasingly clear that this boson resembles very much the one predicted by the standard model (SM). However, there are still many unsolved problems in particle physics that are not answered by the SM. One of them is the existence of the dark matter (DM) in the universe which presence cannot be attributed to any known particles.

Although the measurements of the Higgs couplings are quite demanding for the so-called beyond the standard model (BSM) models, there is still plenty of space in the present results to include new physics. Some of the BSM models can be compatible with the measurements while providing solutions to some of the outstanding questions of particle physics. Such is the case of the models studied in this work. Both the extension with a complex singlet [3–8] and the extension with a new Abelian vector boson together with a complex singlet [9–14] provide DM candidates still compatible with collider bounds, and direct or indirect detection experiments. The models can also undergo a strong first-order phase transition during the era of EWSB [7,15–23] thus explaining electroweak baryogenesis.

Extra scalar singlets are dimension one fields and therefore prone to couple to the SM scalar sector in a renormalizable way without any suppression by inverse powers of the scale of BSM, a concept introduced in [24] and known as the *Higgs portal*. Assuming the scale of new physics is the GUT or the Planck scale we are at present bound to work with minimal theories that are valid up to high energy scales. This theory has in particular to be stable under the renormalization group evolution (RGE) which is an issue already in the SM. The measurement of the Higgs and the top-quark masses show that the SM is either in a marginally stable or in a metastable region of the parameter space [25,26]. However, as shown at two-loop level in

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[14,27], these models not only provide a DM candidate but they also improve the stability of the SM and present a possibility to solve the baryon asymmetry problem.

In this article we explore possibilities of distinguishing the scalar and the vector DM (VDM) models. The minimal VDM requires an extra $U(1)$ gauge symmetry that is spontaneously broken by a vacuum expectation value (vev) of a complex scalar neutral field under the SM symmetries but charged under the extra $U(1)$. This model bears many similarities with a model of scalar DM (SDM) which is a component of an extra complex scalar field (that develops a vev) which is added to the SM. In both cases there are two scalar physical Higgs bosons $h_{1,2}$ that mix in the scalar mass matrix with a mixing angle α . So the goal of this paper is to investigate if those two models could be distinguished. This is a very pragmatic task, both models are attractive candidates for simple DM theories, therefore it is worth knowing if there are observables which can distinguish them.

Using the SCANNERS program [28] we impose the most relevant bounds: theoretical, collider experiment bounds, precision electroweak physics, DM direct and indirect detection experiments, and DM relic density. The parameter space of each model is scanned with all the above constraints providing the regions of the parameter space where the models can indeed be distinguished. Whenever possible these results are presented in terms of physical observables that can be measured at the LHC. Finally we present a direct way to distinguishing the models by looking at the energy distribution in Higgs associated production, with the Higgs decaying to DM, at a future electron-positron collider.

The paper is organized as follows. In Sec. II we present the complex singlet extension of the SM, reviewing its main properties and setting notation. In Secs. II A and II B we discuss the scattering of scalar DM off nuclei and invisible SM-like Higgs boson decays, respectively. In Sec. III we set the review of most relevant aspects of the vector DM model. In Secs. III A and III B constraints from DM direct detection and invisible decays of SM-like Higgs boson are formulated, respectively. In Sec. IV we present a discussion of the possibility to distinguish the models at a future electron-positron collider. The results of the scan showing the allowed parameter space for each model are presented in Sec. VI. In the conclusions, Sec. VII, we summarize our findings. Technical details concerning Goldstone Boson couplings to Higgs bosons are left to the Appendices.

II. SCALAR DARK MATTER

Gauge singlet scalars as candidates for DM were first proposed in [3,4] and then discussed by many authors. Even though the minimal model of scalar DM assumes merely an addition of a real scalar field odd under a $\mathbb{Z}_2$ symmetry, here we are going to consider a model that

requires an extension by a complex scalar field S . The motivation is to compare the VDM with a SDM that are in some sense similar. In order to stabilize a component of S we require an invariance under DM charge conjugation $C: S \rightarrow S^*$, which guarantees stability of the imaginary part of S , $A \equiv \text{Im}S/\sqrt{2}$. The real part, $\phi_S \equiv \text{Re}S/\sqrt{2}$, is going to develop a real vacuum expectation value $\langle \phi_S \rangle = \langle S \rangle = v_S/\sqrt{2}$.¹ Therefore ϕ_S will mix with the neutral component of the SM Higgs doublet H , in exactly the same manner as it happens for the VDM. In order to simplify the potential we impose in addition a $\mathbb{Z}_2$ symmetry $S \rightarrow -S$, which eliminates odd powers of S . Eventually the scalar potential reads:

$$V = -\mu_H^2 |H|^2 + \lambda_H |H|^4 - \mu_S^2 |S|^2 + \lambda_S |S|^4 + \kappa |S|^2 |H|^2 + \mu^2 (S^2 + S^{*2}) \quad (2.1)$$

with μ^2 real, as implied by the C symmetry. Note that the μ^2 term breaks the $U(1)$ explicitly, so the pseudo-Goldstone boson, A is massive. In the limit of exact symmetry, A would be just a genuine, massless Goldstone boson. Since the symmetry-breaking operator $\mu^2 (S^2 + S^{*2})$ is of dimension less than 4, its presence does not jeopardize renormalizability even if noninvariant higher dimension operators were not introduced, see for instance [29]. Note that dimension 3 terms are disallowed by the $\mathbb{Z}_2$'s and gauge symmetries. In other words, we can limit ourselves to dimension 2 $U(1)$ breaking terms preserving the renormalizability of the model. The freedom to introduce solely the soft breaking operators offers a very efficient and economical way to generate mass for the pseudoscalar A without the necessity to introduce dimension 4 terms like S^4 or $|S|^2 S^2$ and keeping the renormalizability of the model. It is also worth noticing that the $\mathbb{Z}_2$ symmetry $S \rightarrow -S$ is broken spontaneously by v_S and therefore ϕ_S , the real part of S , is not stable, making A the only DM candidate.

The requirement of asymptotic positivity of the potential implies the following constraints that we impose in all further discussions:

$$\lambda_H > 0, \quad \lambda_S > 0, \quad \kappa > -2\sqrt{\lambda_H \lambda_S}. \quad (2.2)$$

Hereafter the above conditions will be referred to as the positivity or stability conditions.

The scalar fields can be expanded around the corresponding generic vev's as follows

¹This is a choice that fixes the freedom (phase rotation of the complex scalar) of choosing a weak basis that could be adopted to formulate the model. The model is defined by symmetries imposed in this particular basis in which the scalar vacuum expectation value is real.

$$S = \frac{1}{\sqrt{2}}(v_S + iv_A + \phi_S + iA),$$

$$H^0 = \frac{1}{\sqrt{2}}(v + \phi_H + i\sigma_H) \quad \text{where } H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix}, \quad (2.3)$$

where we have temporarily allowed $\langle S \rangle$ to be complex.

Locations of extrema of the potential (2.1), corresponding values of the potential and corresponding curvatures in the basis (ϕ_H, ϕ_S, A) are as follows

v1:

$$v^2 = \frac{4\lambda_S\mu_H^2 - 2\kappa(\mu_S^2 - 2\mu^2)}{4\lambda_H\lambda_S - \kappa^2},$$

$$v_S^2 = \frac{4\lambda_H(\mu_S^2 - 2\mu^2) - 2\kappa\mu_H^2}{4\lambda_H\lambda_S - \kappa^2}, \quad v_A^2 = 0 \quad (2.4)$$

$$V_1 = \frac{-1}{4\lambda_H\lambda_S - \kappa^2} \{ \lambda_H(\mu_S^2 - 2\mu^2)^2 + \mu_H^2[\lambda_S\mu_H^2 - \kappa(\mu_S^2 - 2\mu^2)] \} \quad (2.5)$$

$$\mathcal{M}^2 = \begin{pmatrix} 2\lambda_H v^2 & \kappa v v_S & 0 \\ \kappa v v_S & 2\lambda_S v_S^2 & 0 \\ 0 & 0 & -4\mu^2 \end{pmatrix}, \quad (2.6)$$

v2:

$$v^2 = \frac{4\lambda_S\mu_H^2 - 2\kappa(\mu_S^2 + 2\mu^2)}{4\lambda_H\lambda_S - \kappa^2}, \quad v_S^2 = 0,$$

$$v_A^2 = \frac{4\lambda_H(\mu_S^2 + 2\mu^2) - 2\kappa\mu_H^2}{4\lambda_H\lambda_S - \kappa^2}, \quad (2.7)$$

$$V_2 = \frac{-1}{4\lambda_H\lambda_S - \kappa^2} \{ \lambda_H(\mu_S^2 + 2\mu^2)^2 + \mu_H^2[\lambda_S\mu_H^2 - \kappa(\mu_S^2 + 2\mu^2)] \} \quad (2.8)$$

$$\mathcal{M}^2 = \begin{pmatrix} 2\lambda_H v^2 & 0 & \kappa v v_S \\ 0 & 4\mu^2 & 0 \\ \kappa v v_S & 0 & 2\lambda_S v_S^2 \end{pmatrix}, \quad (2.9)$$

v3:

$$v^2 = \frac{\mu_H^2}{\lambda_H}, \quad v_S^2 = 0, \quad v_A^2 = 0, \quad (2.10)$$

$$V_3 = -\frac{\mu_H^4}{4\lambda_H} \quad (2.11)$$

$$\mathcal{M}^2 = \begin{pmatrix} 2\mu_H^2 & 0 & 0 \\ 0 & 2\mu^2 + \frac{\kappa\mu_H^2}{2\lambda_H} - \mu_S^2 & 0 \\ 0 & 0 & -2\mu^2 + \frac{\kappa\mu_H^2}{2\lambda_H} - \mu_S^2 \end{pmatrix}, \quad (2.12)$$

v4:

$$v^2 = 0, \quad v_S^2 = \frac{\mu_S^2 - 2\mu^2}{\lambda_S}, \quad v_A^2 = 0, \quad (2.13)$$

$$V_4 = -\frac{(\mu_S^2 - 2\mu^2)^2}{4\lambda_S} \quad (2.14)$$

v5:

$$v^2 = 0, \quad v_S^2 = 0, \quad v_A^2 = \frac{\mu_S^2 + 2\mu^2}{\lambda_S}, \quad (2.15)$$

$$V_5 = -\frac{(\mu_S^2 + 2\mu^2)^2}{4\lambda_S} \quad (2.16)$$

Note that $v_S \neq 0$ and $v_A \neq 0$ may happen only if $\mu^2 = 0$. Since nonzero μ^2 is essential to avoid the appearance of a Goldstone boson, we do not consider those points any more.

Forcing the vacuum v1 to be the global minimum implies that we have to assume $\lambda_H > 0$, $4\lambda_H\lambda_S - \kappa^2 > 0$ and $\mu^2 < 0$. Then for consistency we enforce the conditions

$$2\lambda_S\mu_H^2 > \kappa(\mu_S^2 - 2\mu^2) \quad \text{and} \quad 2\lambda_H(\mu_S^2 - 2\mu^2) > \kappa\mu_H^2 \quad (2.17)$$

It turns out that $V_1 < V_4$ for any choice of parameters, while $V_4 < V_5$ for $\mu^2 < 0$. From (2.17) one can find that the vacuum v3 is never a minimum. Obviously, v2 is not a minimum either for $\mu^2 < 0$. Therefore we conclude that for $\mu^2 < 0$ the vacuum v1 is the global minimum. Note that in this case A is indeed a pseudo-Goldstone boson and its mass vanishes in the limit of exact global $U(1)$ as it was discussed and anticipated below (2.1). The presence of the $U(1)$ breaking term $\mu^2(S^2 + S^{*2})$ implies a trivial shift of the $\mu_S^2 \rightarrow \mu_S^2 - 2\mu^2$ and an addition of the Goldstone boson mass $-4\mu^2$. In fact, an equivalent $U(1)$ breaking would be to add just the Goldstone boson mass without the trivial shift by replacing $\mu^2(S^2 + S^{*2})$ by $\mu^2(S - S^*)^2$.

Similar models have been considered in a more general context including a possibility of fast first order phase transition in [7,20,30]. In the VDM that we consider here, A becomes a longitudinal component of the massive DM vector X , but it remains an independent degree of freedom.

TABLE I. Vertices relevant for the calculation of annihilation cross section in the scalar DM model.

$-i\frac{m_i^2}{v_S}R_{2i}$	$i\frac{2M_Z^2}{v}R_{1i}$	$i\frac{2M_{W^-}^2}{v}R_{1i}$	$i\frac{M_F}{v}R_{1i}$	$-i(R_{2i}R_{2j}\lambda_S - R_{1i}R_{1j}\kappa)$
	$i[\kappa v(R_{1i}R_{2j}R_{2k} + R_{2i}R_{1j}R_{2k} + R_{2i}R_{2j}R_{1k}) + \kappa v_S(R_{2i}R_{1j}R_{1k} + R_{1i}R_{2j}R_{1k} + R_{1i}R_{1j}R_{2k}) + 6\lambda v(R_{1i}R_{1j}R_{1k}) + 6\lambda_S v_S(R_{2i}R_{2j}R_{2k})]$			

There are two mass eigenstates, h_1 and h_2 , in this model. The mass matrix (2.6) can be diagonalized by the orthogonal rotation matrix R acting on the space spanned by the two CP -even scalars ϕ_H and ϕ_S :

$$\begin{pmatrix} \phi_H \\ \phi_S \end{pmatrix} = \begin{pmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{pmatrix} \begin{pmatrix} h_1 \\ h_2 \end{pmatrix}. \quad (2.18)$$

We assume hereafter that h_1 is the 125 GeV boson observed at the LHC.

Note that the third spin-zero state A does not mix with the former ones as the $\mathbb{Z}_2$ dark symmetry remains unbroken by the real vev. We choose as independent parameters of the model the set: v_S , $\sin \alpha$, m_2 , and m_A , while the parameters of the potential can be written as functions of this independent set and $v = 246.22$ GeV and $m_1 = 125.09$ GeV as follows:

$$\begin{aligned} \kappa &= \frac{\sin 2\alpha(m_1^2 - m_2^2)}{2vv_S}, & \lambda_S &= \frac{\cos \alpha^2 m_2^2 + \sin \alpha^2 m_1^2}{2v_S^2}, \\ \lambda_H &= \frac{\cos \alpha^2 m_1^2 + \sin \alpha^2 m_2^2}{2v^2}. \end{aligned} \quad (2.19)$$

The vertices relevant for the calculation of annihilation cross section in the scalar DM model have been collected in Table I.

A. Dark matter direct detections

It is interesting to note that the DM direct detection signals are naturally suppressed in the scalar DM model. It turns out that in the limit of zero DM velocity the tree-level amplitude for DM-nucleon scattering vanishes. The most relevant interaction term in this context is the AAh_i vertex. From the potential Eq. (2.1), one can easily derive the following DM triple-scalar couplings:

$$\begin{aligned} V &\supset \frac{A^2}{2}(2\lambda_S v_S \phi_S + \kappa v \phi_H) \\ &= \frac{A^2}{2v_S}(\sin \alpha m_1^2 h_1 + \cos \alpha m_2^2 h_2), \end{aligned} \quad (2.20)$$

where we have used the relations Eqs. (2.18) and (2.19), and the corresponding Feynman rules are presented in Table I. With these interaction terms, we can write down the corresponding amplitude for the spin-independent DM nuclear recoils as follows:

$$\begin{aligned} i\mathcal{M} &= -i \frac{\sin 2\alpha f_N m_N}{2vv_S} \left(\frac{m_1^2}{q^2 - m_1^2} - \frac{m_2^2}{q^2 - m_2^2} \right) \bar{u}_N(p_4) u_N(p_2) \\ &\approx -i \frac{\sin 2\alpha f_N m_N}{2vv_S} \left(\frac{m_1^2 - m_2^2}{m_1^2 m_2^2} \right) q^2 \bar{u}_N(p_4) u_N(p_2), \end{aligned} \quad (2.21)$$

where q^2 represents the DM momentum transfer when it scatters with nucleons, and m_N and $f_N \approx 0.3$ denote the nucleon mass and its coupling to the SM Higgs. In the limit of zero momentum transfer, $q^2 \rightarrow 0$, the above amplitude vanishes. This behavior is a consequence of the fact that the Goldstone-Higgs coupling is proportional the Higgs mass squared. In the Appendices we explain in a more general context when are the coupling of the form of (2.20), i.e., $\propto m_i^2$. It is interesting to note [31] that a similar cancellation also exists in a DM model with a vector gauge boson mediator which communicates with the SM sector only through kinetic mixings with the SM neutral gauge bosons. As shown in Ref. [31], it is even more remarkable that the cancellation in the vector mediator case does not demand to choose some specific soft breaking terms in the scalar potential, as in the present SDM.

It is shown in Ref. [32] that the leading-order DM-nuclear recoil cross section arises at one-loop order, which is estimated as follows by assuming the one-loop functions to be of $\mathcal{O}(1)$

$$\sigma_{AN} \approx \begin{cases} \frac{\sin^2 \alpha}{64\pi^2} \frac{m_N^4 f_N^2}{m_1^4 v^2} \frac{m_A^8}{m_A^2 v_S^6}, & m_A \geq m_2 \\ \frac{\sin^2 \alpha}{64\pi^2} \frac{m_N^4 f_N^2}{m_1^4 v^2} \frac{m_A^2 m_A^2}{v_S^6}, & m_A < m_2 \end{cases}. \quad (2.22)$$

The above result is a conservative estimate of the upper limit for the one-loop A -nucleon scattering cross section.

It turns out to be of $\mathcal{O}(10^{-49} \text{ cm}^2)$ for $\sin \alpha = 0.1$, $m_2 = 300 \text{ GeV}$ and $m_A \sim 1 \text{ TeV}$, which is much lower than the current XENON1T limits of $\mathcal{O}(10^{-47} \text{ cm}^2)$. Therefore, we expect that the DM direct searches will not impose any relevant constraints on the scalar DM model. In the following, we will use Eq. (2.22) to perform the scan which indeed confirms this expectation.

B. Higgs-boson invisible decays: $h_1 \rightarrow AA$

One strong constraint for DM models comes from invisible decays of the SM-like Higgs boson, the corresponding branching ratio should be less than 24% [33]. In the present scalar DM model with $m_A < m_1/2$, the SM Higgs boson decays invisibly into the stable pseudoscalar DM A , $h_1 \rightarrow AA$, with the decay width given by

$$\Gamma(h_1 \rightarrow AA) = \frac{1}{32\pi} \frac{m_1^2 \sin^2 \alpha}{v_S^2} \sqrt{m_1^2 - 4m_A^2}. \quad (2.23)$$

III. VECTOR DARK MATTER

The model that we want to compare with the SDM is the popular vector DM (VDM) model [9–14] that is an extension of the SM by an additional $U(1)_X$ gauge symmetry and a complex scalar field S , whose vev generates a mass for this $U(1)$'s vector field. The quantum numbers of the scalar field are

$$S = (0, \mathbf{1}, \mathbf{1}, 1) \text{ under } U(1)_Y \times SU(2)_L \times SU(3)_c \times U(1)_X. \quad (3.1)$$

None of the SM fields are charged under the extra gauge group. In order to ensure stability of the new vector boson a $\mathbb{Z}_2$ symmetry is assumed to forbid $U(1)$ -kinetic mixing between $U(1)_X$ and $U(1)_Y$. The extra gauge boson A_μ and the scalar field S transform under the $\mathbb{Z}_2$ as follows

$$A_X^\mu \rightarrow -A_X^\mu, \quad S \rightarrow S^*, \text{ where } S = \phi e^{i\sigma}, \\ \text{so } \phi \rightarrow \phi, \quad \sigma \rightarrow -\sigma. \quad (3.2)$$

All other fields are neutral under the $\mathbb{Z}_2$.

At leading order the vector bosons masses are given by:

$$M_W = \frac{1}{2}gv, \quad M_Z = \frac{1}{2}\sqrt{g^2 + g'^2}v \text{ and } m_X = g_X v_S, \quad (3.3)$$

where g and g' are the $SU(2)$ and $U(1)$ gauge couplings, while v and v_S are H and S vev's: $(\langle H \rangle, \langle S \rangle) = \frac{1}{\sqrt{2}}(v, v_S)$. The scalar potential for this model is given by

$$V = -\mu_H^2 |H|^2 + \lambda_H |H|^4 - \mu_S^2 |S|^2 + \lambda_S |S|^4 + \kappa |S|^2 |H|^2. \quad (3.4)$$

It will also be useful to define, for future reference, the parameter $\lambda_{\text{SM}} \equiv m_1^2/(2v^2) = 0.13$, where $m_1 \equiv 125.09 \text{ GeV}$.

The requirement of positivity for the potential implies the following constraints that we impose in all further discussions:

$$\lambda_H > 0, \quad \lambda_S > 0, \quad \kappa > -2\sqrt{\lambda_H \lambda_S}. \quad (3.5)$$

It is easy to find the minimization conditions for the scalar fields (without losing generality one can assume $v, v_S > 0$):

$$(2\lambda_H v^2 + \kappa v_S^2 - 2\mu_H^2)v = 0 \quad \text{and} \\ (\kappa v^2 + 2\lambda_S v_S^2 - 2\mu_S^2)v_S = 0 \quad (3.6)$$

If $\mu_{H,S}^2 < 0$ the global minimum at (0,0) is the only extremum. For $\mu_{H,S}^2 > 0$ the point (0,0) is a local maximum of the potential, in this case $(0, \frac{\mu_S}{\sqrt{\lambda_S}})$ and $(\frac{\mu_H}{\sqrt{\lambda_H}}, 0)$ are global minima if $\kappa^2 > 4\lambda_H \lambda_S$, otherwise they are saddle points and the global minima are determined by

$$v^2 = \frac{4\lambda_S \mu_H^2 - 2\kappa \mu_S^2}{4\lambda_H \lambda_S - \kappa^2}, \quad v_S^2 = \frac{4\lambda_H \mu_S^2 - 2\kappa \mu_H^2}{4\lambda_H \lambda_S - \kappa^2}. \quad (3.7)$$

For the VDM model only the latter case is relevant, since both vevs need to be non-zero to give rise to the masses of the SM fields and of the dark vector boson. Both scalar fields can be expanded around corresponding vev's as follows

$$S = \frac{1}{\sqrt{2}}(v_S + \phi_S + i\sigma_S), \\ H^0 = \frac{1}{\sqrt{2}}(v + \phi_H + i\sigma_H) \quad \text{where } H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix}. \quad (3.8)$$

The mass squared matrix $\mathcal{M}^2$ for the fluctuations (ϕ_H, ϕ_S) reads

$$\mathcal{M}^2 = \begin{pmatrix} 2\lambda_H v^2 & \kappa v v_S \\ \kappa v v_S & 2\lambda_S v_S^2 \end{pmatrix}, \quad (3.9)$$

where the similarity to the mass matrix (2.6) in the SDM model is obvious. This mass matrix $\mathcal{M}^2$ can be diagonalized by the orthogonal rotation R exactly as in (2.18) for the SDM. Note that here we adopt a convention such h_1 is the observed Higgs particle.

There are 5 real parameters in the potential: $\mu_H, \mu_S, \lambda_H, \lambda_S$, and κ . Adopting the minimization conditions (3.6) μ_H, μ_S can be replaced by v and v_S . Eventually there are 4 independent unknown parameters in the model and a convenient choice in this project is $v_S, \sin \alpha, m_2$, and m_X , which matches the choice made for the SDM model.

TABLE II. The extra vertices relevant for the calculation of annihilation and scattering cross sections in the vector DM model.

$i2m_X g_X R_{2i}$	$i2g_X^2 R_{2i} R_{2j}$

The parameters of the potential can be written as a function of the above set as:

$$\lambda_H = \lambda_{\text{SM}} + \sin^2 \alpha \frac{m_2^2 - m_1^2}{2v^2} \quad (3.10)$$

$$\kappa^2 = 4(\lambda_H - \lambda_{\text{SM}}) \frac{\lambda_S v_S^2 - \lambda_{\text{SM}} v^2}{v_S^2} \quad (3.11)$$

$$\lambda_S = \frac{2\kappa^2}{\sin^2 2\alpha} \frac{v^2}{m_2^2 - m_1^2} \left(\frac{m_2^2}{m_2^2 - m_1^2} - \sin^2 \alpha \right). \quad (3.12)$$

The extra vertices (besides those shown in Table I) needed for further calculations are collected in Table II.

A. Dark matter direct detection

The VDM model is constrained by the direct detection experiments. The spin-independent XN scattering cross section is given by [34]

$$\sigma_{XN} = \frac{\sin^2 2\alpha (m_1^2 - m_2^2)^2}{4\pi} \frac{f_N^2 \mu_{XN}^2 m_X^2 m_N^2}{m_1^4 m_2^2 v^2 v_S^2}, \quad (3.13)$$

where $\mu_{XN} \equiv m_X m_N / (m_X + m_N)$ is the reduced mass in the DM-nucleon system. Note that compared with the pseudoscalar DM case in Eq. (2.22), it is clear that there is no suppression due to additional powers of relative DM

velocity, thus we expect that the DM direct detection to results in a strong constraint to the present VDM model.

B. Higgs-boson invisible decays: $h_1 \rightarrow XX$

When the VDM mass is smaller than half of the SM-like Higgs boson h_1 , $m_X < m_1/2$, the Higgs invisible decay provides another constraint on the VDM scenario. In the present model, the width for invisible decays is provided by the process $h_1 \rightarrow XX$ and can be expressed as follows [34]

$$\Gamma(h_1 \rightarrow XX) = \frac{g_X^2 \sin^2 \alpha}{8\pi} \sqrt{m_1^2 - 4m_X^2} \frac{m_X^2}{m_1^2} \left[2 + \frac{(m_1^2 - 2m_X^2)^2}{4m_X^4} \right]. \quad (3.14)$$

IV. DISENTANGLING THE SCALAR AND VECTOR DM MODELS AT FUTURE LINEAR e^+e^- COLLIDERS

The DM Higgs portal models can be tested by collider experiments [35–38]. The different DM scenarios were discussed using the effective operator approach [39,40], simplified models [41–45] or other simple renormalizable models respecting unitarity and gauge-invariance [46–48]. An especially promising tool to probe the DM models discussed in this paper are future e^+e^- colliders [49]. In particular, they allow for the copious production of DM states associated with a Z boson, what is referred to as Higgsstrahlung process or mono-Z emission [31,50–55] (see the diagram in Fig. 1). We assume that the energy of the Z boson can be reconstructed from data, therefore allowing for the determination of the missing energy, corresponding to the dark particles. The number of events observed for a given energy bin $(E_Z, E_Z + \Delta E_Z)$ allows to measure the value of the differential cross section, $\frac{d\sigma}{dE_Z}(E_Z)$, which is given by the following formula:

$$\frac{d\sigma}{dE_Z}(E_Z) = f(s, E_Z) \cdot \frac{(\frac{\sin 2\alpha}{v_S})^2 \cdot \sqrt{1 - 4\frac{m_{\text{DM}}^2}{Q^2}} \cdot (m_1^2 - m_2^2)^2 \cdot Q^4}{[(Q^2 - m_1^2)^2 + (m_1 \Gamma_1)^2][(Q^2 - m_2^2)^2 + (m_2 \Gamma_2)^2]} \begin{cases} 1 & \text{(SDM)} \\ 1 - 4\frac{m_{\text{DM}}^2}{Q^2} + 12\left(\frac{m_{\text{DM}}^2}{Q^2}\right)^2 & \text{(VDM)} \end{cases} \quad (4.1)$$

where

$$f(s, E_Z) \equiv \frac{(1 - P_+ P_-)(g_v^2 + g_a^2) + 2g_v g_a (P_+ - P_-)}{12 \cdot (2\pi)^3} \sqrt{E_Z^2 - m_Z^2 (2m_Z^2 + E_Z^2)} \left(\frac{g^2}{\cos^2 \theta_W} \frac{1}{s - m_Z^2} \right)^2, \quad (4.2)$$

$$Q^2 = Q^2(s, E_Z) \equiv s - 2E_Z \sqrt{s} + m_Z^2. \quad (4.3)$$

Here $g_v = \frac{1}{2}(1 - 4\sin^2 \theta_W)$ and $g_a = \frac{1}{2}$ are the vector and axial couplings between electrons and the Z boson, g is the weak coupling constant, m_Z is mass of the Z boson and θ_W denotes the Weinberg angle. P_+ and P_-

denote the polarization (defined as in [56]) of the positron and electron beam, respectively. Employing polarized beams can help to reduce the SM background (see Sec. IV D). The mass of the dark particle is denoted by m_{DM} (it is m_A for the SDM and m_X for the VDM) and Q^2 is the squared four-momentum of the decaying

FIG. 1. Feynman diagram for considered channel of DM production. χ denotes the dark particle ($\chi = A, X$).

Higgs particle. Γ_1 and Γ_2 are the total (including SM as well as dark channels) decay widths of h_1 and h_2 , respectively, which must be calculated within each model as follows

$$\Gamma_i = \Gamma_i^{\text{SM}} + \frac{R_{2i}^2 m_i^3}{32\pi v_S^2} \sqrt{1 - \frac{4m_{\text{DM}}^2}{m_i^2}} \cdot \begin{cases} 1 & (\text{SDM}) \\ 1 - 4\frac{m_{\text{DM}}^2}{m_i^2} + 12\left(\frac{m_{\text{DM}}^2}{m_i^2}\right)^2 & (\text{VDM}) \end{cases}, \quad (4.4)$$

where Γ_i^{SM} is the width of h_i into SM final states. Note that the widths in (4.1) were dropped in the numerator as they are higher order terms in the perturbation expansion. Since $Q^2 \geq 4m_{\text{DM}}^2$, the following important inequality holds

$$\max \left[\frac{3}{4}, f(x_{\min}) \right] \geq 1 - 4\frac{m_{\text{DM}}^2}{Q^2} + 12\left(\frac{m_{\text{DM}}^2}{Q^2}\right)^2 \geq \begin{cases} \frac{2}{3} & \text{if } x_{\min} < \frac{2}{3} \\ f(x_{\min}) & \text{if } \frac{2}{3} < x_{\min} < 1 \end{cases}, \quad (4.5)$$

where

$$f(x) \equiv 1 - x + \frac{3}{4}x^2 \quad \text{and} \quad x_{\min} \equiv \left(\frac{2m_{\text{DM}}}{\sqrt{s} - m_Z} \right)^2. \quad (4.6)$$

Therefore from (4.1) we obtain the following solid prediction for the ratio of differential cross sections for SDM and VDM:

$$\left\{ \max \left[\frac{3}{4}, f(x_{\min}) \right] \right\}^{-1} \lesssim \frac{d\sigma_{\text{SDM}}/dE_Z}{d\sigma_{\text{VDM}}/dE_Z} \lesssim \begin{cases} \frac{3}{2} & \text{if } x_{\min} < \frac{2}{3} \\ [f(x_{\min})]^{-1} & \text{if } \frac{2}{3} < x_{\min} < 1 \end{cases}, \quad (4.7)$$

where it was assumed that the decay widths of $h_{1,2}$ are similar in both models. For cases adopted in this section $\max [3/4, f(x_{\min})] \simeq 1$ therefore the left-hand side (lhs) of

inequality (4.7) is very close to 1 while the right-hand side (rhs) is $3/2$. As a consequence of the above inequality, the total number of events predicted for the SDM model must be greater than for the VDM. The maximal deviation of the ratio of the distributions (4.1) from 1 corresponds to $Q^2 = 6m_{\text{DM}}^2$. Hence, it is easy to find that the distance δ between the energy E_Z corresponding to the maximal deviation and the location of the i th pole is

$$\delta = \frac{m_i^2 - 6m_{\text{DM}}^2}{2\sqrt{s}}. \quad (4.8)$$

In turn, this means that the regions where the large ratio of the distributions (4.1) occur, are in the vicinity of a resonance (where the number of events is expected to be large). In particular, if $m_i^2 = 6m_{\text{DM}}^2$ the maximal deviation (50%) appears exactly at the i th pole.

An exemplary plot of $\frac{d\sigma}{dE_Z}$ is presented in Fig. 2. The maximal value of E_Z for this process is

$$E_{\max} = \frac{s - 4m_{\text{DM}}^2 + m_Z^2}{2\sqrt{s}}, \quad (4.9)$$

what corresponds to $Q^2 = 4m_{\text{DM}}^2$. If E_Z was higher, there would not be sufficient energy to produce the dark particles. Note that this threshold is clearly visible on the plot and we therefore assume that the mass of dark particles can be read from data.

The poles, $Q^2 = m_i^2$, correspond to h_i being on-shell. Therefore, the i th pole is present if

$$2 \cdot m_{\text{DM}} < m_i < \sqrt{s} - m_Z. \quad (4.10)$$

FIG. 2. An exemplary plot of $\frac{d\sigma}{dE_Z}$ function for the SDM model. Different curves correspond to different cases: for the purple one, $2 \cdot m_{\text{DM}} < m_1, m_2$; for the brown $m_1 < 2 \cdot m_{\text{DM}} < m_2$; and for the green $m_1, m_2 < 2 \cdot m_{\text{DM}}$.

In this case energy of Z boson is

$$E_Z(Q^2 = m_i^2) = E_i \equiv \frac{s - m_i^2 + m_Z^2}{2\sqrt{s}}, \quad (4.11)$$

which in turn means that the mass of h_2 can be read from the position of the h_2 pole. If the h_2 pole is not present, m_2 has to be determined by an independent measurement.

Recently, two papers [53,55] have discussed similar issues as the one described in this section. Their authors have considered the possibility to disentangle vector, scalar, and fermion DM at e^+e^- colliders. The vector model they adopted is the same as the one discussed here. However, for the scalar DM they used a minimal model with an extension by a real singlet, not by a complex one with softly broken global $U(1)$, which has been adopted here. Note that the scalar model considered here and the one adopted in [53,55] are, in fact, very different. There, the coupling between DM and the mediator (the SM Higgs boson) is just given by the Higgs portal coupling (and the SM vev) and is independent of the mediator mass. In contrast, in the model discussed here, the DM is a pseudo-Goldstone boson of the spontaneously broken $U(1)$ symmetry. It is easy to see that

in the limit of restored symmetry, i.e., when $\mu^2 \rightarrow 0$, the DM A becomes a massless genuine Goldstone boson. As it is shown in the Appendices, in our case, with the $U(1)$ broken softly by the quadratic term $\mu^2(S^2 + S^{*2})$, the coupling between the DM and the mediator (h_i) is proportional to the mediator mass squared m_i^2/v_s . Note that in the VDM model the corresponding degree of freedom is a would-be Goldstone G_X boson which becomes the longitudinal component of the massive vector DM X . Nevertheless, when one compares the ILC potential for those two versions of scalar DM vs VDM, it turns out that our conclusions are slightly less optimistic than those published in [53,55].

In the following subsections we present a comparison of both DM models in a few typical cases for a $\sqrt{s} = 1.5$ TeV collider.

A. Two-pole case

In this section we assume that both poles are present. As already mentioned m_2 and m_{DM} could be determined by the location of the h_2 resonance and by the endpoint of the distribution. We assume that $\sin \alpha$ and v_s are known (deduced from some independent measurements), so that

FIG. 3. Comparison of cross sections for the $e^+e^- \rightarrow Zh_i(\chi\chi)$ process ($\chi = A, X$) for the SDM and for the VDM, in the two-pole case: $2 \cdot m_{DM} < m_1, m_2$. The upper right panel shows $d\sigma/dE_Z$ for both models while the lower one shows the ratio of the distributions between the SDM and the VDM. The parameters chosen for the plot in the right panels are specified in the lower left corner and above the upper left panel. The chosen values for (m_{DM}, m_2) correspond to the point denoted by the star in the left upper panel. The colour bar shows the value of the ratio of total cross sections for $e^+e^- \rightarrow Zh_i(\chi\chi)$. The thick gray line on the upper right panel represents the SM background for our process (see Sec. IV D). Polarizations of the beams are $(P_+, P_-) = (-30\%, 80\%)$.

we can compare the two models at the same points in the parameter space.

Figure 3 presents contours of the ratio of total cross sections $\sigma_{\text{SDM}}/\sigma_{\text{VDM}}$ in the (m_{DM}, m_2) space. The structure expected from (4.8) is visible, we observe the enhancement of the ratio for $m_{\text{DM}} \simeq m_1/\sqrt{6} \simeq 51$ GeV and also for $m_2 \simeq \sqrt{6}m_{\text{DM}}$. In those regions $\sigma_{\text{SDM}}/\sigma_{\text{VDM}}$ reaches its maximal value ~ 1.5 . The right panels show that, for the parameters chosen there, maximal enhancement of $d\sigma/dE_Z$ is observed near the resonance $Q^2 = m_1^2$ and therefore a substantial value for the ratio of the total cross sections (~ 1.40) could be reached. The point in the parameter space adopted in the right panel satisfies all the experimental and theoretical constraints considered here. The region for which a two-pole scenario is not possible is marked in gray.

B. One-pole case

In this scenario we assume that $m_1 < 2 \cdot m_{\text{DM}} < m_2$, therefore only one of the poles could be observed. Figure 4 shows the distribution functions and the ratio of the total cross sections in this case. We also show the contour plot of the ratio of total cross sections $\sigma_{\text{SDM}}/\sigma_{\text{VDM}}$ in the (m_{DM}, m_2) space. Since $m_1 < 2 \cdot m_{\text{DM}}$ only the h_2 resonance appears. Again, for $m_2 \simeq \sqrt{6}m_{\text{DM}}$ the ratio of total cross sections is observed with maximal value close to 1.5, i.e., maximal possible enhancement. There is only one enhancement band present in this case and the point in the

parameter space adopted in the right panels satisfy all the experimental and theoretical constraints considered here. The point has been chosen such that the maximal ratio of the differential cross sections is observed near the resonance, so that the ratio of the total cross sections can reach ~ 1.45 .

C. No-pole case

In this case no pole is present since both Higgs particles are lighter than $2 \cdot m_{\text{DM}}$. Again we adopt a similar strategy to illustrate this case. The difference is that since there is no pole present the mechanism to amplify the ratio of $\sigma_{\text{SDM}}/\sigma_{\text{VDM}}$ does not work. As a result, the contour plots for the ratio of the total cross sections show only very mild enhancement this time. These results are shown in Fig. 5.

D. Expected statistical error and SM background

The expected statistical error for measurements of the total cross section is equal to

$$\Delta\sigma = \sqrt{\frac{\sigma_{\text{sig}} + \sigma_{\text{bg}}}{\eta \int \mathcal{L} dt}}, \quad (4.12)$$

where η stands for the efficiency of detectors, $\int \mathcal{L} dt$ is the luminosity of the collider integrated over the whole data

FIG. 4. As in Fig. 3, however for the one-pole case, i.e., for $m_1 < 2 \cdot m_{\text{DM}} < m_2$.

FIG. 5. As in Fig. 3, however for the no-pole case: $m_1, m_2 < 2 \cdot m_{DM}$.

collection period for a given $\sqrt{s}$, σ_{sig} is the total cross section for the signal and σ_{bg} is the background cross section. Following [57,58], we assume that

$$\eta \approx 1, \quad \int \mathcal{L} dt|_{\text{CLIC}, \sqrt{s}=1.5 \text{ TeV}} \approx 1500 \text{ fb}^{-1},$$

$$\int \mathcal{L} dt|_{\text{CEPC}, \sqrt{s}=240 \text{ GeV}} \approx 3000 \text{ fb}^{-1}. \quad (4.13)$$

In order to decide whether the two models will be experimentally distinguishable, we can compare the difference between the total cross sections for SDM and VDM to its uncertainty

$$\Delta(\sigma_{\text{SDM}} - \sigma_{\text{VDM}}) = \sqrt{(\Delta\sigma_{\text{SDM}})^2 + (\Delta\sigma_{\text{VDM}})^2}$$

$$= \sqrt{\frac{(\sigma_{\text{SDM}} + \sigma_{\text{bg}}) + (\sigma_{\text{VDM}} + \sigma_{\text{bg}})}{\eta \int \mathcal{L} dt}}. \quad (4.14)$$

Production of dark particles can be mimicked by processes that produce missing energy in the form of SM neutrinos that escape detection. The simplest and most relevant example of such a process, with a similar experimental signature, that is the irreducible background, is $e^+e^- \rightarrow ZZ^* \rightarrow Z\nu\bar{\nu}$, where $\nu = \nu_e, \nu_\mu, \nu_\tau$ (see the exemplary

Feynman diagrams in Fig. 6). The background could be reduced to some extent by employing polarized e^+e^- beams. According to the current predictions for the ILC [59], polarization of the beams should be possible at the level of 80% for electrons and 30% for positrons. The expected background, presented in Figs. 3–5, is calculated under that assumption using CALCHEP [60]. The background is a serious obstacle in the determination of parameters for DM particles. One can see that among our examples only in the two-pole case in the vicinity of the SM-like resonance h_1 , the signal is comparable to the background. In order to increase the signal-to-background ratio, one should be collecting only events in a vicinity of a pole for one of the Higgs bosons, i.e., only events with Z boson energy within a certain bin around $E_Z = E_i(\sqrt{s}) \equiv (s - m_i^2 + m_Z^2)/(2\sqrt{s})$. It turns out that for $\sqrt{s} = 1.5$ TeV, $m_{DM} = 44.5$ GeV, $m_2 = 102$ GeV, $v_S = 5$ TeV, and $\sin \alpha = 0.31$ the separation between the cross sections for SDM and VDM at the level of 1σ could be obtained for a bin around $E_Z = E_1(1.5 \text{ TeV})$ with the width ~ 4.5 GeV. To estimate the minimal value of experimental uncertainty of E_Z we assumed that energy of the Z boson is reconstructed from energy of jets that are produced, since the branching ratio of Z into hadrons is almost 70% [61]. The jet energy can be measured in calorimeters with resolution $\sim 3\%$ [58,62]. Hence, the minimal size expected for the resolution of the Z energy near the h_1 pole at, for instance, CLIC is $\sim 3\% \times E_Z|_{E_Z=E_1(1.5 \text{ TeV})} = 22.4$ GeV, and so it seems

FIG. 6. Exemplary diagrams of the standard model background processes. Neutrinos contribute to missing energy and can therefore mimic dark particles. The background cross section could be reduced by polarizing the initial e^+ and e^- beams.

extremely hard to disentangle the two scenarios for the adopted parameters at this collider. On the other hand, for $\sqrt{s} = 240$ GeV expected for the CEPC and the same parameters, for the minimal bin size $\sim 3\% \times E_Z|_{E_Z=E_1(240 \text{ GeV})} = 3.1$ GeV the separation between the two cross sections is at the level of 12σ . Therefore it is fair to conclude that there exist regions of parameters, where the two scenarios might be disentangled at future e^+e^- colliders in resonance regions. However, without a detailed error and background analysis that takes into account all experimental details it is impossible to draw any solid final conclusions.

V. NUMERICAL SIMULATION

The two models described in the previous sections were implemented in the SCANNERS [28,63] code as model classes. The code takes as input any scalar potential that is a polynomial in the fields of order up to four and by considering the VEVs, mixing angle and physical masses as independent parameters, turns the problem of deriving the original potential parameters into a set of linear equations, with a very significant increase in speed of the scanning process (see [28] for details). In the most general cases, the drawback of this method is that a given point is only verified to be a global minimum at the end of the procedure. However, because it is easy to obtain closed conditions for the global minimum for the particular models under study, this problem is avoided. The code is equipped with a set of tools which allow to automatize the parameter scans and also with generic modules that allow to test local vacuum stability and library interfaces to the constraints implemented for each model. SCANNERS is also interfaced with other high energy tools that simplify the implementation of the constraints that will be described shortly.

TABLE III. Independent parameters' range for both models.

Parameter	Range
SM-Higgs— m_1	125.09 GeV
Second Higgs— m_2	[1,1000] GeV
DM— m_{DM}	[1,1000] GeV
Singlet VEV— v_s	[1,10 ⁷] GeV
Mixing angle— α	$[-\frac{\pi}{4}, \frac{\pi}{4}]$

The ranges for the independent parameters are listed in Table III. The ranges are the same for both models under study.

The points generated using SCANNERS have to be in agreement with the most relevant experimental and theoretical constraints. The discovered Higgs boson mass is taken to be $m_h = 125.09$ GeV from the ATLAS/CMS combination [64]. In these models the Higgs couplings to remaining SM particles are all modified by the same factor. Therefore, the bound on the signal strength [64] is used to constrain this parameter. The vacuum expectation value of the Higgs doublet is fixed by the W-mass. The points generated have to comply with the following theoretical constraints: (i) the potential has to be bounded from below; (ii) the vacuum is chosen so that the minimum is the global one and (iii) perturbative unitarity holds. The first two constraints are implemented in the code while perturbative unitarity is imposed through an internal numerical procedure that includes all possible two to two processes and that is available in SCANNERS for a generic model. In these models new contributions to the radiative corrections of the massive gauge-boson self-energies, $\Pi_{WW}(q^2)$ and $\Pi_{ZZ}(q^2)$ appear via the mixing between the neutral components of the doublet and the singlet. We use the variables S, T, U [65] (expressions available in [66]) to guaranty that the models are in agreement with the electroweak precision measurements at the 2σ level.

The phenomenological constraints are imposed either via libraries in the code or with interfaces with other high energy codes. The collider bounds from LEP, Tevatron, and the LHC are all encoded in HIGGSBOUNDS [67]. The program can be used to ensure agreement at 95% confidence level exclusion limits for all available searches for non-standard Higgs bosons. The Higgs decay widths, including the state-of-the-art higher order QCD corrections were calculated with SHDECAY [63].² SHDECAY is based on the implementation of the models in HDECAY [68,69]. In our calculations all electroweak radiative corrections are turned off for consistency. A detailed description of the program can be found in Appendix A of [63].

²The program SHDECAY can be downloaded from the url: <http://www.itp.kit.edu/maggie/sHDECAY>.

Up until run 2 of the LHC only LEP had constraints on resonances below 100 GeV after the discovery of the Higgs boson. New analyses using data from the run 2 of the LHC, at 13 TeV, constrain now the production and decay rates of heavy resonances into gauge bosons [70–72] and into SM-like Higgs pairs [73,74]. These bounds are included in our analysis. The most stringent of the bounds is by far the one from [70]. However, as we will show, even if a substantial number of points are excluded by this analysis, the values of $\sin\alpha$ allowed by the very strong constraints on the 125 GeV Higgs couplings to SM particles (modified by the common factor $\cos\alpha$ that affects all couplings in the same way) are roughly in the same region.

For the DM phenomenology, we consider the constraints from the cosmological DM relic abundance, collider searches, DM direct and indirect detections. The DM relic abundance for each model is calculated with the MICROMEGAS code [75], which is compared with the current experimental result $(\Omega h^2)_{\text{DM}}^{\text{obs}} = 0.1186 \pm 0.002$ from the Planck Collaboration [76]. Note that here we do not restrict the DM relic abundance to be exactly at the experimental value. Rather, we only require the model predicted value be equal to or smaller than the observed one. This way, we can consider both the dominant and subdominant DM cases simultaneously, for which we define the following DM fraction

$$f_{A,X} = \frac{(\Omega h^2)_{A,X}}{(\Omega h^2)_{\text{DM}}^{\text{obs}}}, \quad (5.1)$$

where $(\Omega h^2)_{A,X}$ denote the calculated DM relic abundance for either the pseudoscalar DM A or the VDM X .

The Higgs portal couplings can induce spin-independent DM-nucleon recoils for both scalar and vector DM models and the corresponding expressions have already been presented in Eqs. (2.22) and (3.13). Currently, the LUX [77], PandaX-II [78], and XENON1T [79,80] experiments give the most stringent upper bounds for the DM nucleon scattering. In our work, we apply the latest XENON1T upper bounds [80] for DM mass greater than 6 GeV, while for lighter DM particles, the combined limits from CRESST-II [81] and CDMSlite [82] are used. Note that these experimental DM-nucleon scattering upper limits were derived by assuming that the DM candidate comprises all of DM abundance. Therefore, the proper quantity to be directly compared with experimental limits should be the effective DM-nucleon cross section defined by $\sigma_{AN,XN}^{\text{eff}} \equiv f_{A,X} \sigma_{AN,XN}$.

The DM indirect detection experiments can also impose strong constraints on the DM properties. In the models considered in the present work, the annihilation of DM into visible states may manifest itself in: the temperature anisotropies of CMB radiation, the γ -ray signals in the spheroidal dwarf galaxies or as the $e^\pm$ excesses in the Milky Way what can be probed and constrained by

the observations of Planck [76], Fermi-LAT [83], and AMS-02 [84,85], respectively. According to Ref. [86], it is shown that for the DM mass range of interest, the Fermi-LAT upper bound on the DM annihilations from dwarfs is the most stringent. Note that both for the scalar and vector DM models, most of DM annihilations through the Higgs portal goes into ZZ , W^+W^- , $b\bar{b}$ and light quark pairs. According to Ref. [83], all of these final states give nearly the same upper limits on the DM annihilation cross sections. Thus, we use the Fermi-LAT bound from Ref. [83] on $b\bar{b}$ when $m_{A,X} \geq m_b$, and that on light quarks for $m_{A,X} < m_b$. Also, similar to the DM direct detections, the comparison with the data requires the use of the effective DM annihilation cross sections defined by $\sigma_{AA,XX}^{\text{eff}} = f_{A,X}^2 \sigma_{AA,XX}$, which are computed with the MICROMEGAS code [75] automatically.

Collider searches can provide information on DM particles through the SM-like Higgs h_1 invisible decay, with the corresponding decay width given in Eqs. (2.23) and (3.14) for both DM models. The predicted Higgs invisible decay branching ratios should be compared with the LHC bound on this channel $\text{Br}(h_1 \rightarrow \text{inv}) = 0.24$ [33].

VI. RESULTS

In this section we compare the available parameter space for the two models after applying all the constraints described in Sec. V. Again we note that the models have the same number of independent parameters. From the phenomenological point of view, the experimental measured quantities are the same, the second Higgs mass, the DM mass, the mixing angle α and the singlet VEV. It is clear that the LHC cannot prove the existence of DM if it is not confirmed by direct detection experiments. It is also true that the existence of a second neutral Higgs is predicted in most of the simplest extensions of the scalar sector. However, if a new scalar is discovered while a hint for DM appears in the form of say, mono- X events, it may be possible to exclude some DM models if the events are in a region of the parameter space already excluded. In the remainder of this section the color code in the figures is the following: red is for scalar DM and blue for vector DM, and in both cases relic density is not saturated, meaning that extra DM candidates are needed; on top of those points we present the points that are within 5σ of the central value of the relic abundance value, in pink for the scalar case and in purple for the vector case. The colors are superimposed in the following order: red, blue, pink, and then purple (so for instance a red dot may be hidden behind a blue dot).

We start with Fig. 7 where we present the branching ratio of $h_2 \rightarrow h_1 h_1$ as a function of m_2 for the scalar model and for the vector model. Clearly, there is no significant difference between the two models. Values for the branching ratio reach a maximum of 70% just after the channel opens and then reduces to maximum values of about 40%. However,

FIG. 7. Branching ratio of $h_2 \rightarrow h_1 h_1$ as a function of m_2 for the scalar model and for the vector model (color code in the legend).

if relic density is saturated the branching ratio is mostly below 40% and again indistinguishable for the two models.

In Fig. 8 we plot the branching ratio of the SM-like Higgs (a) and of the second Higgs (b) into DM particles

as a function of the DM mass. Once more no significant deviations can be seen between the models and in this case there is no difference from the saturated to the nonsaturated scenario.

In the left panel of Fig. 9 we plot m_{DM}/v_S , a quantity that reduces to the gauge coupling constant in the VDM model:

$$\frac{m_{\text{DM}}}{v_S} = \begin{cases} g_X & \text{for VDM} \\ \frac{m_A}{v_S} & \text{for SDM} \end{cases}$$

Roughly the same region is populated by both models. Note that points with suppressed m_{DM}/v_S in the range between 10^{-4} and 10^{-2} for $m_{\text{DM}} \lesssim 500$ GeV correspond to h_2 resonances. In our scan m_2 varies between 1 and 1000 GeV, therefore the resonances ($m_{\text{DM}} \sim m_2/2$) are distributed nearly uniformly for $1 \text{ GeV} \lesssim m_{\text{DM}} \lesssim 500 \text{ GeV}$. For those points the requirement of proper DM abundance imply suppression of the coupling between DM and the resonance, so that m_{DM}/v_S must be small.

FIG. 8. Branching ratio of the SM-like Higgs (a) and of the second Higgs (b) into DM particles as a function of the DM mass.

FIG. 9. Left: m_{DM}/v_S as a function of the DM mass; right: $\sin \alpha$ as a function of m_2 .

In the right panel of Fig. 9 we show $\sin \alpha$ as a function of the second Higgs mass. The allowed band between about -0.34 and 0.34 for m_2 above roughly $m_1/2$ is a hard (m_2 -independent) bound on $\sin \alpha$ that comes from the combined signal strength measurements of the production and decay of the SM-like Higgs, h_1 . This bound is weaker than in case of the real singlet model with no DM candidate. The reason is that both $\text{BR}(h_1 \rightarrow VV)$ and $\text{BR}(h_1 \rightarrow f\bar{f})$ might be reduced if $m_{\text{DM}} < m_1/2$. The “pattern” of densely populated points is visible in the right panel of Fig. 9. Those points are originating from constraints imposed by searches for heavy scalar resonances, i.e., h_2 in our case. The pattern originates mainly from the search for $pp(gg) \rightarrow h_2 \rightarrow ZZ$. Let us focus on this final state. The h_2 production cross section is the same as for the SM multiplied by $\sin^2 \alpha$. Therefore, the shape of the pattern reflects mostly the behavior of the SM cross section $\sigma(h_{\text{SM}})$ as a function of m_{SM} that is here replaced by m_2 . That is the reason why the exclusion is maximal close to the $t\bar{t}$ threshold, where the Higgs production cross section via gluon fusion has a local maximum. Notice also the presence of less densely populated regions outside of the pattern with relatively large values of $\sin \alpha$. In order to understand its appearance, one should note that the total width of the second Higgs has an extra contribution $\Gamma(h_2 \rightarrow \text{DMDM})$. Therefore, in contrast to what happens in the singlet extension with no dark matter candidate, here the $\text{BR}(h_2 \rightarrow ZZ)$ might be suppressed implying larger allowed values of $\sin \alpha$ located outside of the pattern. To illustrate this point we plot in Fig. 10 $\text{BR}(h_2 \rightarrow \text{DMDM}) + \text{BR}(h_2 \rightarrow h_1 h_1)$ as a function of m_2 but only for points outside of the pattern. As expected, all those points correspond to large value of $\text{BR}(h_2 \rightarrow \text{DMDM}) + \text{BR}(h_2 \rightarrow h_1 h_1)$. The reason to have much fewer points outside of the pattern is that the decay $h_2 \rightarrow \text{DMDM}$ has to be allowed while the range of

FIG. 10. $\text{BR}(h_2 \rightarrow h_1 h_1) + \text{BR}(h_2 \rightarrow \text{DMDM})$ vs m_2 for points that survive the bounds coming from heavy resonances and in particular $\sigma(pp(gg) \rightarrow h_2 \rightarrow ZZ)$ with still large values $\sin \alpha$. Only points located outside of the pattern in the right panel of Fig. 9 are shown.

FIG. 11. Second Higgs mass (m_2) as a function of the DM mass (m_{DM}).

variation of m_2 and of the m_{DM} is the same. That eliminates 3/4 of points in the considered region.

Finally, one can clearly see the result of the searches for $h_2 \rightarrow h_1 h_1$ close to the cross section threshold and also the much harder bound for $m_2 < m_1/2$. Regarding the comparison of the two models we again see no difference and the same can be said for the projection in the $(\sin \alpha, m_2)$ plane.

In Fig. 11 we show m_2 as a function of the DM mass. This is a projection of the parameter space where a clear difference between the two models can be seen. There are two bands where the models coexist, close to $m_{\text{DM}} \simeq m_1/2$ and to $m_2 \sim 2 \cdot m_{\text{DM}}$. The explanation for the band structure could be easily guessed; these are the two resonances h_1 and h_2 , respectively. In those regions, the kinematical enhancement by a resonance must be compensated by suppressed couplings that govern DM annihilation in the early Universe. This mechanism is nearly the same in both models. However, as seen from the figure there are two distinct regions above and below $m_2 = 2 \cdot m_{\text{DM}}$ where only the scalar model survives. Hence, there are pairs of values (m_2, m_X) that if hinted at the LHC will allow to exclude the vector model in favor of the scalar one. The reverse is not true as can be seen from the figure. The absence of VDM points in those regions is clarified in Fig. 12, where a large suppression of the cross section for scalar DM-nucleon scattering relative to the vector model one can be seen. In fact, a large portion of the parameter space of the VDM is excluded because they are above the Xenon1T bound. Therefore for a given m_{DM} there exist m_2 large enough to be excluded by the Xenon1T bound. On the other hand, for the SDM, even including one-loop corrections,³ all points are below the Xenon1T line. In order to have a clear picture

³Hereafter, in this context, we are referring to the estimate of the upper bound for one-loop radiative corrections as given in (2.22).

FIG. 12. DM-nucleon cross section as a function of the DM mass. Scalar DM-nucleon cross section is computed at one-loop level. The latest results from Xenon1T are shown as the solid line that makes the upper edge of the plot.

of what happens for the SDM we should compare the effect of one-loop vs tree-level result. This is shown in Fig. 13 where in the left panel we show the result for the tree-level cross section and in the right panel we show the one-loop result using Eq. (2.22). At tree-level the cross section are more than orders of magnitude below the Xenon1T line. This is due to the nature of the scalar DM coupling to the Higgs bosons for which a detailed account is given in the Appendix B. The inclusion of the one-loop contributions for the SDM increases the maximum values of the cross section by roughly ten orders of magnitude. Still only a few points are close to Xenon1T represented by the solid line (the upper edge) in the plots. Therefore, the SDM is still not affected by the direct constraints even with the one-loop corrections. Note that the points with maximally suppressed cross section correspond to h_2 resonances scattered in the range $1 \text{ GeV} \lesssim m_{\text{DM}} \lesssim 500 \text{ GeV}$.

Finally, we show in Fig. 14 thermal average DM annihilation cross section into the SM times velocity (at zero temperature) (at

FIG. 14. Thermal average DM annihilation cross section (into the SM) times velocity (at zero temperature) vs DM mass.

zero temperature) vs DM mass. Contrary to the direct bound, the indirect bound affects both the SDM and the VDM. Although the density of points varies, the fact is that there are no major differences between the two models. Furthermore the allowed points for both models span a very large range of cross sections and therefore will most probably not be excluded in the near future.

VII. SUMMARY AND CONCLUSIONS

In this paper we have compared scalar and vector dark matter models, both originating from the extension of the SM with an extra complex scalar S and $U(1)_X$ symmetry. In the first model, the global $U(1)_X$ is softly broken by the term that generates the mass of the DM candidate which otherwise remains a massless Goldstone boson. In the second case, the $U(1)_X$ is local and broken spontaneously therefore this massless mode contributes, within the Higgs mechanism, to the massive vector DM particle.

We have investigated the possibility to differentiate the models by measuring the energy distribution of Z

FIG. 13. Scalar DM-nucleon cross section as a function of the DM mass ($m_{\text{DM}} = m_A$) with the latest result from Xenon1T and relic abundance within 5σ of experimental value.

bosons at the ILC in the process $e^+e^- \rightarrow Z + \text{DM}$. The final conclusion requires a dedicated experimental analysis which takes into account the full background and experimental details of the collider and the detector, a task which is far beyond the scope of this project. However, theoretical predictions show that there are regions in the (m_{DM}, m_2) space for which the total cross section predicted within the SDM is nearly 50% larger than the one for the VDM, so that in those regions, future electron-positron colliders such as the ILC, CLIC or CEPC are likely to be a helpful tool in disentangling the two models. Unfortunately, the standard model $Z\nu_l\bar{\nu}_l$ final states constitute background of considerable magnitude, making e^+e^- collider search for DM very challenging.

We have shown that the direct detection is efficiently suppressed in the SDM model, $\sigma_{\text{DM-N}} \propto v_A^4$, as a consequence of A being a pseudo-Goldstone boson. The inclusion of one-loop corrections in the direct detection cross section increases its maximum values by roughly ten orders of magnitude. Still, the bounds on direct detection do not affect the SDM.

We have determined regions in the (m_{DM}, m_2) space that are excluded for the VDM while being allowed for the SDM. If future measurements point to those regions, the VDM will not be a viable option for DM. Those regions are excluded in the VDM since the DM-nucleon scattering in this case is not particularly suppressed and therefore consistency with Xenon1T eliminates a substantial part of the VDM parameter space. In the SDM the scattering is naturally very much suppressed, and the mechanism of the suppression has been explained in a more general context.

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APPENDIX A: GOLDSTONE-BOSON-HIGGS-BOSON COUPLING IN A LINEAR FORMALISM

In order to gain a better understanding of the cancellation observed in Sec. II A we derive the coupling between two Goldstone bosons and a Higgs boson in a slight more general context. In this Appendix we adopt the linear formalism.

Assume that the potential is composed by an invariant part, V_{inv} , and a softly breaking part V_{soft} , under certain symmetry transformation G

$$\phi_i \rightarrow \phi_i + \delta\phi_i = \phi_i + i\theta^a T_{ij}^a \phi_j, \quad (\text{A1})$$

where T^a are the generators of the Lie algebra of the group G and θ^a are the corresponding parameters. So that

$$\delta V = \frac{\partial V}{\partial \phi_i} \delta\phi_i = \frac{\partial V_{\text{soft}}}{\partial \phi_i} \delta\phi_i, \quad (\text{A2})$$

with $V = V_{\text{inv}} + V_{\text{soft}}$. We assume ϕ_i are real fields. Explicitly one can write

$$\frac{\partial V}{\partial \phi_i} \theta^a T_{ij}^a \phi_j = \frac{\partial V_{\text{soft}}}{\partial \phi_i} \theta^a T_{ij}^a \phi_j. \quad (\text{A3})$$

Differentiating twice with respect to ϕ_k and ϕ_l and evaluating the final expression at a minimum $\phi_n = \langle \phi_n \rangle = v_n$ of the full theory, i.e., for $V = V_{\text{inv}} + V_{\text{soft}}$, one obtains

$$\begin{aligned} & V_{lki} \theta^a T_{ij}^a v_j + \{M_{ki}^2 \theta^a T_{il}^a + (k \leftrightarrow l)\} \\ &= \frac{\partial^3 V_{\text{soft}}}{\partial \phi_l \partial \phi_k \partial \phi_i} \Big|_{\phi_n=v_n} \theta^a T_{ij}^a v_j \\ &+ \left\{ \frac{\partial^2 V_{\text{soft}}}{\partial \phi_k \partial \phi_i} \Big|_{\phi_n=v_n} \theta^a T_{il}^a + (k \leftrightarrow l) \right\}, \quad (\text{A4}) \end{aligned}$$

where

$$\begin{aligned} V_{lki} &\equiv \frac{\partial^3 V}{\partial \phi_l \partial \phi_k \partial \phi_i} \Big|_{\phi_n=v_n} \quad \text{and} \\ M_{ki}^2 &\equiv \frac{\partial^2 V}{\partial \phi_k \partial \phi_i} \Big|_{\phi_n=v_n}. \quad (\text{A5}) \end{aligned}$$

We shall specialize to the case of a complex singlet S charged under a $U(1)$ symmetry

$$\begin{aligned} \phi &= \begin{pmatrix} \phi_1 \\ \vdots \\ \phi_{N-2} \\ s \equiv \frac{\text{Re}S}{\sqrt{2}} \\ a \equiv \frac{\text{Im}S}{\sqrt{2}} \end{pmatrix} \quad v = \langle \phi \rangle = \begin{pmatrix} v_1 \\ \vdots \\ v_{N-2} \\ v_S \\ 0 \end{pmatrix} \\ M^2 &= \begin{pmatrix} M_{1,1}^2 & \cdots & M_{1,N-1}^2 & 0 \\ \vdots & \ddots & \vdots & \vdots \\ M_{N-1,1}^2 & \cdots & M_{N-1,N-1}^2 & 0 \\ 0 & \cdots & 0 & m_a^2 \end{pmatrix} \quad (\text{A6}) \end{aligned}$$

Note that the mass matrix M^2 is, in general, nondiagonal. Since we assume invariance under $S \rightarrow S^*$, there is no mixing between $\text{Im}S$ and other states in the mass matrix if $\langle a \rangle = 0$. Since the $U(1)$ is softly broken the a mass could be nonzero, i.e., a pseudo-Goldstone boson.

The $U(1)$ generator in this basis reads

$$T = \begin{pmatrix} 0 & \cdots & 0 & 0 \\ \vdots & \ddots & \vdots & \vdots \\ 0 & \cdots & 0 & i \\ 0 & \cdots & -i & 0 \end{pmatrix}. \quad (\text{A7})$$

In other words

$$T_{il} = i(\delta_{i,N-1}\delta_{l,N} - \delta_{i,N}\delta_{l,N-1}), \quad (\text{A8})$$

so that

$$M_{ki}^2 T_{il} = i(M_{k,N-1}^2 \delta_{l,N} - M_{k,N}^2 \delta_{l,N-1}), \quad \text{and} \\ T_{ij} v_j = -i \delta_{i,N} v_S. \quad (\text{A9})$$

Replacing the above in (A4) and choosing the V_{INN} component one finds

$$V_{INN} v_S = M_{l,N-1}^2 - \delta M_{l,N-1}^2 \\ - (m_a^2 - \delta M_{N,N}^2) \delta_{l,N-1} + \left. \frac{\partial^3 V_{\text{soft}}}{\partial \phi_l \partial \phi_N \partial \phi_N} \right|_{\phi_n=v_n} v_S, \quad (\text{A10})$$

where

$$\delta M_{k,i}^2 \equiv \left. \frac{\partial^2 V_{\text{soft}}}{\partial \phi_k \partial \phi_i} \right|_{\phi_n=v_n}. \quad (\text{A11})$$

Note that if $V_{\text{soft}} \neq 0$ then m_a^2 receives contributions from the symmetric part of the potential as well⁴ and therefore

$$m_a^2 \equiv \left. \frac{\partial^2 V}{\partial \phi_N \partial \phi_N} \right|_{\phi_n=v_n} \neq \left. \frac{\partial^2 V_{\text{soft}}}{\partial \phi_N \partial \phi_N} \right|_{\phi_n=v_n} \equiv \delta M_{N,N}^2 \quad (\text{A12})$$

In the symmetric limit of $V_{\text{soft}} \rightarrow 0$ one obtains $V_{INN} v_S = M_{l,N-1}^2$. Note that the contribution $m_a^2 - \delta M_{N,N}^2$ might be written also in the following way

$$m_a^2 - \delta M_{N,N}^2 = \left. \frac{\partial^2 V_{\text{inv}}}{\partial \phi_N \partial \phi_N} \right|_{\phi_n=v_n}, \quad (\text{A13})$$

where v_i in the vacuum of the full theory, i.e., for $V = V_{\text{inv}} + V_{\text{soft}}$.

The mass matrix M^2 could be diagonalized by an orthogonal rotation R as follows

$$M^2 = R \mathcal{M}^2 R^T, \quad (\text{A14})$$

where $\mathcal{M}^2$ is the diagonal matrix. The mass eigenstates are $\varphi = R^T \phi$. The rotation matrix is of the form

$$R = \begin{pmatrix} R_{1,1} & \cdots & R_{1,N-1} & 0 \\ \vdots & \ddots & \vdots & \vdots \\ R_{N-1,1} & \cdots & R_{N-1,N-1} & 0 \\ 0 & \cdots & 0 & 1 \end{pmatrix}. \quad (\text{A15})$$

The cubic coupling that is relevant for us could be written in terms of the mass eigenstates as follows

$$V = \cdots + V_{lki} R_{ll'} \varphi_{l'} R_{kk'} \varphi_{k'} R_{ii'} \varphi_{i'} + \cdots \quad (\text{A16})$$

We are interested in the V_{INN} vertex and therefore we choose $k' = i' = N$. We also limit ourself to $l' \neq N$. Since $R_{k,N} = \delta_{k,N}$ and $R_{i,N} = \delta_{i,N}$,

$$V = \cdots + V_{INN} R_{ll'} \varphi_{l'} \varphi_N \varphi_N + \cdots \quad (\text{A17})$$

The term $M_{l,N-1}^2 R_{ll'}$ from (A10) together with (A17) can be expressed by mass eigenvalues and mixing angles as $M_{l,N-1}^2 R_{ll'} = m_{l'}^2 R_{N-1,l'}$. Then the coefficient of $\varphi_{l'} \varphi_N \varphi_N$ (with $l' \neq N$) reads

$$\frac{1}{v_S} \left\{ m_{l'}^2 R_{N-1,l'} + \left[\frac{\partial^3 V_{\text{soft}}}{\partial \phi_l \partial \phi_N \partial \phi_N} v_S R_{l,l'} - \frac{\partial^2 V_{\text{inv}}}{\partial \phi_N \partial \phi_N} R_{N-1,l'} \right. \right. \\ \left. \left. - \frac{\partial^2 V_{\text{soft}}}{\partial \phi_l \partial \phi_{N-1}} R_{l,l'} \right] \right|_{\phi_n=v_n} \}. \quad (\text{A18})$$

The above equation allows to calculate corrections to the $U(1)$ -symmetric relation $V_{l'NN} = M_{l',N-1}^2 / v_S$ for a given symmetry-breaking potential V_{soft} . For instance for $V_{\text{soft}} = \mu^2(S^2 + S^{*2})$ the first term in the bracket is trivially zero while the remaining ones sum to zero

$$\left[-\frac{\partial^2 V_{\text{inv}}}{\partial \phi_N \partial \phi_N} R_{N-1,l'} - \frac{\partial^2 V_{\text{soft}}}{\partial \phi_l \partial \phi_{N-1}} R_{l,l'} \right] \Big|_{\phi_n=v_n} \\ = (4\mu^2 - 2\mu^2 - 2\mu^2) R_{N-1,l'} = 0 \quad (\text{A19})$$

That way we have reproduced the result of (2.20). It is also worthwhile to consider a linear $U(1)$ breaking, by $M^3(S + S^*)/\sqrt{2}$. In this case, even though derivatives of V_{soft} do not contribute to corrections to $V_{l'NN} = M_{l',N-1}^2 / v_S$, the derivative of V_{inv} , as it is evaluated at the minimum of the full theory, does contribute:

$$\left. \frac{\partial^2 V_{\text{inv}}}{\partial \phi_N \partial \phi_N} \right|_{\phi_n=v_n} = -\frac{M^3}{v_S} \quad (\text{A20})$$

⁴Of course, those contributions vanish in the limit $V_{\text{soft}} \rightarrow 0$.

Therefore we conclude that soft $U(1)$ breaking terms other than the quadratic ones may spoil the proportionality of the coupling to the Higgs mass squared observed in (2.20).

APPENDIX B: PSEUDO-GOLDSTONE-BOSON-HIGGS-BOSON COUPLINGS IN THE NONLINEAR FORMALISM

In this Appendix we rederive the above effective pseudo-Goldstone-Higgs couplings within the nonlinear realization of the same Lagrangian. Here we write down the complex field S in the following form:

$$S = \frac{1}{\sqrt{2}}(v_s + s)e^{ia/v_s}, \quad (\text{B1})$$

so that the $U(1)$ symmetric part of the potential does not contain couplings involving the Goldstone boson a any more. Since a is odd under the Z_2 symmetry transformation $S \leftrightarrow S^*$, it can be an appropriate DM candidate. The only terms that a appears in are the kinetic and the $U(1)$ softly-breaking terms. We will consider linear and quadratic breaking as follows:

$$\begin{aligned} \mathcal{L}_a &= \partial^\mu S^* \partial_\mu S - \frac{M^3}{\sqrt{2}}(S + S^*) - \mu^2(S^2 + S^{*2}) \\ &= \frac{(v_s + s)^2}{2v_s^2} \partial^\mu a \partial_\mu a - M^3(v_s + s) \cos\left(\frac{a}{v_s}\right) \\ &\quad - \mu^2(v_s + s)^2 \cos\left(\frac{2a}{v_s}\right) \\ &\supset \frac{1}{2} \partial^\mu a \partial_\mu a + \frac{1}{2} \left(4\mu^2 + \frac{M^3}{v_s}\right) a^2 + \frac{s}{v_s} \partial^\mu a \partial_\mu a \\ &\quad + \left(\frac{4\mu^2}{v_s} + \frac{M^3}{2v_s^2}\right) sa^2, \end{aligned} \quad (\text{B2})$$

from which we can easily read off the pseudoscalar DM mass squared as $m_a^2 = -4\mu^2 - M^3/v_s$, which is the same as that obtained within the linear realization of the $U(1)$ symmetry.

Now we are going to show that the pseudo-Goldstone-Higgs vertex agrees with the result obtained in the

Appendix A. We focus on the following vertex involving partial derivatives of a

$$\begin{aligned} \frac{1}{v_s} s \partial^\mu a \partial_\mu a &= -\frac{1}{v_s} (\partial^\mu s \partial_\mu a) a - \frac{1}{v_s} sa \square a \\ &= -\frac{1}{2v_s} \partial^\mu s \partial_\mu (a^2) + \frac{m_a^2}{v_s} sa^2 \\ &= \frac{1}{2v_s} (\square s) a^2 + \frac{m_a^2}{v_s} sa^2 \\ &= \frac{1}{2v_s} (\sin \alpha \square h_1 + \cos \alpha \square h_2) a^2 + \frac{m_a^2}{v_s} sa^2 \\ &= -\frac{1}{2v_s} (\sin \alpha m_1^2 h_1 + \cos \alpha m_2^2 h_2) a^2 + \frac{m_a^2}{v_s} sa^2, \end{aligned} \quad (\text{B3})$$

where we have repeatedly used the integration by parts and exploited free equations of motion for a and $h_{1,2}$, i.e., $\square a = -m_a^2 a$ and $\square h_i = -m_i^2 h_i$. By putting the final expression of Eq. (B3) into Eq. (B2), we obtain

$$\begin{aligned} \mathcal{L}_a &\supset \frac{1}{2} (\partial^\mu a \partial_\mu a - m_a^2 a^2) - \frac{1}{2v_s} (\sin \alpha m_1^2 h_1 + \cos \alpha m_2^2 h_2) a^2 \\ &\quad + \frac{1}{v_s} \left(4\mu^2 + \frac{M^3}{2v_s} + m_a^2\right) sa^2 \\ &= \frac{1}{2} (\partial^\mu a \partial_\mu a - m_a^2 a^2) - \frac{1}{2v_s} (\sin \alpha m_1^2 h_1 + \cos \alpha m_2^2 h_2) a^2 \\ &\quad - \frac{M^3}{2v_s^2} sa^2 \\ &= \frac{1}{2} (\partial^\mu a \partial_\mu a - m_a^2 a^2) - \frac{1}{2v_s} (\sin \alpha m_1^2 h_1 + \cos \alpha m_2^2 h_2) a^2 \\ &\quad - \frac{M^3}{2v_s^2} (\sin \alpha h_1 + \cos \alpha h_2) a^2. \end{aligned} \quad (\text{B4})$$

So indeed, the coupling is the same as obtained in the Appendix A and in Sec. II A.

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